Simple Polynomial Controller Design by the Coefficient Diagram Method

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Abstract: - In this paper, Coefficient Diagram Method (CDM) is used to design of simple controllers for stable time delay systems using polynomial approach. For this, first order plus time delay (FOPTD) model is used. The presented procedure is based on various approximations of the time delay. The explicit tuning formulae of the CDM controllers were determined according to the FOPTD plant model. Obtained control algorithms were compared and analysed for various first order time delay systems.

Key-Words: - Coefficient Diagram Method, Time delay, Disturbance rejection, Stable systems, Standard Manabe form.

1 Introduction

The presence of a time delay in input-output relation of the plants is an important case in system modelling and control. Time delay which is also known to be the dead time can be originated from the system itself, the use of system outcomes at the plant input again, or the impossibility of the synchronous measurements of the input and output signals [1]. Undesired effects may occur in the stability and transient characteristics of the system as a result of the delay effect. Therefore it is usually improper to use the usual control methods in which the time delay is not considered for the design [2].

There exist various methods to control of time delay systems. Many possible approaches for determining or tuning the parameters of an appropriate controller have been given in literature from simple PID control systems to Smith predictor structures [3-4]. In all methods, the main purpose of the designer is to design a controller for the plant such that the control system achieves the desired performance. In this paper, one method for short time-delays compensation based on approximations of the time delay is presented. Three time delay approximations are used: the Taylor numerator (TN), The Taylor denominator (TD) expansions and Padé approximation (PA). The polynomial approach [5] is used to obtain the controllers in the 2DOF control system structure. For a tuning of the controller parameters, CDM control technique is used. Coefficient Diagram Method, recently developed and introduced by Manabe [6], can be extended to the time delay processes. The most important properties of the method are adaptation of the polynomial representation for both the plant and controller, use of the 2DOF control system structure, non-existence (or very small) of the overshoot in the step response of the closed loop system, determination of the settling time at the start and to continue the design accordingly, robustness of the control system with respect to the parameter changes, and sufficient gain and phase margins for the closed-loop system [7]. The most important advantages of CDM for the time delay systems can be listed as follows [1]:

1. The design procedure is easily understandable and systematic. Therefore, the coefficients of the CDM controller polynomials can be determined more easily than those of the PID or the other types of the controller. This creates the possibility of an easy realisation for a new designer to control any kind of system.
2. There are explicit relations between the performance parameters specified before the design and the coefficients of the controller polynomials as described in Section 2. Thus, the designer can easily realise many control systems having various performance properties for a given control problem in a wide range of freedom.
3. It is needed to develop different tuning methods for time delay processes of different properties in PID control. But it is sufficient to use a single design procedure given in Section 3 in CDM technique. This is an outstanding advantage [8].
4. It is particularly hard to design robust controllers realising the desired performance properties for oscillatory processes having poles near the imaginary axis. It has been reported that successful designs can be achieved even in these cases by using CDM [9].
5. It is theoretically proven that the CDM design is equivalent to LQ design with proper state augmentation. Thus, CDM can be considered as “improved LQG”, because order of the controller is smaller and weight selection rules are also given [10].

The paper is organised as follows. The next section gives the fundamental properties of CDM. In Section 3, derivation of the controller tuning formulae for the FOPTD plant model of the stable processes is given. Then simulation examples in Section 4 illustrate the effectiveness of the CDM design proposed. Finally, concluding remarks are given in Section 5.

2 Coefficient Diagram Method

The controller must be designed under some practical limitations when a control problem is considered. The controller is desired to be of minimum degree, minimum phase (if possible) and stable. It must have a sufficiently narrow bandwidth and power rating limitations. If the controller is designed without considering these limitations, the robustness property will be very poor, although the stability and time response requirements are met. When all of these mentioned properties are considered together, the controller designed by using CDM proposed by Manabe [9] will have the smallest degree, the smallest bandwidth and will have the closed loop time response without an overshoot. These properties guarantee the sufficient damping of the disturbance effects and the low economic property [11].

CDM is a polynomial algebraic method. The advantages of the classical and modern control techniques are integrated with the basic principles of this method which is derived by making use of the previous experience and knowledge about the controller design. This way an effective and efficient design method, namely CDM is constructed. Without confronting with serious difficulties and necessitating much experience, CDM now makes possible to design very good controllers with less effort and relative ease when compared with the other existing methods [6]. Many control systems have been designed successfully using CDM [9,12]. Comparing designs done by CDM and other design methods, it is seen that CDM can give a controller design which is both stable and robust, and it has the desired system response speed. Also, CDM is less sensitive to disturbances and bounded uncertainties resulted from parameter variations. Therefore CDM is an important method for controller design.

