Intelligent Control for Robot Manipulators by Learning

CHAE-WOOK CHUNG
Department of Electronic Communication
Ansan College of Technology
170 Choji-dong Ansan
KOREA

TAE-YONG KUC
School of Information and Communication Engineering
Sungkyunkwan University
300 Chunchun-dong Suwon
KOREA

Abstract : - An intelligent control method is proposed for control of rigid robot manipulators which achieves exponential tracking of repetitive robot trajectory under uncertain operating conditions such as parameter uncertainty and unknown deterministic disturbance. In the learning controller, exponentially stable learning algorithms are combined with stabilizing computed error feedforward and feedback inputs. It is shown that all the error signals in the learning system are bounded and the repetitive robot motion converges to the desired one exponentially fast with guaranteed convergence rate. An engineering workstation based control system is built to verify the effectiveness of the proposed control method.

Key-Words : - Intelligent control, Learning control, Adaptive control, Exponential learning rule, Convergence rate, Robot manipulator, SCARA

1 Introduction

In recent years adaptive control schemes which accommodate the complete nonlinear dynamics of manipulator have been developed [4], [8], [10], [13], [14], [15], [17]–[19]. Among the recent works on adaptive control of robot manipulators, the so called computed torque based control techniques take into account for the nonlinearity of robot dynamic equations as well as the parametric uncertainties caused by imperfect modeling of robot dynamic system [4], [10], [13], [14], [15], [17], [19]. Three crucial features regarding the structure of manipulator dynamics which make the global stability proofs of these controllers possible are the positive definiteness of inertia matrix, the linear parameterization property and the passivity property of robot systems. On the other hand, a set of learning schemes based on the repetition property of robot motion trajectory has appeared in recent robotics literatures [1], [2], [3], [5], [6], [9], [11], [15]. Among the numerous learning control approaches, the *computed torque based learning control* schemes use some additional computed torque inputs for stabilization of robot dynamics at the beginning stage of learning. However, these learning control schemes differ from the *computed torque based adaptive control* schemes [4], [10], [13], [14], [15], [17], [19] in that they improve the performance of robot motion in the transient phase as well as in the steady state. The learning control can be also considered as an asymptotic optimal control method in the sense that it learns the optimal control input along the desired robot trajectory along with the lapse of trials. Since each of the two control methods, adaptive control and learning control, has its own usefulness and merits in adaptability, optimality, robustness, etc., it is expected that a more practical and efficient control in terms of robot intelligence can be obtained when they are merged into a unified controller.

This paper extends the computed torque error based learning control technique in [6] to the synthesis and analysis of optimal learning controller for robot manipulators. Inspired by the tuning algorithm of convergence rate for the learning controller [7], we develop an optimal learning controller whose rate of convergence can be set arbitrarily fast. The proposed optimal learning controller not only can estimate system parameters but learn the optimal control inputs which is the inverse dynamics solution along the desired trajectory of robot system. With the aid of the standard Lyapunov stability theory for adaptive control techniques, it is shown that all the signals in the learning control system are bounded and the robot motion converges globally exponentially to the desired one. One of remarkable features of the proposed optimal learning controller is that exponential convergence of the learning system with predefined convergence rate is guaranteed which is independent of the parameter uncertainty and unknown deterministic disturbances.

2 Problem Formulation

Consider an *n-dof* rigid robot system:

$$D(q)\ddot{q} + B(q,\dot{q})\dot{q} + g(q,\dot{q}) + d = \tau, \qquad (1)$$

where $q \in R^n$ is the generalized joint coordinate vector, $D(q) \in R^{n\times n}$ the positive definite inertia matrix, and $B(q,\dot{q})\dot{q}$ is the centripetal and Coriolis force vector. $g(q,\dot{q})$, d, and τ are the gravity and friction, the deterministic disturbances, and the input vector, respectively. The unknown deterministic disturbance vector $d(t) = d_1(t) + d_2(t)$ where d_1 is periodic disturbance with periodicity δ and d_2 satisfies $||d_2|| \leq d_0$ for positive constant d_0. Then, the purpose of our control design is to find an optimal learning controller which drives the trajectory of robot system to the desired one $q_d \in C^2$ with given convergence rate, where C^2 is the set of twice continuously differentiable functions. We assume that the set of desired trajectories and optimal control input are δ-periodic:

$$\begin{array}{ll} q_d(t) = q_d(t+\delta), & \dot{q}_d(t) = \dot{q}_d(t+\delta), \\ \ddot{q}_d(t) = \ddot{q}_d(t+\delta), & \tau_d(t) = \tau_d(t+\delta), \end{array}$$

where the desired control input τ_d is defined as:

$$\tau_d = D(q_d)\ddot{q}_d + B(q_d, \dot{q}_d)\dot{q}_d + g(q_d, \dot{q}_d) + d_1,$$

