Time-Optimal System Design Generalized Methodology

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Abstract: - The new general methodology for the electronic system design was elaborated by means of the optimum control theory formulation in order to improve the characteristics of the system design process. This approach generalizes the design process and generates a set of the different design strategies that serves as the structural basis to the optimal strategy construction. The principal difference between this new methodology and before elaborated theory is the more general approach on the system parameters definition. The main equations for the system design process were elaborated. These equations include the special control functions that are introduced into consideration artificially to generalize the total design process. Numerical results demonstrate the efficiency and perspective of the proposed approach.

Key-Words: - Time-optimal design algorithm, control theory formulation.

1 Introduction

One of the main problems of the total quality design improvement is the problem of the computer time reduction for a large system design. This problem has a special significance for the VLSI electronic circuit design. The traditional system design methodology includes two main parts: the model of the system that can be described as algebraic equations or differential-integral equations and a parametric optimization procedure that achieves the cost function optimal point. By this conception it is possible to change optimization strategy and use different models and different analysis methods. However, the time of the large-scale circuit analysis and the time of optimization procedure increase when the network scale increases.

There are some powerful methods that reduce the necessary time for the circuit analysis. Because a matrix of the large-scale circuit is a very sparse, the special sparse matrix techniques are used successfully for this purpose [1]-[2]. Other approach to reduce the amount of computational required for the linear and nonlinear equations is based on the decomposition techniques. The partitioning of a circuit matrix into bordered-block diagonal form can be done by branches tearing as in [3], or by nodes tearing as in [4] and jointly with direct solution algorithms gives the solution of the problem. The extension of the direct solution methods can be obtained by hierarchical decomposition and macromodel representation [5]. Other approach for achieving decomposition at the nonlinear level consists on a special iteration techniques and has been realized in [6] for the iterated timing analysis and circuit simulation. Optimization technique that is used for the circuit optimization and design, exert a very strong influence on the total necessary computer time too. The numerical methods are developed both for the unconstrained and for the constrained optimization [7] and will be improved later on. The practical aspects of these methods were developed for the electronic circuits design with the different optimization criterions [8]-[9]. The fundamental problems of the development, structure elaboration, and adaptation of the automation design systems have been examine in some papers [10]-[13].

The above described system design ideas can be named as the traditional approach or the traditional strategy because the analysis method is based on the Kirchhoff laws.

The other formulation of the circuit optimization problem was developed in heuristic level some decades ago [14]. This idea was based on the Kirchhoff laws ignoring for all the circuit or for the circuit part. The special cost function is minimized instead of the circuit equation solving. This idea was developed in practical aspect for the microwave circuit optimization [15] and for the synthesis of high-performance analog circuits [16] in extremely case, when the total system model was eliminated. The last papers authors affirm that the design time was reduced significantly. This last idea can be named as the modified traditional design strategy.

Nevertheless all these ideas can be generalized to reduce the total computer design time for the system design. This generalization can be done on the basis of the control theory approach and includes the special control function to control the design process. This approach consists of the reformulation of the total design problem and generalization of it to obtain a set of different design strategies inside the same optimization procedure [17]. The number of the different design strategies, which appear in the generalized theory, is equal to 2^M for the constant value of all the control functions, where M is the number of dependent parameters. These strategies serve as the structural basis for more strategies construction with the variable control functions. The main problem of this new formulation is the unknown optimal dependency of the control function vector that satisfies to the time-optimal design algorithm.

However, the developed theory [17] is not the most general. In the limits of this approach only initially dependent system parameters can be transformed to the independent but the inverse transformation is not supposed. The next more general approach for the system design supposes that initially independent and dependent system parameters are completely equal in rights, i.e. any system parameter can be defined as independent or dependent one. In this case we have more vast set of the design strategies that compose the structural basis and more possibility to the optimal design strategy construct.

