Beam Space-Time of a BER Minimized OFDM Systems With Bezout Precoders

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ABSTRACT

We consider the performance analysis of beam space-time of a multiple-input multiple output (MIMO) orthogonal frequency-division multiplexing (OFDM) based systems for wireless local area network (LAN) system transmission with Bezout precoders [1]. Employing multiple antennas at both the transmitter and receiver end offers a promising channel capacity. To facilitate the performance complexity tradeoff design in practice, this paper proposes the Bezout precoder design for space-time OFDM system. Finally, we deploy our system with a high-data rate ultrawideband OFDM based wireless system for IEEE 802.15.3a to see its performance.

Keywords: MIMO, OFDM, Space-Time Coding, WHT, singular value decomposition (SVD).

1. INTRODUCTION

To achieve the information rate and/or the diversity gain afforded by the increasing hardware complexity, appropriate precoding and modulation techniques are necessary. Two main approaches emerged from the effort of defining such effective transmission strategies. Mainly, one uses appropriate mapping of the information symbols in space and time so that full diversity gains become possible. This is without channel state information (CSI) at the transmitter and with low complexity at the receiver. The other is to address specifically the optimization of the information rate in the class of flat fading [4] and frequency-selective channels [2]-[3].

Orthogonal Frequency Division Multiplexing (OFDM) has been standardized for a variety application, such as digital audio broadcasting (DAB) and terrestrial digital video broadcasting (DVB-T). The transmitter/receiver diversities in multi-input multi-output (MIMO) channels will play a key role in future high-rate wireless communication. In a practical environment, the impairment introduced by multipath propagation and limited bandwidth can cause server receiver performance degradation. Intersymbol interference (ISI) has become a very critical problem in high-speed telecommunication systems. In this paper, we combine the Bezout inverse (zero-forcing) theory and Walsh-Hadamard Transform (WHT) with the space-time block coding (STBC) techniques. Finally, via the maximum likelihood decoding rule analysis, it can be shown that the Bezout WHT outperforms the OFDM precoder in power, rate, and receiver implementation [1].

2. PROBLEM DESCRIPTION

The WHT-OFDM receiver shown at Fig.1, is performed the OFDM-related FFT-aided demodulation of the incoming signal samples, followed by subcarrier-based channel transfer function equalization and dispreading with the aid of inverse WHT (IWHT). For practical MIMO OFDM systems with spatial (antenna) correlations, the frequency domain channel response matrix at the kth ($k = 0, \ldots, n_T-1$) subcarrier and pth ($p = 0, \ldots, \tilde{n}-1$) OFDM slot is given by [5]

$$\mathbf{x}[p,k] = \sqrt{\frac{SNR}{N}}\mathbf{H}[p,k]\mathbf{W}_{\mathrm{WHT}}\mathbf{s}[p,k] + \mathbf{n}[p,k] \quad (1)$$

where

$$\mathbf{H}[p,k] \in C^{n_T \times n_R},\ k = 0,\cdots,n_T,\ p = 0,\cdots,\tilde{n}-1$$

is the matrix of complex channel frequency responses defined as

$$\mathbf{H}[p,k] = \sum_{l=0}^{L-1} R_l^{1/2} H_l[p] S_l^{1/2} \exp\left(\frac{-j2\pi lk}{N_c}\right) \quad (2)$$

where $R_l = R_l^{1/2} R_l^{1/2}$, and $S_l = S_l^{1/2} S_l^{1/2}$ represent the receive and transmit spatial-correlation matrices, which are determined by the spacing and the angle spread of MIMO antennas, L is the number of resolvable paths of the frequency-selective fading channels; $\mathcal{N}_c$ is the number of block circular matrix; in addition, the power of $H_l[p], \forall l$ is normalized by letting $\sum_{l=1}^{L-1} \beta_l^2 = 1.$