With this setting, the proposed optimal learning controller is constructed as follows:

$$\tau(t) = \tau_{fb}(t) + \tau_{ce}(t) + \tau_o^+(t), \qquad (2)$$

where τ_{fb}, τ_{ce} and τ_o denote the feedback error input, the computed-torque-error input and the asymptotic optimal control input, respectively. The projection $(\cdot)^+$ is defined as

$$x_i^+ = \Pr(x_i) = \begin{cases} \bar{x}_i, & x_i > \bar{x}_i \\ x_i, & \underline{x}_i \leq x_i, x_i^* \leq \bar{x}_i \\ \underline{x}_i, & \text{otherwise,} \end{cases}$$

where x_i represents the *i-th* element of x and x_i^* the desired or true value of x_i. The stabilizing error inputs are given by

$$\begin{aligned} \tau_{fb} &= \Gamma z + \hat{d}_0^+ sgn(z) + \gamma(e, \dot{e}), \qquad (3) \\ \tau_{ce} &= Y_e \hat{\theta}^+, \end{aligned}$$

where $e = q_d - q$, $z = \dot{e} + ae (a > 0)$, and the feedback gain Γ and nonlinear function γ are defined in the next section. $\hat{d}_0$ is an estimate of d_0 and $sgn(z)$ is defined as $sgn(z) = \frac{z}{|z|}$ for $|z| \neq 0$ and $sgn(z) = 0$ for $z = 0$. $\hat{(\cdot)}$ represents an estimated system with estimated parameters and $Y_e\hat{\theta} = \hat{D}_e(q)\ddot{q}_d + \hat{B}_e(q, \dot{q})\dot{q}_d + \hat{g}_e(q, \dot{q}) + a\left(\hat{D}(q)\dot{e} + \hat{B}(q, \dot{q})e\right)$, where

$$\begin{aligned} \hat{D}_e(q) &= \hat{D}(q) - \hat{D}(q_d), \\ \hat{B}_e(q, \dot{q}) &= \hat{B}(q, \dot{q}) - \hat{B}(q_d, \dot{q}_d), \\ \hat{g}_e(q, \dot{q}) &= \hat{g}(q, \dot{q}) - \hat{g}(q_d, \dot{q}_d). \end{aligned}$$

Applying control input (2) to uncertain system (1), we obtain an error system as follows:

$$\begin{aligned} & D(q)\dot{z} + B(q, \dot{q})z + \Gamma z \\ & \quad = d_2 - \hat{d}_0^+ sgn(z) + Y_e\tilde{\theta}^+ + \tilde{\tau}_o^+ - \gamma(e, \dot{e}), \qquad (4) \end{aligned}$$

where the parameter error vector and asymptotic optimal input error vector are defined as $\tilde{\theta}^+ = \theta - \hat{\theta}^+ \in R^l$ and $\tilde{\tau}_o^+ = \tau_d - \tau_o^+$, respectively. Here, the regression matrix $Y_e \in R^{n \times l}$ is derived from the equation $Y_e\tilde{\theta}^+ = \tau_{ce}^* - \tau_{ce}$, where τ_{ce}^* is the computed error input τ_{ce} with replacement of the estimated parameter $\hat{\theta}^+$ with the system parameter vector θ. Now, the posed problem of optimal learning control design will be converted to a problem of finding a set of learning rules which makes the error dynamic system (4) converge with predefined convergence rate. In what follows, we show that the converted problem is easily solvable by using a performance index which reflects the desired rate of convergence.

3 Nonadaptive and Adaptive Optimal Learning

To begin with, we consider the case where the parameters of robot system are known, i.e., $\hat{\theta} = \theta$. Define a functional of exponentially weighted functions as a performance index $V(t)$.

$$V(t) = \frac{1}{2\beta}\int_{t-\delta}^{t} W d\eta + \frac{1}{2}w^T(t-\delta)Dw(t-\delta). \qquad (5)$$

where $w(t) = e^{\lambda t}z(t)(\lambda \geq 0)$ and the constant $\beta > 0$ and $W(t) = \tilde{\tau}_{ow}^T L^{-1}\tilde{\tau}_{ow} + \tilde{d}_{0w}^2$ with positive gain $L = L^T > 0$. Here, the exponentially weighted error terms are defined as $\tilde{\tau}_{ow} = \tau_d - e^{\lambda t}\tau_o$ and $\tilde{d}_{0w} = d_0 - e^{\lambda t}\hat{d}_0$, respectively. Multiplying both sides of equation (4) by $e^{\lambda t}$ yields an exponentially weighted error system.