2 Problem Formulation

In accordance with the new design methodology [17] the design process is defined as the problem of the cost function $C(X)$ minimization for $X \in R^N$ by the optimization procedure, which can be determined in continuous form as:

$$\frac{dx_i}{dt} = f_i(X,U), \quad i=1,2,\ldots,N \tag{1}$$

and by the analysis of the electronic system model in the next form:

$$\left(1-u_j\right) g_j(X) = 0, \quad j=1,2,\ldots,M \tag{2}$$

where $N=K+M$, K is the number of independent system parameters, M is the number of dependent system parameters, X is the vector of all variables $X=\left(x_1,x_2,\ldots,x_K,x_{K+1},x_{K+2},\ldots,x_N\right)$; U is the vector of control variables $U=\left(u_1,u_2,\ldots,u_M\right)$; $u_j \in \Omega$; $\Omega=\{0;1\}$.

The functions of the right part of the system (1) are depended from the concrete optimization algorithm and, for instance, for the gradient method are determined as:

$$f_i(X,U) = -b\frac{d}{dx_i}\left\{C(X)+\frac{1}{\varepsilon}\sum_{j=1}^{M} u_j g_j^2(X)\right\} \tag{3}$$

for $i=1,2,\ldots,K$,

$$f_i(X,U) = -b\cdot u_{i-K}\frac{d}{dx_i}\left\{C(X)+\frac{1}{\varepsilon}\sum_{j=1}^{M} u_j g_j^2(X)\right\} + \frac{\left(1-u_{i-K}\right)}{dt}\left\{-x'_i+h_i(X)\right\} \tag{3'}$$

for $i=K+1,K+2,\ldots,N$,

where b is the iteration parameter; the operator $\frac{d}{dx_i}$ hear and below means $\frac{d}{dx_i}\varphi(X) = \frac{\partial\varphi(X)}{\partial x_i} + \sum_{p=K+1}^{K+M}\frac{\partial\varphi(X)}{\partial x_p}\frac{\partial x_p}{\partial x_i}$, x'_i is equal to $x_i(t-dt)$; $h_i(X)$ is the implicit function $(x_i = h_i(X))$ that is determined by the system (2), $C(X)$ is the cost function of the design process.

The problem of the optimal design algorithm searching is determined now as the typical problem of the functional minimization of the control theory. The total computer design time serves as the necessary functional in this case. The optimal or quasi-optimal problem solution can be obtained on the basis of analytical [18] or numerical [19]-[22] methods. By this formulation the initially dependent parameters for $i=K+1,K+2,\ldots,N$ can be transformed to the independent ones when u_j=1 and it is independent when u_j=0. On the other hand the initially independent parameters for $i=1,2,\ldots,K$, are independent ones always.

We have been developed in the present paper the new approach that permits to generalize more the above described design methodology. We suppose now that all of the system parameters can be independent or dependent ones. In this case we need to change the equation (2) for the system model definition and the equation (3) for the right parts description.

The equation (2) defines the system model and is transformed now to the next one:

$$(1-u_i)g_j(X)=0 \tag{4}$$

$i = 1,2,\ldots, N$ and j Î J

where J is the index set for all those functions $g_j(X)$ for which $u_i = 0$, $J = \{j_1, j_2, \ldots, j_z\}$, j_s Î P with $s = 1, 2, \ldots, Z$, P is the set of the indexes from 1 to M, $P = \{1, 2, \ldots, M\}$, Z is the number of the equations that will be left in the system (4), $Z \in \{0, 1 \ldots, M\}$.

The right hand side of the system (1) is defined now as:

$$f_i(X,U) = -b \cdot u_i \frac{d}{dx_i} F(X,U) + \frac{(1-u_i)}{dt}\{-x_i(t-dt)+h(X)\} \tag{5}$$

for $i = 1,2,\ldots, N$,

where $F(X,U)$ is the generalized objective function and it is defined as:

$$F(X,U) = C(X) + \frac{1}{e}\sum_{j\in\Pi\backslash J} g_j^2(X) \tag{6}$$

This new definition of the design process is more general than in [17]. It generalizes the methodology for the system design and produces more representative structural basis of different design strategies. The total number of the different strategies, which compose the structural basis, is equal to $\sum_{i=0}^{M} C_{K+M}^{i}$. We expect the new possibilities to accelerate the design process in this case.