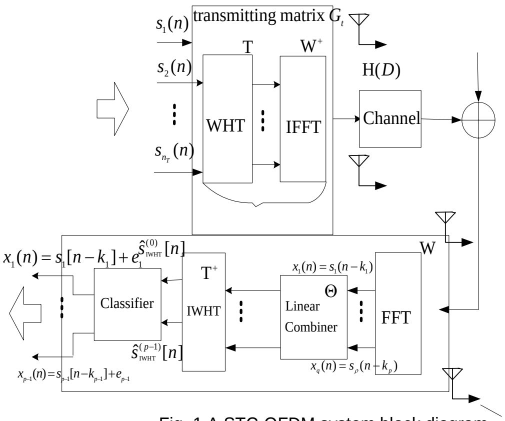

Fig. 1 A STC-OFDM system block diagram

Definition 1[1]–Perfect Recoverability: A transfer function H(*D*) is perfectly recovered (PR) *if and only if* (*iff*) there exists a polynomial matrix G(D) such that

$$G(D)\mathbf{H}(D) = Diag[D^{k_j}] \quad (3)$$

where k_j denotes the necessary delays incurred on the recovered *j*th source signal. The equality in (3) is referred to as (generalized) Bezout identity.

Definition 2. Given a MIMO system in (1), the *j*th source signal is said to be *PR* with a ρ_j -tap equalizer (i.e. degree = ρ_j -1) if there exists a (row) polynomial vector g(*D*) with degree lower than ρ_j such that

$$G(D)x(D) = s_j(D)D^k{}_j. \quad (4)$$

Theorem 1- Signal recoverability for MIMO Channel [1].

1) A MIMO system with transfer function **H**(*D*) is perfectly recovered (*PR*) *iff* **H**(*D*) is right coprime, except for a common factor with determinant D^k.
2) A MIMO system with transfer function **H**(*D*) is *PR iff* **H**(λ) has full column rank for any (complex value) λ ≠ 0.

Theorem 2- Bezout Equalizer and Precoder for MIMO Channel [1].

The Bezout system theory serves as a theoretical foundation for flexible transceiver design of a MIMO systems. Mainly it can be stated as two folds:

1) When $p < q$ and the channel is known to receiver: According to Theorem 1, there exists a PR Bezout equalizer *iff* the $q \times p$ transfer function **H**(*D*) is delay-permissive right coprime. More exactly, **F**(*D*) is a *PR* Bezout precoder *iff* **F**(*D*) satisfies
$$G(D)H(D) = \text{Diag}[D^{k_j}]$$
2) When $p > q$ and the channel is known to transmitter: According to Theorem 1, there exists a PR Bezout equalizer *iff* the $q \times p$ transfer function **H**(*D*) is delay-permissive left coprime. More exactly, **F**(*D*) is a *PR* Bezout precoder *iff* **F**(*D*) satisfies
$$H(D)F(D) = \text{Diag}[D^{k_j}].$$

With such a Bezout precoder, the symbols received by the receiver will not only be ISI-free but also ICI-free (containing only the desired stream's information).

The singular value decomposition (SVD) of $\mathbf{H} = U_H \Lambda_H V_H^*$, with the *n*th singular value denoted by $\lambda_{H,n}$. The spatio-temporal covariance matrix for z(k) is $\mathbf{R}_Z$ with eigenvalue decomposition $U_Z \Lambda_Z V_Z^*$, and eigenvalue of $\lambda_{Z,n}$.

The associated capacity after equalization for SISO (Single-Input Single Output) channel by $\mathbf{H}$ is then

$$C_{ZF,n} = \frac{1}{N} \log_2 \left(1 + \frac{\lambda_{Z,n} |\lambda_{H,n}|^2}{\sigma^2}\right), \qquad (5a)$$

where $\lambda_{Z,n}$ is found from the spatio-temporal Bezout zero-forcing solution

$$\lambda_{Z,n} = (\xi - \frac{\sigma^2}{\left|\lambda_{H,n}\right|^2})^+ \quad (5b)$$

where

$$\xi_n = \frac{\left\| H_n(k,p) e_n^{(j_1-j_2)} \right\|_2^2}{\sigma_H^2 \left\| e_n^{(j_1-j_2)} \right\|^2} \quad (5b)$$

and where $e_n^{(j_1-j_2)}$ is the transmitted space-time error sequence between code words j_1 and j_2. The function $(\cdot)^+$ is equal to the argument if the argument is positive and is zero if the argument is negative. The MIMO system is

$$C_{ZF,n} = \frac{1}{N}\sum_{n=1}^{K}\log_2(1+\frac{\lambda_{Z,n}\left|\lambda_{H,n}\right|^2}{\sigma^2}). \quad (6)$$