$$\begin{aligned} & D(q)\dot{w} + B(q, \dot{q})w + \Gamma_0 w \\ & \quad = e^{\lambda t}\left(d_2 - \hat{d}_0^+ sgn(z) + \tilde{\tau}_o^+ - \gamma(e, \dot{e})\right), \qquad (6) \end{aligned}$$

where $\Gamma_0 = \Gamma - \lambda D$. Then, the converted problem of optimal learning control design is simply written as: for the given control system (2) and (3), find a set of learning rules for the optimal control input τ_o and estimate of disturbance bound $\hat{d}_0$ so that it minimizes the index $V(t)$ asymptotically subject to the constraint of error dynamics equation (6). We now show by minimizing the quadratic index $V(t)$ in the sense of Lyapunov stability that the converted problem has a solution. Indeed, differentiate the index $V(t)$ along the error system (6) to obtain

$$\begin{aligned} \dot{V}(t) &= w^T\left(D\dot{w} + \frac{1}{2}\dot{D}w\right) + \Delta W(t) \\ &= -w^T(t-\delta)\Gamma_0 w(t-\delta) \\ &\quad + w^T(t-\delta)e^{\lambda(t-\delta)}(\tilde{\tau}_o^+ - \gamma) \\ &\quad + w^T(t-\delta)e^{\lambda(t-\delta)}\left(d_2 - \hat{d}_0^+ sgn(z(t-\delta))\right) \\ &\quad + \Delta W(t), \end{aligned}$$

where $\Delta W(t) = \frac{1}{2\beta}\left(W(t) - W(t-\delta)\right)$.

Let $W^+(t) = \tilde{\tau}_{ow}^{+T} L^{-1} \tilde{\tau}_{ow}^{+} + \tilde{d}_{0w}^{+2}$, where $\tilde{\tau}_{ow}^{+} = \tau_d - e^{\lambda t}\tau_o^+$ and $\tilde{d}_{0w}^{+} = d_0 - e^{\lambda t}\hat{d}_0^+$. Then, since $W^+(t) \leq W(t)$, $\Delta W(t)$ is computed as

$$\begin{aligned}
&\Delta W(t) \leq \frac{1}{2\beta}\left(W(t) - W^+(t-\delta)\right) \\
&= \frac{1}{2\beta}\left(e^{2\lambda t}|\tau_o(t)|^2_{L^{-1}} - e^{2\lambda(t-\delta)}|\tau_o^+(t-\delta)|^2_{L^{-1}}\right) \\
&\quad - \frac{1}{\beta}\tau_d^T L^{-1}\left(e^{\lambda t}\tau_o(t) - e^{\lambda(t-\delta)}\tau_o^+(t-\delta)\right) \\
&\quad + \frac{1}{2\beta}\left(e^{2\lambda t}|\hat{d}_0(t)|^2 - e^{2\lambda(t-\delta)}|\hat{d}_0^+(t-\delta)|^2\right) \\
&\quad - \frac{1}{\beta}d_0^T\left(e^{\lambda t}\hat{d}_0(t) - e^{\lambda(t-\delta)}\hat{d}_0^+(t-\delta)\right),
\end{aligned}$$

where $|(\cdot)|^2_M = (\cdot)^T M(\cdot)$.

Hence, we obtain

$$\begin{aligned}
\dot{V}(t) \quad &\leq \quad -|w(t-\delta)|^2_{\Gamma_0} + \frac{\beta}{2}|w(t-\delta)|^2_{L_0} \\
&\quad + \left(1 - e^{-\lambda(t-\delta)}\right)e^{\lambda(t-\delta)} \\
&\quad\quad \cdot \left(w^T(t-\delta)\tau_d + |w(t-\delta)|d_0\right) \\
&\quad - e^{\lambda(t-\delta)}w^T(t-\delta)\gamma + E(t),
\end{aligned}$$

where

$$\begin{aligned}
L_0 \quad &= \quad I + L, \\
E(t) \quad &= \quad \frac{1}{2\beta}\Bigg(|e^{\lambda t}\tau_o(t)|^2_{L^{-1}} \\
&\quad\quad - \left|e^{\lambda(t-\delta)}\tau_o^+(t-\delta) + \beta L w(t-\delta)\right|^2_{L^{-1}}\Bigg) \\
&\quad + \quad \frac{1}{2\beta}\Bigg(|e^{\lambda t}\hat{d}_0(t)|^2 \\
&\quad\quad - \left|e^{\lambda(t-\delta)}\hat{d}_0^+(t-\delta) + \beta|w(t-\delta)|\right|^2\Bigg) \\
&\quad - \quad \frac{1}{\beta}\tau_d^T L^{-1}\Bigg(e^{\lambda t}\tau_o(t) \\
&\quad\quad - e^{\lambda(t-\delta)}\tau_o^+(t-\delta) - \beta L w(t-\delta)\Bigg) \\
&\quad - \quad \frac{1}{\beta}d_0^T\Bigg(e^{\lambda t}\hat{d}_0(t) \\
&\quad\quad - e^{\lambda(t-\delta)}\hat{d}_0^+(t-\delta) - \beta|w(t-\delta)|\Bigg).
\end{aligned}$$

