Slip Ratio Control of Anti-lock Brake System: Comparison of Sliding Mode and Bang-bang Controllers

H. EBRAHIMIRAD*, M. J. YAZDANPANAH*, R. KAZEMI**
*Control and Intelligent Processing Center of Excellence
Department of Electrical and Computer Engineering
University of Tehran, Iran
**K.N. Toosi University of Technology, Tehran, Iran

ABSTRACT- Sliding mode controller is applied to an ABS system in this paper. A mathematical model is used for controller design and evaluation. Finally, the proposed control action is compared with that of a bang-bang controller. Simulation results reveal the effectiveness of the proposed sliding mode controller.

Key-Words: - sliding mode control, ABS, slip, bang –bang control.

1 INTRODUCTION

Variable structure control systems (VSCS) have been started in Russia by many researches, like Barbashin [1] and Utkin [2] as a special class of nonlinear systems. At the very beginning, VSCS were studied for solving several specific control tasks in second-order liner and nonlinear systems. The most distinguishing property of VSCS is that the closed loop system is completely insensitive to system uncertainties and external disturbances. A great deal of efforts has been put on establishing both theoretical VSCS concepts and practical applications. Some of the concepts and theoretical advances of VSCS are covered in [3,4,5]. Due to its excellent invariance and robustness properties, variable structure control has been developed into a general design method and extended to a wide spectrum of system types including multivariable, large-scale, infinite-dimensional and stochastic systems. The ideas have successfully been applied to problems as divers as automatic flight control, control of electric motors, congestion control and robots [6-10].

Sliding mode control [11] is a particular type of VSCS. In sliding mode control, VSCS are designed to drive and then constrain the system to lie within a neighborhood of the switching function [12,13]. There are two main advantages of this approach. Firstly, the dynamic behavior of the system may be tailored by the particular choice of switching functions. Secondly, the closed-loop response becomes totally insensitive to a particular class of uncertainty. In addition, the ability to specify performance directly makes sliding mode control attractive from the design perspective. This design approach consists of two components. The first, involves the design of a switching function so that the sliding motion satisfies design specifications. The second is conserved with the selection of a control law, which will make the switching function attractive to the system state. In this paper, a sliding mode control applied to a Anti-lock Braking System (ABS). The simulation results show a better control action of the sliding mode controller versus bang-bang controller.

The organization of the paper is as follows: In Section II the approximate model of the ABS is presented. In Section III the design procedure of sliding mode control is considered. The comparison of two control actions using simulation results on the wheel slip control of a ABS is presented in section IV. Finally, Section V concludes the paper.

2 Wheel Dynamics

The problem of wheel slip control is best explained by looking at Wheel Dynamics as shown in figure 1. The model consists of a single wheel attached to a mass *m*. As the wheel rotates, driven by the inertia of the mass m in the direction of the velocity v, a tyre reaction force F_x is generated by the friction between the tyre surface and the road surface. The tyre reaction force will generate a torque that results in a rolling motion of the wheel causing an angular velocity ω. A brake torque applied to the wheel will act against the spinning of the wheel causing a negative angular acceleration.

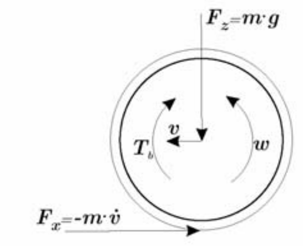

Figure 1. Wheel dynamics

The dynamic equations for the motion of the vehicle are[14],

$$\dot{v} = \frac{-F_X}{m} \tag{1}$$

$$\dot{\omega} = \frac{rF_X - T_b}{J} \tag{2}$$

where, m is mass of the vehicle, v is vehicle speed, ω is angular speed of the wheel, F_x is tyre friction force, T_b is brake torque, r is wheel radius and J is wheel inertia. F_z is vertical force as follows,

$$F_Z = mg \tag{3}$$

The tyre friction force F_x is given by

$$F_X = F_Z \mu(\lambda) \tag{4}$$

where the friction coefficient μ is a nonlinear function of λ longitudinal tyre slip that is defined by

$$\lambda = 1 - \frac{r\omega}{v} \tag{5}$$

and describes the normalized difference between the vehicle speed v and the speed of the wheel perimeter ωr. The slip value of $\lambda = 0$ characterizes the free motion of the wheel where no friction force F_x is exerted. If the slip attains the value $\lambda = 1$; then the wheel is locked ($\omega = 0$).