SNR is defined here as the mean *SNR* per SISO channel dimension averaged over all matrix sub channels $H_{i,j}$ within ***H***, i.e.,

$$SNR \equiv (P_T / N n_T n_R \sigma^2); \sum_{i=1}^{n_R}\sum_{j=1}^{n_T}\sum_{n=1}^{N}\left|\lambda_{H_{i,j,n}}\right|^2,$$

where P_T is received signal power in the *i*th sub-channel.

Theorem 3: With a white space-time transmission distribution, the capacity per transmitted symbol for the time-varying, discrete-time, spatio-temporal channel (*J* = *S*, *V*, *M*) subject to a transmitted-power limit for each channel use is

$$\begin{aligned} C &= \frac{1}{N}\sum_{k=0}^{N-1} I_k \\ &= E_H\left[W\log_2(\mathbf{I}+\chi_2^2\frac{P_T}{\sigma^2})\right] \\ &= E_H\left[W\sum_{n=1}^{K}\log_2(\mathbf{I}+\left|\lambda_{J,n}\right|^2\frac{P_T}{n_T\sigma^2})\right] \\ &= E_H\{W\log_2\det[(\mathbf{I}+\frac{P_T}{\sigma^2 n_T}Q)]\} \\ &= \frac{1}{N}\sum_{k=0}^{N-1}\log_2[\det(\mathbf{I}_{n_R}+SNR\,\mathbf{H}(e^{j2\pi(k/N)})\cdot(\mathbf{H}^H(e^{j2\pi(k/N)}))] \end{aligned} \quad (7)$$

where the quantity I_k is the mutual information of the *k*th MIMO OFDM subchannel. Note that since $\mathbf{H}(e^{j2\pi(k/N)})$ is random, I_k is random as well. *W* is the bandwidth of each sub-channel and $E_H[\cdot]$ denotes the expectation of channel **H** with respect to the random variable χ_2^2, $\sqrt{\lambda_{J,n}}$ are the singular values of the channel matrix, Q is defined as $Q = \begin{cases} \mathbf{H}\mathbf{H}^H, n_R < n_T \\ \mathbf{H}^H\mathbf{H}, n_R > n_T \end{cases}$, $\mathbf{I}_{n_R}$ is the identity matrix of size n_R, $\mathbf{H}^k$ is an $n_R \times n_T$ channel matrix with its (*j*, *i*)-th entry $H_{j,i}^{t,k}$, and *SNR* is the signal-to-noise ratio per receive antenna.

Proof Sketch: The proof follows directly from the definition of mutual information (Theorem 1) and the white transmitter-covariance matrix assumption. The reader is referred to [2]-[3] for the complete proof.

Proposition 1: The distribution of I_k (k = 0, 1, …, N-1) is independent of *k* and given by

$$I_k \approx \log[\det(\mathbf{I}_{n_R} + \rho\Lambda\mathbf{H}_w\mathbf{H}_w^H)], \text{ for } k = 0,1,...,N-1.$$

where $\Lambda = diag\{\lambda_i(\mathbf{R})\}_{i=0}^{n_R-1}$, $\mathbf{H}_w$ is an $n_R \times n_T$ i.i.d. random matrix with $\mathcal{CN}(0, 1)$ entries, and $\mathbf{R} = \sum_{l=0}^{L-1}\mathbf{R}_l$. Finally, $\lambda_i(\mathbf{R})$ denotes the *i*th eigenvalue of **R**.