If we introduce the following learning rules

$$\begin{aligned}
e^{\lambda t}\tau_o(t) \quad &= \quad e^{\lambda(t-\delta)}\tau_o^+(t-\delta) + \beta L w(t-\delta), \\
e^{\lambda t}\hat{d}_0(t) \quad &= \quad e^{\lambda(t-\delta)}\hat{d}_0^+(t-\delta) + \beta|w(t-\delta)|
\end{aligned}$$

then it is obvious that $E(t) = 0$. Further, let $\Gamma = \beta L_0$ and γ be a saturation-type control input such that $\gamma = \left(1 - e^{-\lambda(t-\delta)}\right)\sigma_0 sgn(z)$, where σ_0 satisfies $||\tau_d|| + d_0 \leq \sigma_0$. Substituting the feedback gain Γ and nonlinear function γ chosen, we obtain

$$\begin{aligned}
\dot{V}(t) \quad &\leq \quad -\frac{\beta}{2}w^T(t-\delta)\bar{L}w(t-\delta) \\
&\leq \quad 0,
\end{aligned} \tag{7}$$

where the matrix $L = L^T > 0$ is chosen to satisfy $\bar{L} = I + L - \frac{2\lambda}{\beta}D > 0$.

In the above construction, the chosen learning rules and feedback inputs minimize asymptotically the index $V(t)$ in the sense of Lyapunov stability solving the optimal learning control problem along with the lapse of time. Now, let $\bar{\beta} = \alpha\beta$ and $\alpha = e^{-\lambda\delta}$. Then, the learning rules turn out to be

$$\tau_o(t) \quad = \quad \alpha\tau_o^+(t-\delta) + \bar{\beta}Lz(t-\delta), \tag{8}$$

$$\hat{d}_0(t) \quad = \quad \alpha\hat{d}_0^+(t-\delta) + \bar{\beta}|z(t-\delta)| \tag{9}$$

where α $(0 < \alpha \leq 1)$ can be considered as the forgetting factor. Here, the initial conditions at $t \in [0, \delta]$ are set to $\tau_o(t) = \tau_d(t) - d_1$ and $\hat{d}_0(t) = \bar{d}$, where $\bar{d}$ is the nominal disturbance bound. Then, using the learning rules (9) and (10), convergence property of the proposed optimal learning controller is obtained as follows:

Proposition 1 Non-Adaptive Optimal Learning: *Suppose that the nonadaptive optimal learning system consists of the equations of system (1), the control inputs (2) and (3) and the learning rules (8) and (9). Assume that β and L satisfy $\bar{L} = \left(I + L - \frac{2\lambda}{\beta}D\right) > 0$. Then, the nonadaptive optimal learning control system converges as follows:*

i) $\lim_{t\to\infty} z(t) = 0$ *globally asymptotically if* $\lambda = 0$,

ii) $\lim_{t\to\infty} z(t) = 0$ *globally exponentially with envelope* $e^{-\lambda t}$, *if* $\lambda > 0$.

Remarks:

1) *Proposition 1* implies that the rate of convergence can be tuned via assignment of λ.

2) The magnitude of additive input γ which takes into account for the effect of nonzero λ reduces to zero if $\lambda = 0$.

3) Since $\left(1 - e^{-\lambda t}\right) \leq 1$, the magnitude of nonlinear input γ can be made independent of the size of λ. This implies that exponential rate of convergence λ can be assigned arbitrarily large, as long as the feedback gain L and learning gain β satisfy the inequality condition $\bar{L} > 0$, where the size of βL is proportional to λD.

4) In practice, the forgetting factor α is chosen less than unity in that $0.95 \leq \alpha \leq 1$ implying the exponential rate constant λ is also bounded, since

$\lambda = -\frac{1}{\delta}\ln\alpha$, where δ is the period of robot motion trajectory.

5) The interpretation of a in the composite error $z = \dot{e} + ae$ is twofold : one is the proportional gain of PD error and the other is the inverse time-constant of a 1st-order filter, $e(t) = e^{-at}e(0) + \int_0^t e^{-a(t-\eta)}z(\eta)d\eta$. In the second interpretation, once z converges to zero, so does the position error term e. The larger a means the faster convergence, but the larger control effort and sensitivity to sensor noise. These interpretation has been not included in the paper for brevity of presentation.

When the parameters are not known, the following learning rule is used for parameter estimation.

$$\hat{\theta}(t) = \alpha\hat{\theta}^{+}(t-\delta) + \bar{\beta}S^{-1}Y_e^T(t-\delta)z(t-\delta), \quad (10)$$

where S is symmetric positive definite matrix and α and $\bar{\beta}$ are defined as in (8) and (9).