3 Numerical Results

Some non-linear passive electronic circuits have been analyzed to demonstrate the new, more general system design approach. The circuits have various nodal numbers from 1 to 4, $(M \in [1,4])$. The numerical results correspond to the variable optimized step for the system (1) integration.

3.1 Example 1

The simplest nonlinear circuit in Fig. 1 is analyzed. The nonlinear element has the following dependency: $R_n = r_0 + bV_1$. Using the Laws of Kirchhoff we can obtain the following equation for

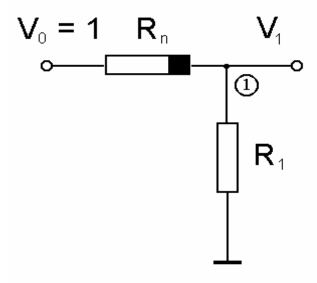

Fig. 1. Simplest one node circuit

the function $g(X)$ definition:

$$g(X) \equiv (x_1^2 + r_0 + bx_2)x_2 - x_1^2 = 0 \tag{7}$$

where the coordinates of the vector X are defined by means of $x_1^2 = R_1$, $x_2 = V_1$. This definition overcomes the problem of the positive restriction for the resistance. Only one control function is defined in the limits of the previously defined methodology [17] and only two different design strategies compose the structural basis, for u=0 and for u =1. However, we need to introduce two control functions and three different design strategies for the new generalized formulation. We have the vector of the control functions $U(u_1, u_2)$ and three different design strategies: (1,0), (1,1), (0,1). The last strategy is the new.

3.1.1 Strategy (1,0)

This is the traditional design strategy. In this case the parameter x_1 is an independent one and x_2 is a dependent one. The control vector has the next form: (1,0). The optimization procedure is done by the following equation:

$$\frac{dx_1}{dt} = -\frac{dF}{dx_1}$$

with the cost function $F(X) \equiv C(X) = (x_2 - k)^2$ and x_2 can be calculated by the analytic formula:

$$x_2 = \left[-(x_1^2 + r_0) + \sqrt{(x_1^2 + r_0)^2 + 4bx_1^2}\right] / 2b$$

3.1.2 Strategy (1,1)

This is the modified traditional design strategy. Both parameters x_1 and x_2 are independent and two equations for the optimization procedure can be defined now in the next now:

$$\frac{dx_1}{dt} = -\frac{dF}{dx_1}, \quad \frac{dx_2}{dt} = -\frac{dF}{dx_2}$$

with the objective function $F(X) \equiv C(X) + g^2(X)$.

3.1.3 Strategy (0,1)

This is the new strategy, which did not appear in previously developed theory. In this case x_1 is a dependent parameter and x_2 is independent one. The optimization procedure is defined by the following equation:

$$\frac{dx_2}{dt} = -\frac{dF}{dx_2}$$

with the objective function $F(X) \equiv C(X) = (x_2 - k)^2$. The dependent parameter x_1 is calculated now from the equation (7) as $x_1 = \sqrt{(r_0 + bx_2)x_2 / (1 - x_2)}$. We have an analytical solution due to the very simple example. We need to solve the system (4) by means of the Newton-Raphson method for all others examples.

3.1.4 Results

The numerical results for three above mentioned strategies are shown in Table 1.

Table 1. The total set of design strategy structural basis.

N	Control functions vector U (u1, u2)	Calculation results: Iterations number	Total design time (sec)
1	(1 0)	9	0.000131
2	(1 1)	26	0.002353
3	(0 1)	5	0.000075

It is very interesting that the new design strategy (01), which appears in generalized theory, has the iteration number and the total design time lesser than others. This new design strategy gives the time gain 1.75 times with respect to the traditional strategy (10).

3.2 Example 2

The two-node circuit (Fig. 2) is analyzed by means of the new generalized methodology.