The basic block diagram of the CDM control system is shown in Fig 1. In this figure, y is the output, r is the reference input, u is the control and d is the external disturbance signal. The transfer function of the plant $G(s)=N(s)D^{-1}(s)$ where $N(s)$ and $D(s)$ are the numerator and the denominator of the $G(s)$, respectively. $A(s)$ is the denominator polynomial of the controller transfer function while $F(s)$ and $B(s)$ are called the reference numerator and the feedback numerator polynomials of the controller transfer function. Since the transfer

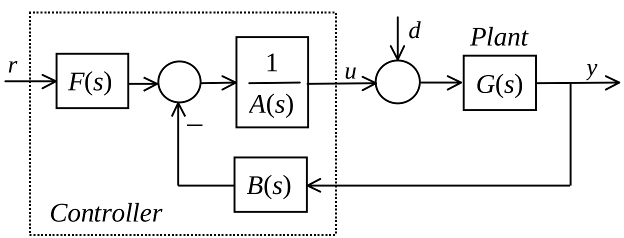

Figure 1. A block diagram of CDM control system.

function of the controller has two numerators, it resembles to a 2DOF system structure. Better performance can be expected when using a 2DOF structure, because it can focus on both tracking the desired reference signal and disturbance rejection. Unstable pole-zero cancellation and use of more number of integrators are also avoided in implementations with this structure.

The output of the controlled closed-loop system is

$$y=\frac{N(s)F(s)}{P(s)}r+\frac{A(s)N(s)}{P(s)}d \tag{1}$$

where P(s) is the characteristic polynomial and given by

$$P(s)=D(s)A(s)+N(s)B(s)=\sum_{i=0}^{n}a_i s^i \tag{2}$$

According to Manabe [6], CDM design parameters, namely equivalent time constant (τ), stability indices (γ_i) and stability limit indices (γ_i^*) are given by

$$\tau=a_1/a_0\,, \tag{3a}$$

$$\gamma_i=\frac{a_i^2}{a_{i+1}a_{i-1}}\,,\quad i=1\sim(n-1),\quad \gamma_0=\gamma_n=\infty\,, \tag{3b}$$

$$\gamma_i^*=\frac{1}{\gamma_{i-1}}+\frac{1}{\gamma_{i+1}}\,. \tag{3c}$$

From Eq.3a-c, the coefficients a_i can be written as

$$a_i=\frac{\tau^i}{\prod_{j=1}^{i-1}\gamma_{i-j}^{j}}a_0=Z_i a_0\,. \tag{4}$$

Finally, the design parameters are replaced into Eq.2 and the target characteristic polynomial is obtained as

$$P_{t\arg et}(s)=a_0\left[\{\sum_{i=2}^{n}(\prod_{j=1}^{i-1}\frac{1}{\gamma_{i-j}^{j}})(\tau s)^i\}+\tau s+1\right]. \tag{5}$$

The equivalent time constant specifies the time response speed. The stability indices and the stability limit indices affect the stability and the time response. The variation of the stability indices due to plant parameter variation specifies the robustness.

3 Controller Design Using CDM

CDM uses "simultaneous approach" [6] to obtain the controller and closed loop transfer function. In this approach, the type and degree of the controller polynomials and the characteristic polynomial of the closed-loop system are defined at the beginning. Considering the design specifications, coefficients of the polynomials are found later in the design procedure. Because of simultaneous design structure, the designer is able to keep a good balance between the rigor of the requirements and the complexity of the controller.

The processes encountered in industry can be mostly described as FOPTD model

$$G_m(s) = \frac{K}{Ts+1} e^{-\theta s} \qquad (6)$$

where K is the gain, T is the time constant and θ is the time delay .The experimental identification of this model using many techniques are well described in [3,13]. The term $e^{-\theta s}$ which represents the time delay in Eq.6 is approximated by

- the Taylor numerator expansion

$$e^{-\theta s} \approx 1 - \theta s \qquad (7)$$

- the Taylor denominator expansion

$$e^{-\theta s} \approx \frac{1}{1+\theta s} \qquad (8)$$

- the Padé approximation

$$e^{-\theta s} \approx \frac{2-\theta s}{2+\theta s} . \qquad (9)$$

Especially, if the ratio of the time delay to time constant is small, these approximations can successfully be used for the time delay. The first order approximations are sufficient because their higher number lead to a higher order of the approximative transfer function of a controlled system and consequently to more complex resulting controllers. The results obtained in the next section show that the first order approximation for time delay is sufficiently acceptable and gives good results. The equivalent linear time-invariant models of Eq.6 are obtained as in Table 1.