Now, let the robust control input $\gamma(t)$ is replaced by $\gamma(t) = (1 - e^{-\lambda(t-\delta)})\,(\sigma_0 + \sigma(e,\dot{e}))\,sgn(z)$, where σ_0 is defined as in *Proposition 1* and $\sigma(e,\dot{e})$ is bounding function of $Y_e\theta$ as $||Y_e\theta|| \le \sigma(e,\dot{e})$(*See e.g.*, [14]). Then, with the learning rules (8), (9), and (10) and the control input (2), we obtain the following results.

Proposition 2 Adaptive Optimal Learning: *Assume that the feedback gain L is chosen as*

$$\bar{L} = I + L - Y_e S^{-1} Y_e^T - \frac{2\lambda}{\beta}D > 0.$$

Then, the adaptive optimal learning control system which consists of the learning rules (8), (9), and (10) and the control input (2) converges as follows:

i) $\lim_{t\to\infty} z(t) = 0$ *globally asymptotically if* $\lambda = 0$,

ii) $\lim_{t\to\infty} z(t) = 0$ *globally exponentially with rate* $e^{-\lambda t}$, *if* $\lambda > 0$.

Proof Proposition 1 and Proposition 2:
In *Proposition 1*, define $V_0(t)$ as

$$V_0(t) = \frac{1}{2\beta}\int_0^t W(\eta)d\eta + \frac{1}{2}\sum_{k=1}^{n} w^T(t-k\delta)Dw(t-k\delta),$$

for $(n-1)\delta \le t \le n\delta \quad (n = 1, 2, 3, \cdots)$.
Then, $V_0(t)$ can be written as

$$\begin{aligned} V_0(t) &\le V_n(t) \\ &= \frac{1}{2\beta}\sum_{k=1}^{n}\Big(\int_{t-k\delta}^{t-(k-1)\delta} W(\eta)d\eta \\ &\quad + w^T(t-k\delta)Dw(t-k\delta)\Big). \end{aligned}$$

Following the same procedure as in the above construction of (8) for all the n-integrals on their own time-intervals, we obtain

$$\dot{V}_n(t) \le -\frac{\beta}{2}\sum_{k=1}^{n} w^T(t-k\delta)\bar{L}w(t-k\delta) \le 0,$$

where $\bar{L}$ is defined as in (7). The inequality implies that $V_0(t)$ is bounded and $w(t) \in L_2 \cap L_\infty$. When $\lambda = 0$, since the learning signals τ_o^+ and $\hat{d}_0^+$ are bounded in the error equation (4), we have $\dot{z} \in L_\infty$. This confirms the uniform boundedness of z, implying $\lim_{t\to\infty} z(t) = 0$ globally asymptotically from Barbalat's lemma [16]. Further, if $\lambda > 0$, since $w(t) \in L_\infty$, we obtain ii). This completes the proof of *Proposition 1*. *Proposition 2* can be proven similarly by replacing $W(t)$ by

$$W(t) = \tilde{\tau}_{ow}^T L^{-1}\tilde{\tau}_{ow} + \tilde{\theta}_w^T S\tilde{\theta}_w + \tilde{d}_{0w}^2,$$

where $\tilde{\theta}_w = \theta - e^{\lambda t}\hat{\theta}$. ■

It is observed that due to parametric uncertainty represented by matrix $Y_e S^{-1} Y_e^T$ the feedback gain L of *Proposition 2* is larger than *Proposition 1*. However, it can be shown that if $z(t)$ is used in the learning rules (8), (9), and (10) instead of $z(t-\delta)$, the same feedback gain L can be used as in *Proposition 1*. (*See e.g.*, [12])

4 Experimental Results

For demonstration of the proposed learning control method, experiments are carried out using the SCARA-type robot manipulator shown Fig.1. It has four degrees of freedom, however, only the first two links have beeb utilized in the experiments, since the third and forth links, which are devoted to move and orient the end-effector, respectively, are completely decoupled from the others with respect to the dynamics of the SCARA manipulator. Therefore, in the experiments, we use the dynamic model of the first two joints.

The dynamic motion of the manipulator is described by the differential equations with the following entries [20]:

$$\begin{aligned} D &= \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}, B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \\ g &= [g_1 \quad g_2]^T \\ d_{11} &= m_1 l_{c_1}^2 + m_2 l_1^2 + m_2 l_{c_2}^2 + 2m_2 l_1 l_{c_2} C_2 + I_1 + I_2 \end{aligned}$$

