Approximation of the Coherent-mode Structure of an Optical Field According to the Results of Physical Experiment

ANDREY S. OSTROVSKY, ALEXANDRE I. GREBENNIKOV, EDGAR HERNÁNDEZ-GARCÍA
Facultad de Ciencias Físico Matemáticas
Benemérita Universidad Autónoma de Puebla
A. P. 1152, 70000, Puebla, Pue.
MEXICO

Abstract: The problem of reconstructing the coherent-mode structure of an optical field with unknown cross-spectral density function is solved by means of optimal approximating the measured radiant intensity. An alternative field is defined with the coherent-mode structure calculated for the chosen orthonormal basis. An effciency of the proposed technique is illustrated with the results of a numerical simulation experiment.

Key-Words: Coherent-mode structure, Generalized radiant intensity, Gaussian Schell-model source

1 Introduction

As is well known, the transverse cross-spectral density function $W(\mathbf{x}_1, \mathbf{x}_2)$ of an optical field may be represented by the Mercer expansion [1]:

$$W(\mathbf{x}_1, \mathbf{x}_2) = \sum_{n=0}^{\infty} \lambda_n \varphi_n^*(\mathbf{x}_1) \varphi_n(\mathbf{x}_2), \qquad (1)$$

where λ_n and $\varphi_n(\mathbf{x})$ are, respectively, the eigenvalues and the eigenfunctions of the Fredholm integral equation

$$\int_D W(\mathbf{x}_1, \mathbf{x}_2) \varphi_n(\mathbf{x}_1) \mathrm{d}\mathbf{x}_1 = \lambda_n \varphi_n(\mathbf{x}_2). \qquad (2)$$

Each of summands in Eq. (1) represents the cross-spectral density function of a field that is completely coherent in the space-frequency domain and obeys the same propagation law as the cross-spectral density $W(\mathbf{x}_1, \mathbf{x}_2)$ does. Hence, it may be associated with the perfectly coherent mode of field oscillation, and the infinite set of coherent modes

$$\Lambda = \{\lambda_n, \varphi_n(\mathbf{x})\}, \qquad n = 0,1,2,..., \qquad (3)$$

may be referred to as *the coherent-mode structure* of the field. The latter is an essential tool in describing the processes and systems in optics. Recently we employed the concept of the coherent-mode structure for representation of generalized radiometric characteristics [2], for representation and generation of propagation invariant optical fields [3,4], and for representation of partially coherent optical systems [5,6]. However, the practical value of this tool is essentially restricted for the following two reasons. Firstly, in practice, the analytical expression for cross-spectral density function $W(\mathbf{x}_1, \mathbf{x}_2)$, as a rule, is unknown, and hence, the Fredholm equation (2) cannot be solved in the closed form. Secondly, even when the cross-spectral density function may be approximated by a definite analytical function such a solution may be obtained only for a very limited number of field models. Clearly, an alternative approach to calculating the coherent-mode structure of the field, which does not involve the solution of the Fredholm equation (2), is desired. In this paper, we propose such an approach based on approximation of the coherent mode structure according to the results of the physical experiment.

2 Radiant intensity and alternative coherent-mode structure of the field

As it was shown in Ref [2], the radiant intensity of the field may be expressed in terms of the coherent-mode structure as follows:

$$J(\mathbf{s}) = \left(\frac{k}{2\pi}\right)^2 \cos^2\theta \sum_{n=0}^{\infty} \lambda_n |\Phi_n(\mathbf{s})|^2, \qquad (4)$$

where

$$\Phi_n(\mathbf{s}) = \int_D \varphi_n(\mathbf{x}) \exp(ik\mathbf{s}\cdot\mathbf{x}) \mathrm{d}\mathbf{x}. \qquad (5)$$

In Eqs. (4) and (5) k is the wave number, $\mathbf{s}$ is the unit vector which points into the half-space $z > 0$, θ is the angle between **s** and the *z* axis. It is important to stress here that quantity defined by Eq. (4) may be directly measured in physical experiment.

Now, let us introduce a function

$$\widetilde{W}(\mathbf{x}_1,\mathbf{x}_2)=\sum_{m=0}^{M-1}\mu_m\psi_m^*(\mathbf{x}_1)\psi_m(\mathbf{x}_2), \quad (6)$$

where μ_m are some real positive variables bounded from above and $\psi_m(\mathbf{x})$ are some continuous functions which form an orthonormal set. This function, according to its construction, is square integrable, Hermitian and nonnegative definite, and hence, may be considered as the cross-spectral density function of some *alternative field* in the plane $z=0$. Each of summands in Eq. (6) may be associated with the coherent mode of oscillation. Hence, the finite set

$$\widetilde{\Lambda}=\{\mu_m,\psi_m(\mathbf{x})\}, \quad m=0,1,\ldots,M-1, \quad (7)$$

may be referred to as the coherent-mode structure of the alternative field or, for brevity, *the alternative coherent-mode structure*. Changing formally λ_n and $\varphi_n(\mathbf{x})$ in Eqs. (4) and (5) for μ_m and $\psi_m(\mathbf{x})$, respectively, one may obtain the following finite-sum expressions for the radiant intensity:

$$\mathcal{J}(\mathbf{s})=\left(\frac{k}{2\pi}\right)^2\cos^2\theta\sum_{m=0}^{M-1}\mu_m\left|\Psi_m(\mathbf{s})\right|^2, \quad (8)$$

where

$$\Psi_m(\mathbf{s})=\int_D\psi_m(\mathbf{x})\exp(ik\mathbf{s}\cdot\mathbf{x})\mathrm{d}\mathbf{x}. \quad (9)$$