- Determination of the nominal plant and the controller polynomials:

Since CDM is a polynomial-based method, the transfer function of the plant is thought to be two independent polynomials; one is the numerator polynomial $N(s)$ of degree m and the other is the denominator polynomial $D(s)$ of degree n ($m \leq n$).The explicit forms of the controller polynomials $A(s)$ and $B(s)$ appearing in the CDM control system structure shown in Fig.1 are represented by

$$A(s) = \sum_{i=0}^{p} l_i s^i \ \text{ and } \ B(s) = \sum_{i=0}^{q} k_i s^i \qquad (10)$$

where the condition $p \geq q$ must be satisfied for the sake of practical realisations. For a good performance, the degrees of the controller polynomials to be chosen get importance. The most important fact that effects the degrees is the existence of a disturbing signal and its type. It is advised that the minimum degree polynomials are chosen depending on the type of the disturbance. In this paper, the controller polynomials are chosen for the step disturbance signal. The controller polynomials then have forms

$$A(s) = l_1 s \ , \qquad (11a)$$

$$B(s) = k_1 s + k_0 \qquad (11b)$$

for the numerator approximation and

$$A(s) = l_2 s^2 + l_1 s \ , \qquad (12a)$$

$$B(s) = k_2 s^2 + k_1 s + k_0 \ . \qquad (12b)$$

for the denominator and Padé approximations.

- Computation of the coefficients of the controller polynomials during the design:

Pole-placement method is a straightforward design method much used in control engineering basically made use of to compute the controller polynomials in CDM. A feedback controller is chosen by pole-placement technique and then, a feedforward controller is determined so as to match the steady-state gain of closed loop system. According to this, the controller polynomials which are determined by Eq. 11 and 12 are replaced in Eq. 2. Hence, a polynomial depending on the parameters k_i and l_i is obtained. Then, a target characteristic polynomial $P_{target}(s)$ is determined by placing the design parameters into Eq. 5. Equating these two polynomials

$$A(s)D(s) + B(s)N(s) = P_{target}(s) \qquad (13)$$

is obtained, which is known to be Diophantine equation. Solving this equation, the following explicit formulae are found for the coefficients of the controller polynomials $A(s)$ and $B(s)$ in Eq. 11

$$l_1 = \frac{\theta^2 + \tau\theta + Z_2}{T+\theta} \qquad (14a)$$

$$k_1 = \frac{1}{K}(\tau + \theta - l_1) , \qquad (14b)$$

$$k_0 = 1/K \ . \qquad (14c)$$

The numerator polynomial $F(s)$ which is defined as the pre-filter element is chosen to be

$$F(s) = P(s)/N(s)\big|_{s=0} = P(0)/N(0) . \qquad (15)$$

This way, the value of the error that may occur in the steady-state response of the closed loop system is reduced to zero. Thus, $F(s)$ is computed by

$$F(s) = P(s)/N(s)\big|_{s=0} = 1/K \ . \qquad (16)$$

The parameters of Eq. 12a and b can be derived as

$$l_2 = \frac{1}{T\theta} Z_4 , \qquad (17a)$$

$$l_1 = \frac{1}{T\theta}(Z_3 - (T+\theta)l_2) , \qquad (17b)$$

Table 1. Equivalent transfer Functions for Eq. 6

	Equivalent transfer Functions of FOPTD model
TN exp.	$G_{sn}(s) = K\frac{1-\theta s}{Ts+1} = \frac{-K\theta s + K}{Ts+1}$
TD exp.	$G_{sd}(s) = \frac{K}{(Ts+1)(1+\theta s)} = \frac{K}{Ts^2 + (T+\theta)s + 1}$
PA	$G_{sp}(s) = K\frac{2-\theta s}{(Ts+1)(2+\theta s)} = \frac{-K\theta s + 2K}{Ts^2 + (2T+\theta)s + 2}$