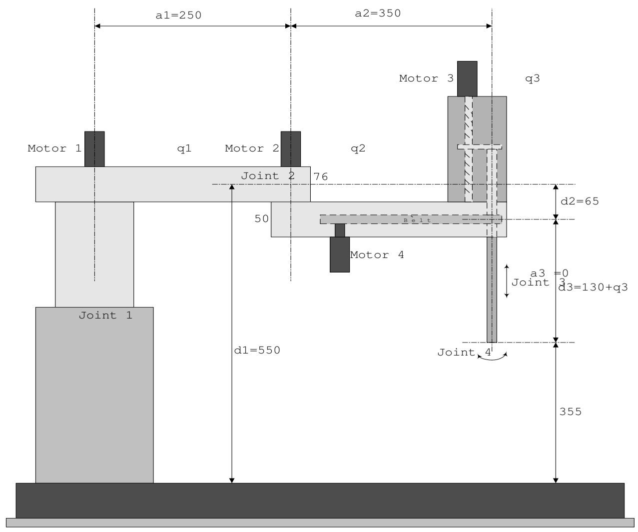

Fig. 1. SCARA-type Robot Manipulator used in the experiment

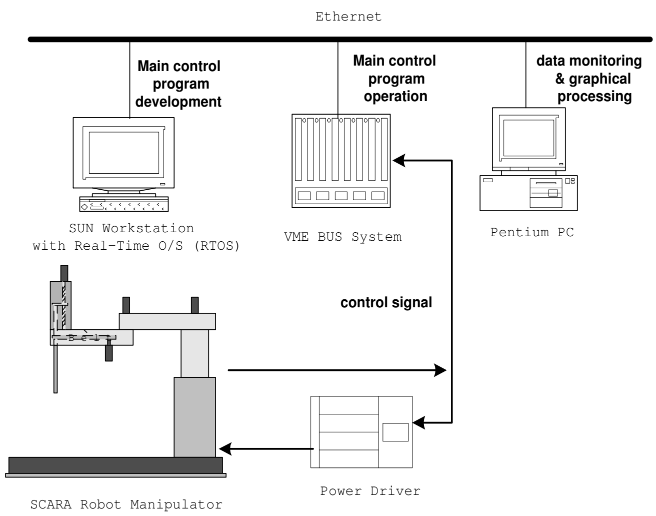

Fig. 2. The experimental setup

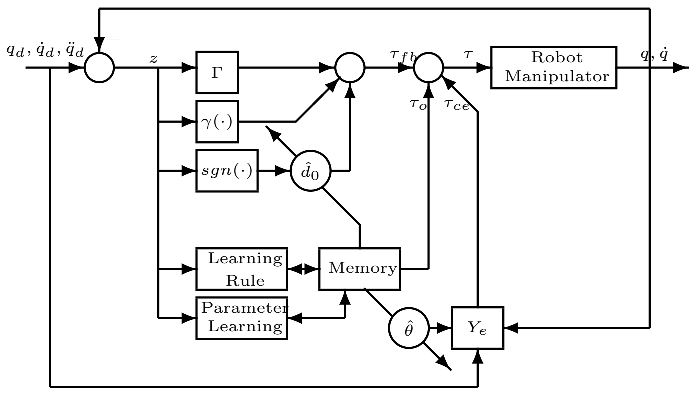

Fig. 3. Schematic diagram of the proposed learning controller

$$
\begin{aligned}
d_{12} &= d_{21} = m_2 l_{c_2}^2 + m_2 l_1 l_{c_2} C_2 + I_2 \\
d_{22} &= m_2 l_{c_2}^2 + I_2 \\
b_{11} &= -m_2 l_1 l_{c_2} S_2 \dot{q}_2 \\
b_{12} &= m_2 l_1 l_{c_2} S_2 (\dot{q}_1 + \dot{q}_2) \\
b_{21} &= m_2 l_1 l_{c_2} S_2 \dot{q}_1 \\
b_{22} &= 0 \\
g_1 &= k_1 \dot{q}_1 + p_1 sgn(\dot{q}_1) \\
g_2 &= k_2 \dot{q}_2 + p_2 sgn(\dot{q}_2)
\end{aligned}
$$

where $C_i = cos(q_i)$ and $S_i = sin(q_i)$. k_i and p_i stand for viscous and Coulomb friction coefficients, respectively. Defining parameters $\theta_1, \cdots, \theta_{10}$ as

$$
\begin{aligned}
&\theta_1 = m_1 l_{c_1}^2, \quad \theta_2 = m_2 l_1^2, \quad \theta_3 = m_2 l_{c_2}^2, \\
&\theta_4 = m_2 l_1 l_{c_2}, \quad \theta_5 = I_1, \quad \theta_6 = I_2, \\
&\theta_7 = k_1, \quad \theta_8 = k_2, \quad \theta_9 = p_1, \\
&\theta_{10} = p_2,
\end{aligned}
$$

we can obtain the regression matrix Y_e in the parameterized error equations as in [6].

The system is controlled by means of an engineering workstation based controller which consists of a SUN SPARC Classic, VME Bus system, and power driver. The control program operates under a UNIX with RTOS(Real-Time OS) operating system, which assures real-time control. The schematic diagrams of the experimental apparatus and the proposed learning controller are shown in Fig.2 and Fig.3, respectively.