3 SLIDING MODE CONTROL DESIGN

The A Sliding Mode Controller is a Variable Structure Controller (VSC). Basically, a VSC includes several different continuous functions that map plant state to a control surface, and the switching among different functions is determined by plant state that is represented by a switching function. Without lost of generality, consider the design of a sliding mode controller for the following second order system:

$$\dot{x} = f(x,t) + bu(t) \tag{6}$$

Here, we assume $b > 0$. $u(t)$ is the input to the system. The following is a possible choice of the structure of a sliding mode controller [9]:

$$u = -k\,\mathrm{sgn}(s) + u_{eq} \tag{7}$$

where u_{eq} is called equivalent control which is used when the system state is in the sliding mode [10]. k is a constant, representing the maximum controller output. s is called switching function because the control action switches its sign on the two sides of the switching surface $s = 0$. s is defined as [9]:

$$s = e \tag{8}$$

where $e = x - x_d$ and x_d is the desired state. λ is a constant. The definition of e here requires that k in (1) be positive. $\mathrm{sgn}(s)$ is a sign function, which is defined as:

$$\mathrm{sgn}(s) = \begin{cases} -1 & if \quad s < 0 \\ 1 & if \quad s > 0 \end{cases} \tag{9}$$

The control strategy adopted here will guarantee a system trajectory move toward and stay on the sliding surface $s = 0$ from any initial condition if the following condition meets:

$$s\dot{s} \le -\eta|s| \tag{10}$$

where η is a positive constant that guarantees the system trajectories hit the sliding surface in finite time [9].

Using a sign function often causes chattering in practice. One solution is to introduce a boundary layer around the switch surface [10]:

$$u = -ksat(s) + u_{eq} \tag{11}$$

$sat(s)$ is a saturation function that is defined as:

$$sat(s) = \begin{cases} s & if \quad |s| \le 1 \\ \mathrm{sgn}(s) & if \quad |s| > 1 \end{cases} \tag{12}$$

This controller is actually a continuous approximation of the ideal relay control [9]. The consequence of this control scheme is that invariance property of sliding mode control is lost. The system robustness is a function of the width of the boundary layer.

A variation of the above controller structures is to use a hyperbolic tangent function instead of a saturation function [11]:

$$u = k\tanh(s/0.05) + u_{eq} \tag{13}$$

It is proven that if k is large enough, the sliding model controllers of (7), (11) and (13) are guaranteed to be asymptotically stable [10].

4 SIMULATION RESULTS

In this section, the closed-loop responses will be presented. In Figs. 1, 3 and 5 the main results are depicted for bang-bang controller according to [14] and in Figs. 2, 4 and 6 the results of the system using siding mode controller are depicted. Slip ratio are depicted in figure 1 and 2 for the bang-bang and sliding mode controller, respectively. An overshoot of fifty percent is observed for the bang-bang controller, while using sliding mode controller, overshoot does not occur. Figure 3 and 4 show the torque brake of the proposed controller and bang-bang controller. As it is seen, the proposed controller is better than another one, remarkably. Angular vehicle and wheel speed are shown in figure 5 and 6. It is obvious that the performance of the proposed sliding mode controller is much better than bang-bang controller one.

5 Conclusion

In this paper a simple mathematically model of an ABS system was considered and a slip-ratio based on sliding mode controller was proposed. The control performance between bang-bang and sliding mode controller was compared and analyzed. In the simulation, it is concluded that proposed sliding mode controller has better braking performance and the percent overshot for slip ratio is decrease than bang-bang controller has. In addition, the sliding mode controller with tangent hyperbolic function has reduced the chattering problem significantly than a simple sliding mode controller.