$$k_2 = \frac{1}{K}(Z_2 - (T+\theta)l_1 - l_2), \quad (17c)$$

$$k_1 = \frac{1}{K}(\tau - l_1), \quad (17d)$$

$$k_0 = 1/K. \quad (17e)$$

$$F(s) = P(s)/N(s)\big|_{s=0} = 1/K \quad (18)$$

for the denominator approximation and

$$l_2 = \frac{1}{T\theta} Z_4, \quad (19a)$$

$$l_1 = \frac{\theta^3 + 2Z_1\theta^2 + 4(Z_2 - 4l_2)\theta + 8(Z_3 - 2Tl_2)}{8\theta(2T+\theta)}, \quad (19b)$$

$$k_2 = \frac{1}{K\theta}(\theta T l_1 + (2T+\theta)l_2 - Z_3), \quad (19c)$$

$$k_1 = \frac{1}{2K}(\tau + 0.5\theta - 2l_1), \quad (19d)$$

$$k_0 = 0.5/K. \quad (19e)$$

$$F(s) = P(s)/N(s)\big|_{s=0} = 0.5/K \quad (20)$$

for the Padé approximation.

- Choice of the key parameters for the design and test to be done after the design:

- Choice of the equivalent time constant: One of the most important properties of CDM is that the desired settling time (t_s) is determined at the beginning before starting to design. Considering the Standard Manabe form [6], the equivalent time constant is chosen to be $\tau = t_s/2.5$.

There is an implicit relation between the equivalent time constant and the magnitude of the control signal. If τ increases then the time response becomes slow and the control signal gets smaller. If τ gets smaller, the time domain response gets faster but the control signal increases. For this reason, the value of τ must be chosen by considering the above mentioned relation in practical applications.

- Choice of the stability indices and the stability limit indices: In general, the stability indices are chosen as γ_i={2.5, 2, 2, ..., 2}, since the Standard Manabe form is used for the controller design in CDM. For the time delay systems, if the approximations instead of the time delay are used, the numerator of the plant transfer function is transformed into first order polynomial in TN and PA approximations. An overshoot can be occurred when Standard Manabe form is used in this case [11]. Thus, the values of the stability indices can be changed so as to decrease the overshoot.

The control system shown in Fig.2 is simulated by using the actual plant and the CDM controller. It is decided whether some adjustments are needed in the controller parameters by considering the time domain and the frequency response characteristics of the control system

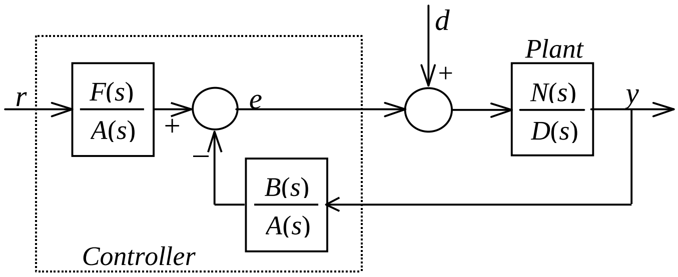

Figure 2. Simulation block diagram for the CDM control system.

and the desired performance characteristics. If any change is needed, the key parameters for the design are modified and repeating the processes.

4 Simulation Examples

In this section, two examples are given in order to illustrate the performance of the CDM in the design of simple controllers for time delay systems. First, the CDM control systems are designed using three approximations for the various ratio between the time delay and the time constant of the FOPTD system. Then, a second order plus time delay (SOPTD) system with an external disturbance is considered. In this example, the results are compared with some PID control methods.

4.1 Example 1

Consider a FOPTD system with transfer function in Eq. 6 where K=1 and T=1. The time delay is varied between 0.1 and 1. The aim in this example is to investigate the applicability of three approximations, and the effect of the parameter τ upon control responses from aspect of the control quality. The step disturbance $d(t)=-0.5u(t)$ has been injected into plant input.

Standard Manabe form is used to design of the CDM controllers. The maximum overshoot (M_p) and the settling time (t_s) of the closed loop step response according to the time delay (θ) and the equivalent time constant (τ) in an interval are obtained and separately plotted. The effect of the type of the approximation in the design is seen in Fig.3. From the figures, optimum value of τ for a certain time delay can be chosen considering the shortest settling time and the smallest overshoot simultaneously.

Fig. 3 illustrates following results for the approximations:

- The control systems designed with all three approximations are generally successful for small values of the θ/τ. But TD expansion is completely unsuccesful for the higher values of the rate.
- The TN expansion produced step responses with too small overshoot than the other approximations.
- The Padé approximation designs the CDM control systems with more suitable settling times.