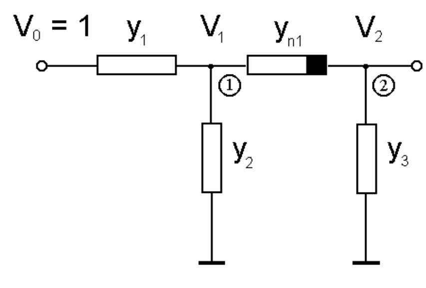

Fig. 2. Two-node circuit topology.

The nonlinear element has the following dependency: $y_{n1} = y_0 + b(V_1 - V_2)^2$. The vector X includes 5 components: $x_1^2 = y_1$, $x_2^2 = y_2$, $x_3^2 = y_3$, $x_4 = V_1$, $x_5 = V_2$. The model of this circuit (4) includes two equations (*M*=2) and the optimization procedure (5) includes five equations. The total structural basis contains $\sum_{i=0}^{2} C_5^i = 16$ different strategies. The system (4) is solved by the Newton-Raphson method. The cost function *C(X)* is defined by the following form: $C(X) = (x_4 - k_1)^2 + (x_5 - k_2)^2$. The design results for some of these strategies are presented in Table 2.

Table 2. Some strategies of the structural basis for two-node circuit.

N	Control functions vector U (u1, u2, u3, u4, u5)	Calculation results: Iterations number	Total design time (sec)
1	(0 1 0 1 1)	5	0.000851
2	(0 1 1 1 1)	178	0.016671
3	(1 0 0 1 1)	201	0.026235
4	(1 0 1 1 1)	3162	0.300012
5	(1 1 0 0 1)	23	0.002205
6	(1 1 0 1 0)	49	0.100011
7	(1 1 0 1 1)	49	0.002405
8	(1 1 1 0 0)	107	0.010365
9	(1 1 1 0 1)	1063	0.170011
10	(1 1 1 1 0)	143	0.013115
11	(1 1 1 1 1)	243	0.006215

Four last strategies of the table are the same that had been defined inside the previously formulated methodology. We can name these strategies as the “old” ones. It is very interesting that some new strategies have the computer time significantly lesser than all the “old” strategies. The strategy number 1 has the minimal computer time and the maximum time gain 12.2 with respect to the traditional design strategy 8. At the same time the best "old" strategy 11 has the time gain 1.67 only.

3.3 Example 3

In Fig. 3 there is a circuit that has seven parameters, i.e. four admittances y_1, y_2, y_3, y_4 and three nodal voltages V_1, V_2, V_3.

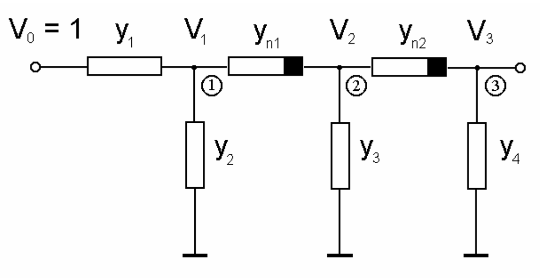

Fig. 3. Three-node circuit topology.

The nonlinear elements were defined by the following dependencies: $y_{n1} = a_{n1} + b_{n1} \cdot (V_1 - V_2)^2$, $y_{n2} = a_{n2} + b_{n2} \cdot (V_2 - V_3)^2$. The vector X includes 7 components: $x_1^2 = y_1$, $x_2^2 = y_2$, $x_3^2 = y_3$, $x_4^2 = y_4$, $x_5 = V_1$, $x_6 = V_2$, $x_7 = V_3$ The mathematical model of this circuit (4) includes now three equations (M=3) and the optimization procedure (1), (5) includes seven equations. The cost function $C(X)$ is defined by the following expression: $C(X) = (V_1 - V_2 - k_1)^2 + (V_2 - V_3 - k_2)^2 + (V_3 - k_3)^2$.