Proof Sketch: The proof similarly follows directly from [2]- [3].

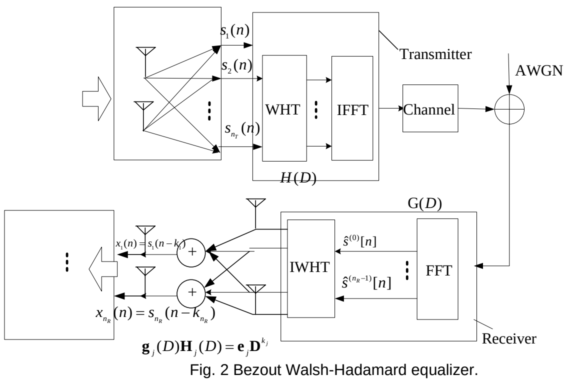

Fig. 2 Bezout Walsh-Hadamard equalizer.

3. MIMO Detection Algorithms

MIMO detection is applied to these signals on a subcarrier basis. When a packet is detected and the synchronization and channel estimation are performed, the FFT begins retrieving the subcarrier signals. A spatial multiplexing maximum likelihood detection (MLD) and per-antenna-coding (PAC) soft-decision output maximum likelihood detection (PAC-SOMLD) performs well. The disadvantage of this scheme is that its complexity grows exponentially with n_T [7]-[8].

Per-Antenna-Coded SOMLD: In case of per-antenna-coded soft-decision output MLD, the detection and detection block of Fig. 2 consists of $\mathcal{N}_c$ maximum likelihood detectors that produce soft estimates (i.e., soft-decision output values) of the coded bits transmitted on the respective subcarrier. To find these soft-decision outputs, we can use the same approach as [9], where the log likelihood ratio (*LLR*) is used as an indication for the reliability of a bit. Suppose that at a given time instance, on a certain subcarrier, $K = n_T\, m$ bits are sent, where $m = log_2\, M$ denotes the amount of bits used per M-QAM constellation point. Then, (omitting the subcarrier index), if b_k is the *k*th bit of the transmitted vector to estimate, the *LLR* for this bit is

$$LLR(b_k) = \ln\frac{\Pr(b_k = +1\,|\,\mathbf{x})}{\Pr(b_k = -1\,|\,\mathbf{x})} = \ln\frac{\sum_{s_j|b_k=+1}\Pr(\mathbf{s}_j|\mathbf{x})}{\sum_{s_j|b_k=-1}\Pr(\mathbf{s}_j|\mathbf{x})} \qquad (8)$$

where the ensemble $\boldsymbol{s}_j$ with $j = \{1, \ldots, J\}$ denotes all possible transmitted vectors at a given time instance on a certain subcarrier, and thus, $J = M^{n_T}$. When we apply Bayes' rule, the LLR becomes

$$LLR(b_k) = \ln\frac{\sum_{s_j|b_k=+1}\Pr(\mathbf{x}|\mathbf{s}_j)}{\sum_{s_j|b_k=-1}\Pr(\mathbf{x}|\mathbf{s}_j)}\,\frac{\Pr(\mathbf{s}_j)}{\Pr(\mathbf{s}_j)}. \qquad (9)$$

Because the vectors $\boldsymbol{s}_j$ are equally likely to be transmitted, $\Pr(\mathbf{s}_j)$ is equal for all vectors $\boldsymbol{s}_j$. Since we assume that the vector **x** is the result of a MIMO transmission over a flat-fading Rayleigh channel, we know that this vector **x** has a complex multivariate normal distribution. Therefore, for a given channel matrix **H**, the conditional probability density function can be shown to be

$$p(\mathbf{x}|\mathbf{H},\mathbf{s}_j) = \frac{1}{\det(\pi\Sigma_{nn})}\cdot e^{-(\mathbf{x}-\mathbf{H}\mathbf{s}_j)^H\Sigma_{nn}^{-1}(\mathbf{x}-\mathbf{H}\mathbf{s}_j)} \qquad (10)$$

where Σ_{nn} is the noise covariance matrix and equals

$$\begin{aligned}\Sigma_{in} &= E[\mathbf{x}-E[\mathbf{x}])(\mathbf{x}-E[\mathbf{x}])^H \\ &= E[\mathbf{n}\mathbf{n}^H] = \sigma_n^2\mathbf{I}_{n_R}\end{aligned} \qquad (11)$$