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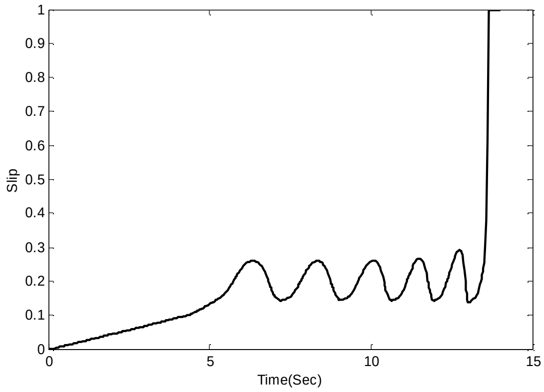

Fig. 1. Slip ratio response using bang-bang controller.

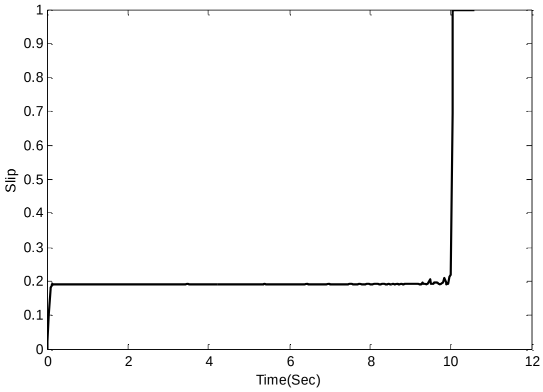

Fig. 2. Slip ratio response using SMC controller.

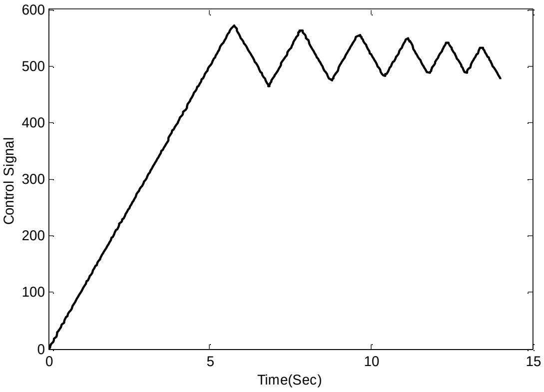

Fig. 3. Control signal for bang-bang controller.

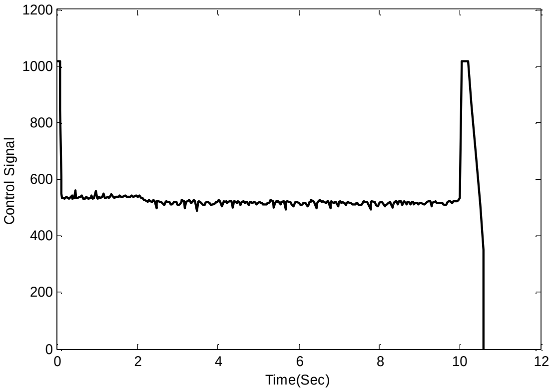

Fig. 4. Control signal for SMC controller.

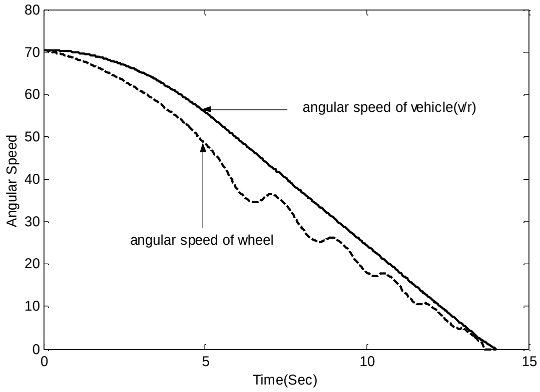

Fig. 5. Speed response using bang-bang controller.

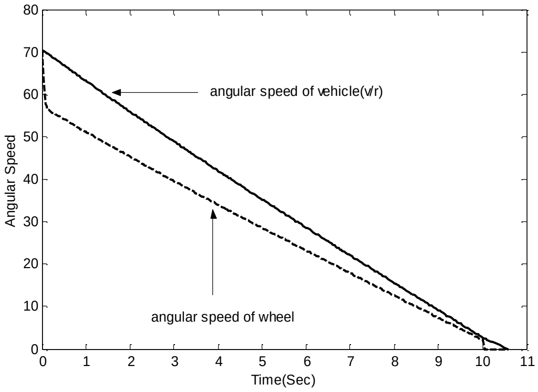

Fig. 6. Slip response using SMC controller.