The total structural basis contains $\sum_{i=0}^{3} C_7^i = 64$ different strategies. For instance, the structural basis of the previous developed methodology includes only $2^3 = 8$ different strategies. The design results for the "old" strategies and for some of the new strategies are presented in Table 3. Among the "old" strategies (14-21) there are three strategies (17, 18, and 21) that have the design time lesser than the traditional strategy 14. However, the time gain is not very large. The best strategy 18 among all the "old" strategies has the time gain 1.86 only. At the same time, among the new strategies there are many (2, 6, 10, 11, 12, 13) that have the design time significantly lesser than the traditional one and have the time gain more than 14. The optimal strategy among all the presented is the number 11. It has the computer time gain 23.1 times with respect to the traditional design strategy. Further analysis may be focused on the problem of the optimal design strategy searching by means of the control vector manipulation. It is clear intuitively that we can obtain the time gain much more by means of the new structural basis.

Table 3. Some strategies of the structural basis for three-node circuit.

N	Control functions vector U (u1,u2,u3,u4,u5,u6,u7)	Calculation results	
		Iterations number	Total design time (sec)
1	(0101111)	1127	0.8414
2	(0110111)	63	0.0122
3	(0111010)	2502	1.8411
4	(0111101)	1390	0.9731
5	(0111110)	224	0.3571
6	(0111111)	43	0.0125
7	(1011110)	354	0.5205
8	(1011111)	2190	1.1601
9	(1100111)	326	0.5042
10	(1110011)	23	0.0161
11	(1110101)	14	0.0099
12	(1110110)	27	0.0103
13	(1110111)	51	0.0102
14	(1111000)	59	0.2291
15	(1111001)	167	0.2732
16	(1111010)	174	0.2911
17	(1111011)	185	0.1543
18	(1111100)	63	0.1228
19	(1111101)	198	0.2451
20	(1111110)	228	0.2582
21	(1111111)	293	0.1765

3.4 Example 4

The four-node circuit is analyzed below (Fig. 4). The problem includes five parameters on the basis of the admittances $(x_1, x_2, x_3, x_4, x_5)$, where $x_1^2 = y_1$, $x_2^2 = y_2$, $x_3^2 = y_3$, $x_4^2 = y_4$, $x_5^2 = y_5$, and four parameters on the basis of the nodal voltages

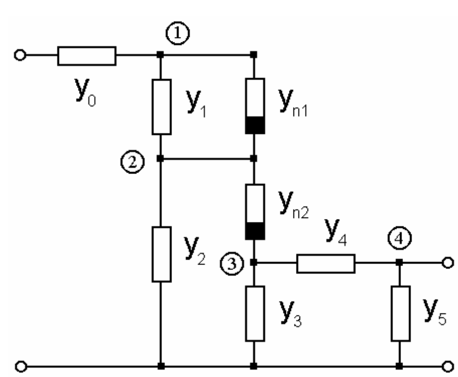

Fig. 4. Four-node circuit topology.

(x_6, x_7, x_8, x_9), where $x_6 = V_1$, $x_7 = V_2$, $x_8 = V_3$, $x_9 = V_4$. The control vector U includes nine components $(u_1, u_2, ..., u_9)$. The nonlinear elements were defined by the same dependencies like in the previous example. The circuit model includes five equations of the system (4) and the optimization procedure includes nine equations (1), (5). The system (9) is solved by the Newton-Raphson method. The cost function *C(X)* of the design process is defined by the following form: $C(X) = (x_9 - k_0)^2 + (x_6 - x_7 - k_1)^2 + (x_7 - x_8 - k_2)^2$. The total number of the different design strategies that compose the structural basis of the generalized theory is equal to $\sum_{i=0}^{4} C_9^i = 256$. At the same time the structural basis of the previous developed theory includes 16 strategies only. The results of the structural basis strategies that include all the "old" strategies (the last 16 strategies) and some new strategies are shown in Table 4.

Table 4. Some strategies of the structural basis for four-node circuit.