Finally, we obtain the LLR of b_k

$$LLR(b_k) = \frac{\sum_{\mathbf{s}_j|b_k=+1}\exp(-c\cdot\frac{D}{\sigma_n^2})}{1+\sum_{\mathbf{s}_j|b_k=-1}\exp(-c\cdot\frac{D}{\sigma_n^2})} \qquad (12)$$

where $D = \left\|\mathbf{x}-\mathbf{H}\mathbf{s}_j\right\|^2, c\in\{0,1\}.$ When applying the max-log approximation this results in

$$LLR(b_k) \approx \frac{1}{\sigma_n^1}\left(\min_{\mathbf{s}_j|b_k=-1} D - \min_{\mathbf{s}_j|b_k=+1} D\right). \qquad (13)$$

Once the LLRs are computed, they are deinterleaved and decoded. Since the TX streams are encoded separately, the deinterleaving and decoding also has to be performed per stream. Hence, n_T deinterleavers and Viterbi decoders are required to produce the final output.

4. Simulation Results

This section shows the results of a number of simulations that are performed to obtain the results of IEEE 802.15.3a. The main simulation parameters are based on the IEEE 802.15.3a standard and summarized in Table I, shown in Fig. 3. Model simplifications/ assumptions for 802.15.3a are as follows:

Table I: Simulation parameters, Based on the IEEE 802.15.3a Multiband OFDM -200 Mb/x Mode

System Parameter	Parameter value
Modulation	QPSK, rate-5/8 forward error correction coding (convolution + puncturing)
Data rate	200 Mb/s
Number of subcarriers N_c	122 subcarriers, 128-pt FFTs
Number of data subcarriers	22 pilots, zero prefix,
Frequency hopping	Mode 1 & 3 bands
L, *K*, interleaver	2, 512, 16-random interleaver
Channel Model	CM1: Line-of-sight, distance 0-4 m CM2: Non-Line-of-sight, distance 0-4 m CM3: Non-Line-of-sight, distance 4-10 m CM4: Extreme Non-Line-of-sight, distance 1-100 m

Model assumptions:

- Baseband-equivalent model (no up/down conversion)
- Random data transmission (no data scrambling used)
- Fixed (selectable) number of data symbols per packet (no pad bits used)
- Continuous frame-to-frame operation
- Fixed transmit power level; link-SNR specified (on-the-fly)
- Idealized timing/frequency acquisition
- Not modeled:
  - Other mandatory and optional data rates (55, 80, 110, 160, 200, 320, 480 Mb/s)
  - MAC/PHY interface and PLCP header (TX VECTOR/RX VECTOR, HCS/FCS)
  - Time windowing of OFDM symbols.

  IEEE 802.15.3a Multiband OFDM – 200 Mb/s Mode [6].

The simulation results are shown in Fig. 4 (a) – (h) with the equalizer of QPSK & QAM rate 5/8 forward error correction coding (convolution + puncturing) for 8, 10bit & 12bit -FFT of UWB – Multiband OFDM 200Mb/s Mode, respectively.

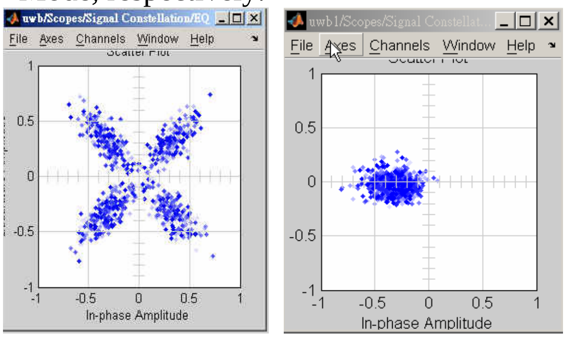

Fig. 4 (b) QPSK & (c) QAM The Simulation result of signal constellation of equalizer with QPSK & QAM rate 5/8 forward error correction coding (convolution + puncturing).