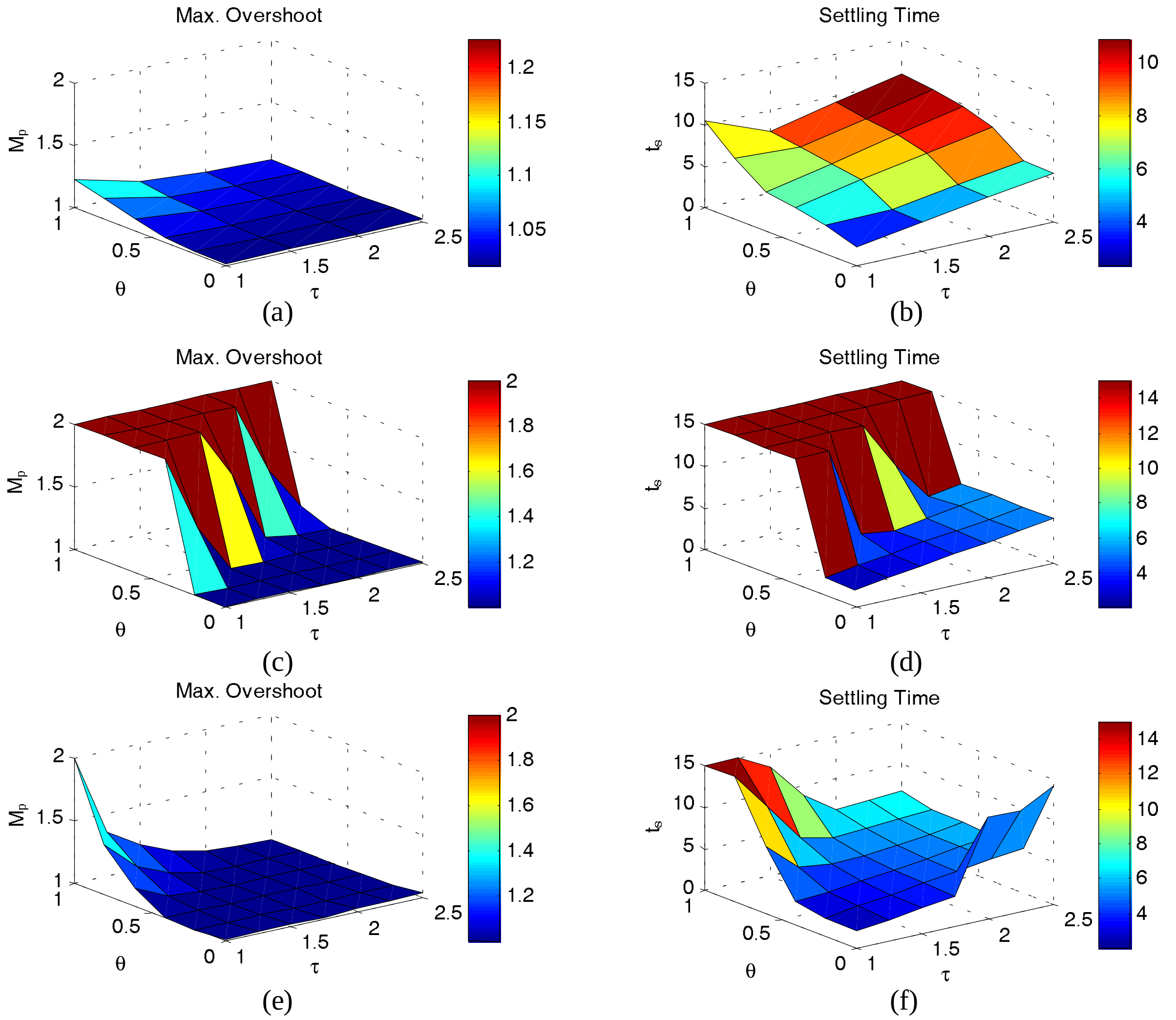

Figure 3. The maximum overshoot (M_p) and the settling time (t_s) values for the various time delay approximations: (a)-(b) TN expansion, (c)-(d) TD expansion, (e)-(f) Padé approximation.

- Especially, CDM designs the control systems which have step responses without overshoot and the small settling times for the small values of the time delay

More simulations shown that the CDM control systems are also successful using the bigger τ values for the long time delays. But, it is suggest that the designer use Smith predictor for this case [8].