For the desired joint trajectories, we choose

$$q_{d_1}(t) = \frac{\pi}{6}\sin(2\pi t) \quad q_{d_2}(t) = -\frac{\pi}{6}\sin(2\pi t)$$

for $t \in [0, 1]$. The feedback gains are set to a=3.0 and $L = diag[40\ 20]$ and the sampling period to 2.5 ms. As a physical constraint, the actuator input torques are limited to the maximum value of 400Nm for joint 1 and 200Nm for joint 2, respectively.

Fig.4 shows the tracking performance of PD controller, while Fig.5 is the response of learning system with the training factor $\beta = 0.6$ and the parameter $\lambda = 0.0$, *i.e.* $\alpha = 1$, at the first trial. Fig.6 shows asymptotic convergence of the learning system after the 25th trial. Finally, Fig.7 demonstrates the exponential convergence of learning system with three different λ's.

5 Conclusion

We present in this paper an adaptive optimal learning controller for exponential tracking of repetitive robot motion within the framework

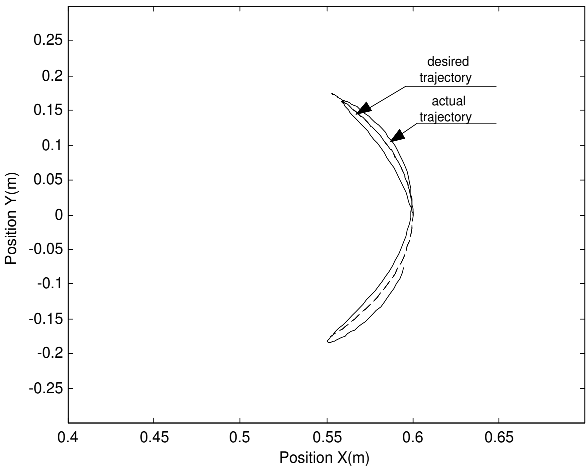

Fig. 4. Response of PD controller

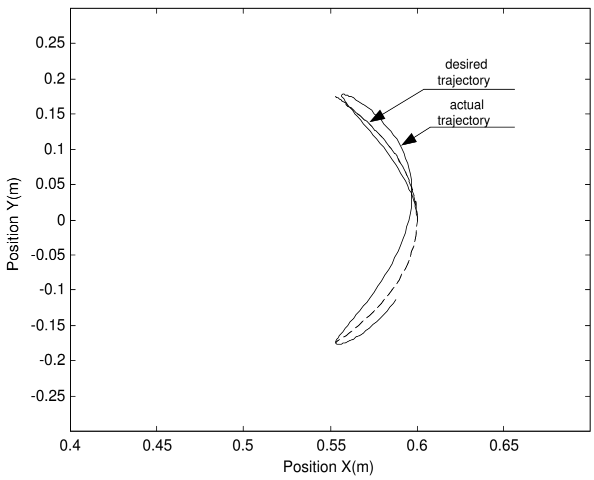

Fig. 5. Response of the proposed learning controller at the 1st trial

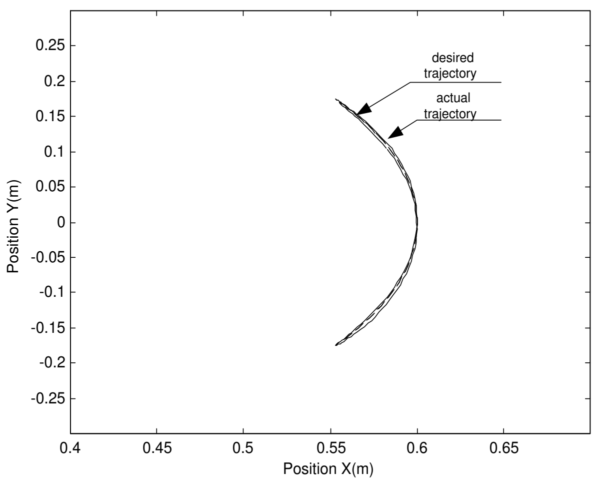

Fig. 6. Response of the proposed learning controller at the 25th trial

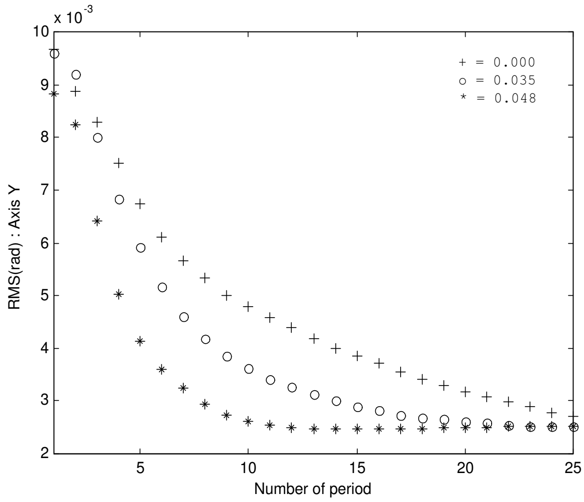

Fig. 7. Exponential convergence of the proposed learning controller with three different λ's