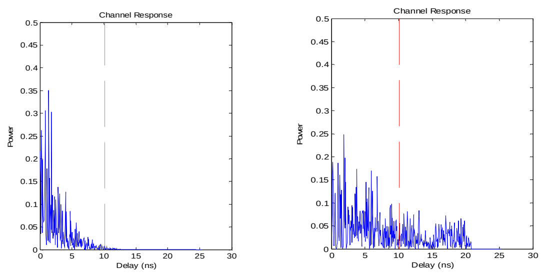

Fig. 4 (d) Channel 1 Response: CM1 (e) Channel 2 Response: CM2.

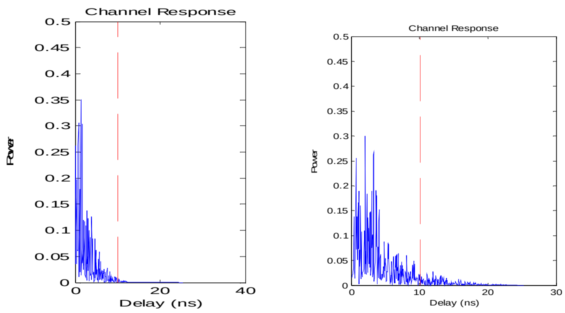

Fig. 4 (f) Channel 3 Response: CM3 (g) Channel 4 Response: CM4.

6. CONCLUDING REMARKS

In this paper, we have considered the performance analysis, Bezout precoder, and detection algorithm of MIMO OFDM systems for high data-rate wireless transmission. The main contributions are listed as follows:

1) As a test case, the OFDM-based wireless local area network (WLAN) standard IEEE 802.15.3a is considered, but the results are applicable more generally.
2) The bit-error probability is determined as a function of average channel SNR, with time-domain channel taps that fade with identical independent complex Gaussian distribution, deep frequency-domain interleaving, and ideal channel estimation. The impact of channel-estimation errors on the bit error rate (BER) performance of the system is studied.
3) From simulation, it is concluded that the MIMO detection for the Bezout Walsh-Hadamard scheme with STBC technique, outperforms the more complex PAC soft output MLD per-antenna-coding for low SNRs.

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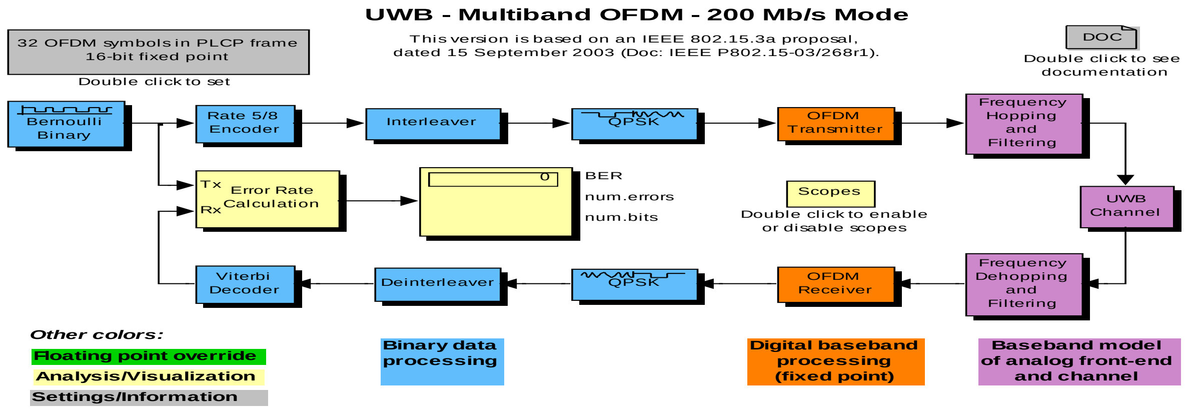

Fig. 3 The Simulink model of UWB – Multiband OFDM 200Mb/s Mode.