4.2 Example 2

The design of CDM controllers for a SOPTD system with transfer function $G(s) = e^{-0.5s}/(s+1)^2$ is considered using the PID controller method of Ziegler-Nichols, ISE optimization and the ISTE optimization. The FOPTD model obtained using the model fit method given in [13] is $G_m(s) = e^{-0.99s}/(1.65s+1)$. The controller parameters for the design methods of Ziegler-Nichols, the ISE and ISTE optimization, respectively, are K_p=2.813, T_i=1.636, T_d=0.409; K_p=1.657, T_i=1.694, T_d=0.513; K_p=1.648, T_i=1.955, T_d=0.4. For CDM design, the equivalent time constant is chosen as τ=2.4 for TN expansion and τ=2.5 PA approximation, and stability indices is selected as in the Manabe form. Fig. 4 shows responses to a unit step change in the input and to a disturbance with magnitude of 0.5 at the time of t=15s for the five design methods. The CDM design gives average performance for the standard form when compared to other design methods. The stability indices can be changed to obtain better control system performance as mentioned in Section 3. For this, some design studies were shown in Figs. 5 and 6. The stability indices effect a little the performance of the CDM control system using the TN expansion as shown in Fig.5. But Fig.6 show that the better performance is obtained with modifying the stability indices for padé approximation. Finally, when τ=1.6 and γ_i=[2.5 8 5] are chosen with PA, the CDM control system gives the better overall response compared to the others.

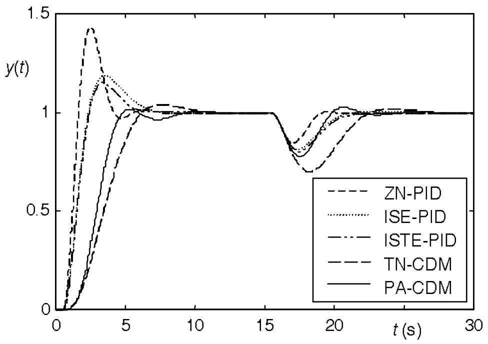

Figure 4. Step responses for Example 2

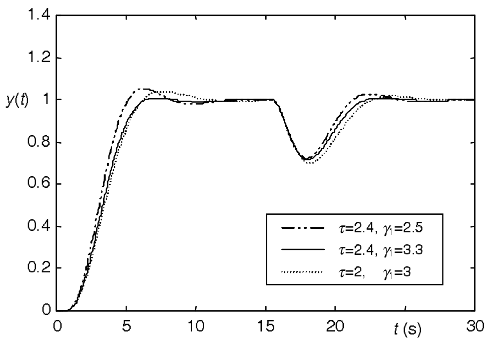

Figure 5. Some various designs for TN expansion.

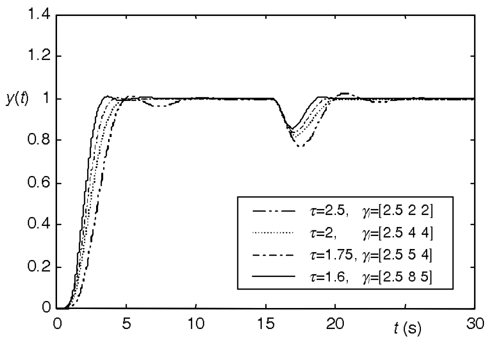

Figure 6. Some various designs for Padé Approximation.

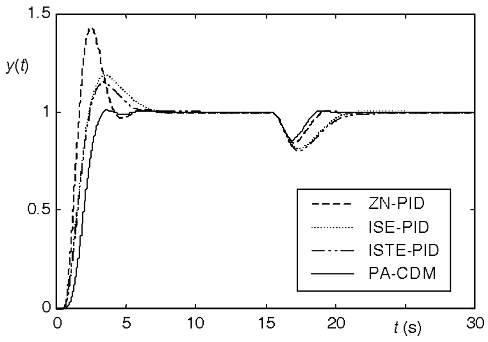

Figure 7. Final step responses for Example 2

5 Conclusion

The approach to control of time delay systems based on approximations has been introduced. The presented results prove that the padé approximation is the most suitable approximation for the time delay element. However the TN expansion can also be used for the control system design. But the TD approximation generally is not recommended to success good performance. Moreover, the studies shows that the especially Pade Approximation provides robust controllers for an estimation error in the plant parameters.

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