N	Control functions vector U (u1,u2,u3,u4,u5,u6,u7,u8,u9)	Calculation results: Iterations number	Total design time (sec)
1	(1 1 1 0 1 0 0 0 1)	5	0.0031
2	(1 1 1 1 1 0 0 0 1)	397	0.4312
3	(1 1 1 0 1 1 0 0 1)	5	0.0029
4	(1 1 0 1 1 1 1 1 0)	119	0.0209
5	(1 1 1 1 0 0 1 0 1)	101	0.0232
6	(1 1 1 0 1 0 0 1 1)	15	0.0134
7	(1 1 1 0 1 1 1 0 1)	5	0.0009
8	(1 1 1 0 1 1 1 1 1)	101	0.0243
9	(1 1 1 1 0 0 1 1 1)	185	0.0324
10	(1 1 1 1 0 1 0 0 1)	74	0.0102
11	(1 1 1 1 0 1 0 1 1)	121	0.0254
12	(1 1 1 1 0 1 1 1 1)	159	0.0127
13	(1 1 1 1 1 0 0 0 0)	33	0.0263
14	(1 1 1 1 1 0 0 0 1)	397	0.4317
15	(1 1 1 1 1 0 0 1 0)	6548	7.1392
16	(1 1 1 1 1 0 0 1 1)	76	0.0122
17	(1 1 1 1 1 0 1 0 0)	456	0.5113
18	(1 1 1 1 1 0 1 0 1)	24	0.0052
19	(1 1 1 1 1 0 1 1 0)	3750	4.3661
20	(1 1 1 1 1 0 1 1 1)	90	0.0095
21	(1 1 1 1 1 1 0 0 0)	68	0.0354
22	(1 1 1 1 1 1 0 0 1)	596	0.6213
23	(1 1 1 1 1 1 0 1 0)	5408	6.2191
24	(1 1 1 1 1 1 0 1 1)	78	0.0255
25	(1 1 1 1 1 1 1 0 0)	238	0.2104
26	(1 1 1 1 1 1 1 0 1)	77	0.0227
27	(1 1 1 1 1 1 1 1 0)	139	0.0131
28	(1 1 1 1 1 1 1 1 1)	131	0.0103

The strategy 13 is the traditional one. There are seven different strategies in the "old" group that have the design time less that the traditional strategy. These are the strategies 16, 18, 20, 24, 26, 27 and 28. The strategy 18 is the optimal one among all the "old" strategies and it has the time gain 5.06 with respect to the traditional design strategy. On the other hand the best strategy among all the strategies (number 7) of the Table 4 has the time gain 29.2. So, we have the additional acceleration 5.77 times. This effect was obtained on the basis of the more extensive structural basis and servers as the principal result of the new generalized methodology. The posterior analysis and the optimization of the control vector U can increase this time gain as shown in [23].

4 Conclusion

The traditional method for the analog circuit design is not time-optimal. The problem of the optimum algorithm construction can be solved more adequately on the basis of the optimal control theory application. The time-optimal design algorithm is formulated as the problem of the functional optimization of the optimal control theory. In this case it is necessary to select one optimal trajectory from the quasi-infinite number of the different design strategies, which are produced. The new and more complete approach to the electronic system design methodology has been developed now. We have checked that this approach generates more broadened structural basis of the different design strategies. The total number of the different design strategies, which compose the structural basis by this approach, is equal to $\sum_{i=0}^{M} C_{K+M}^i$. This new structural basis serves as the necessary set for the optimal design strategy search. This basis includes very perspective strategies that can be used for the time-optimal design algorithm construction. Some new strategies have better convergence and lesser computer time than the strategies that appeared in before developed methodology. This approach can reduce considerably the total computer time for the system design. Analysis of the different electronic systems gives the possibility to conclude that the potential computer time gain that can be obtain on the basis of the broadening of the structural basis is significantly more than for the previous developed methodology.

Acknowledgment

This work was supported by the Universidad Autonoma de Puebla, under Grant VIEPIII05G02.

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