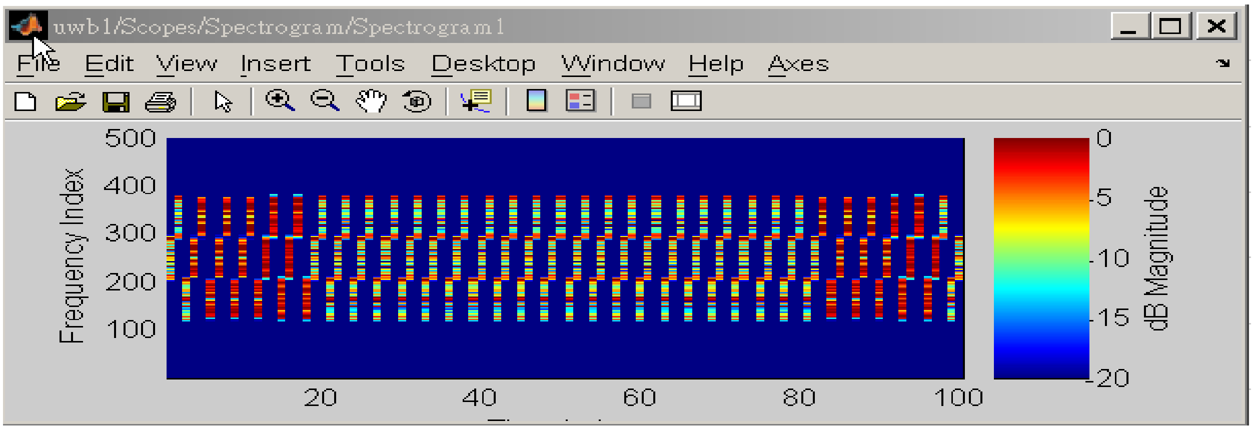

Fig. 4 (a) The Simulation result of spectral gram of UWB – Multiband OFDM 200Mb/s Mode.

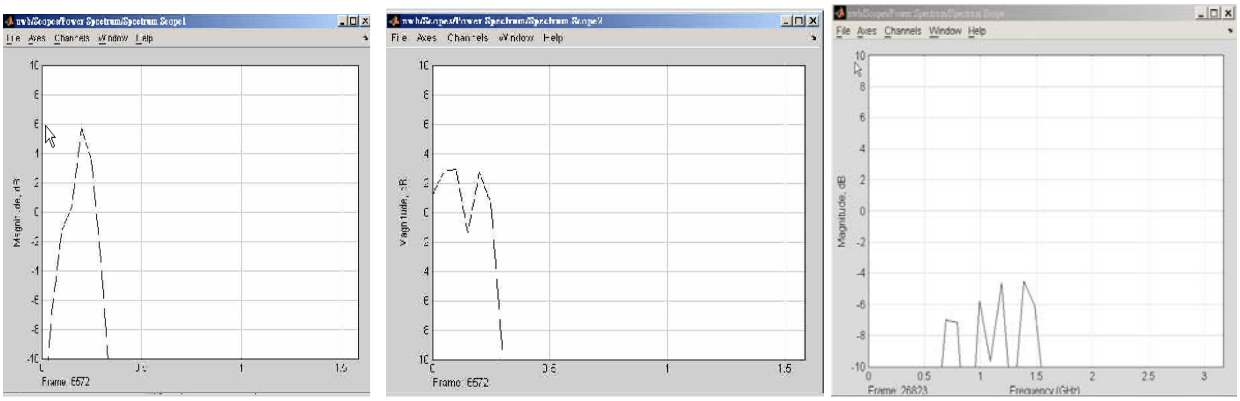

Fig. 4 (h) The Simulation result of 8, 10bit & 12bit -FFT of UWB – Multiband OFDM 200Mb/s Mode.