of computed torque based control system. In the proposed controller, the learning algorithm includes the parameter learning rules to estimate the uncertain parameters of robot system, the unknown disturbance bound and the input learning rule for the desired optimal control input. It is shown that all the error signals in the optimal learning control system are bounded and the robot motion converges to the desired one with predefined convergence rate. The experiments are also shown the convergence property and effectiveness of the proposed controller.

References:

[1] S. Arimoto, S. Kawamura, and F. Miyazaki, "Bettering operation of robots by learning," *Journal of Robotic Systems*, vol. 1, no. 2, pp. 123-140, 1984.

[2] Z. Bien and J. X. Xu, *Iterative learning control - Analysis, Design, Intergration and Applications*, Boston: Kluwer Academic Pub., 1998.

[3] P. Bondi, G. Casalino, and L. Gambardella, "On the iterative learning control theory for robotic manipulators," *IEEE J. Robot. Automat.*, vol. 4, no. 1, pp. 14-22, 1988.

[4] J.J. Craig, P. Hsu and S.S. Sastry, "Adaptive control of mechanical manipulators," *IEEE Conf. on Robotics and Automation*, CA, USA, pp. 190-195, 1986.

[5] R. Horowitz, W. Messener, and J. Moore, "Exponential convergence of a learning controller for robot manipulators," *IEEE Trans. Automat. Contr.*, vol. 36, no. 7, pp. 890-894, 1991.

[6] T. Kuc and J. Lee, "An adaptive learning control of robot manipulators," *IEEE Conf. Dec. Contr.*, U.K, pp. 1206-1211, 1991.

[7] T. Kuc, J. Lee and B. Park, "Tuning convergence rate of a robust learning controller for robot manipulators," *IEEE Conf. Dec. Contr.*, New Orleans, USA, pp. 1714-1719, 1995.

[8] M. Liu, "Computed torque scheme based adaptive tracking for robot manipulators," *IEEE Int. Conf. on Robotics and Automation*, pp. 587-590, 1995.

[9] W. Messner, R. Horowitz, W. Kao and M. Boals, "A new adaptive learning rule," *IEEE Conf. on Robotics and Automation*, pp. 1522-1527, 1990.

[10] R.H. Middelton and G.C. Goodwin, "Adaptive computed torque control for rigid-link manipulations," *System & Control Letters*, vol. 10, pp. 9-16, 1988.

[11] K.L. Moore, M. Dahleh, and S.P. Bhattacharyya, "Iterative learning control: a survey and new results," *Journal of Robotic Systems*, vol. 9, no. 5, pp. 563-594, 1992.

[12] B. Park, T. Kuc, and J. Lee, "Adaptive learning control of uncertain robot manipulators", *Int. J. Contr.*, vol. 65 , no. 5, pp. 725-744, 1996.

[13] J.S. Reed and P.A. Iaonnou, "Instability analysis and robust adaptive control of robot manipulators," *IEEE Trans. on Robotics and Automation*, vol. 5, pp. 381-385, 1989.

[14] N. Sadegh, R. Horowitz "Stability and robustness analysis of a class of adaptive controllers for robotic manipulators," *Int. J. Robot. Res.*, vol. 9, no. 3, pp. 74-92, 1990.

[15] N. Sadegh, R. Horowitz, W.-W. Kao and M. Tomizuka, "A unified approach to the design of adaptive and repetitive controllers for robotic manipulators," *ASME J. of Dynamic Systems, Measurement and Control*, vol. 112, pp. 618-629, 1990.

[16] S. Sastry and M. Bodson, *Adaptive Control: Stability, Convergence and Robustness*, New Jersey: Prentice-Hall Inc., 1989.

[17] J.-J.E. Slotine and W. Lee, "Composite adaptive control of robot manipulators," *Automatica*, vol. 25, no. 4, pp. 509-519, 1989.

[18] Y.D. Song, R.H. Middleton, and J.N. Anderson, "Study on the exponential path tracking control of robot manipulators via direct adaptive methods," *IEEE Int. Conf. on Robotics and Automation*, Sacramento, USA, Apr. 1991.

[19] M.W. Spong and R. Ortega, "On adaptive inverse dynamics control of rigid robots," *IEEE Trans. on Automat. Contr.*, vol. 35, no. 1, pp. 92-95, 1990.

[20] M.W. Spong and M. Vidyasagar, *Robot Dynamics and Control*, New York: Wiley, 1989.