Once the quantity *J* of the original field has been measured for discrete points $\mathbf{x}_p$ and discrete directions $\mathbf{s}_q$, its good approximation may be obtained by solving the following problem of the conditional optimization:

$$\sum_q\left[\mathcal{J}(\mathbf{s}_q)-J(\mathbf{s}_q)\right]^2\to\min_{\mu_m}, \quad \mu_m\geq 0, \quad (10)$$

The optimal solution $(\mu_m)_{\mathrm{opt}}$ may be obtained numerically by well known methods of quadratic programming. Substituting the obtained solution into Eq. (6), one may find the function $\widetilde{W}_{\mathrm{opt}}(\mathbf{x}_1,\mathbf{x}_2)$ that is an approximation of the cross-spectral density function $W(\mathbf{x}_1,\mathbf{x}_2)$ in the chosen basis of the coherent-mode functions $\psi_m(\mathbf{x})$. Hence, the alternative coherent-mode structure (7) with $\mu_m=(\mu_m)_{\mathrm{opt}}$ may be accepted as a finite approximation of the coherent mode structure of the original field.

3 Numerical simulation

To demonstrate the justifiability of the proposed technique we realized the numerical simulation experiment in which we calculated the alternative coherent-mode structure of the field with the known cross-spectral density function and the coherent-mode structure obtained as an exact solution of the Fredholm equation (2). In the experiment, as an original field, we considered the one-dimensional Gaussian Schell-model secondary source which is characterized by a cross-spectral density function of the form

$$W(x_1,x_2)=I_0\exp\left(-\frac{x_1^2+x_2^2}{4\sigma_I^2}\right)\times\exp\left[-\frac{(x_1-x_2)^2}{2\sigma_\gamma^2}\right], \quad (11)$$

where I_0 is the spectral intensity of the source at the center, σ_I is the rms width of the intensity distribution across the source, and σ_γ is the rms width of the complex degree of coherence of the source. The coherent-mode structure of such a source is defined as follows [7]:

$$\varphi_n(x)=\left(\frac{2c}{\pi}\right)^{1/4}\frac{1}{\left(2^n n!\right)^{1/2}}\times H_n\left(x\sqrt{2c}\right)\exp\left(-cx^2\right), \quad (12)$$

$$\lambda_n=I_0\left(\frac{\pi}{a+b+c}\right)^{1/2}\left(\frac{b}{a+b+c}\right)^n, \quad (13)$$

where

$$a\equiv\frac{1}{4\sigma_I^2}, \quad b\equiv\frac{1}{2\sigma_\gamma^2}, \quad c\equiv\left(a^2+2ab\right)^{1/2}. \quad (14)$$

It may be readily shown that for this source

$$\Phi_n(s)=i^n\sqrt{2}\left(\frac{\pi}{2c}\right)^{1/4}\frac{1}{\left(2^n n!\right)^{1/2}}$$

$$\times H_n\left(\frac{k}{\sqrt{2c}}s\right)\exp\left(-\frac{k^2}{4c}s^2\right). \quad (15)$$

To calculate the radiant intensity $J(\theta)$, we truncated the summation in Eq. (4) with the effective number of coherent modes defined by Starikov [8] as

$$N \approx \left(1+4\frac{\sigma_I^2}{\sigma_\gamma^2}\right)^{1/2}. \quad (16)$$

In our experiment we chose σ_I/σ_γ =10 (the case of truly partially coherent source) and hence, $N \approx 20$.

As the alternative mode functions we used the Hermitians functions

$$\psi_m(x) = \left(\frac{1}{2^m m! \sigma_I \sqrt{2\pi}}\right)^{1/2}$$
$$\times H_m\left(\frac{x}{\sqrt{2\sigma_I}}\right)\exp\left(-\frac{x^2}{4\sigma_I}\right), \quad (17)$$

for which

$$\tilde{J}(\theta) = \frac{2k\sigma_I}{\sqrt{2\pi}}\cos^2\theta\exp\left(-2k^2\sigma_I\sin^2\theta\right)$$
$$\times\sum_{m=0}^{M-1}\mu_m\frac{1}{2^m m!}\left[H_m\left(k\sqrt{2\sigma_I}\sin\theta\right)\right]^2 \quad (18)$$

We calculated the quantities given by Eqs. (4) and (17) in 10 discrete directions θ_q, and solved the optimization problem (10) using the standard program included in MatLab package. Using the obtained solution, we evaluated the normalized rms error *z* of the approximation of cross-spectral density function $W(x_1, x_2)$ of the original field. The results of calculation for two different values of *M* are given in Table 1.

$(\mu_m)_{opt}$	μ_0	μ_1	μ_2	μ_3	μ_4	ε
$M=6$	2.3	1.8	1.4	0.9	0.5	0.073
$M=11$	2.3	2.1	1.8	1.6	1.4	-

$(\mu_m)_{opt}$	μ_5	μ_6	μ_7	μ_8	μ_9	ε
$M=6$	-	-	-	-	-	-
$M=11$	1.2	0.9	0.7	0.5	0.3	0.022

Table 1. The results of numerical experiment

4 Conclusions

The problem of approximating the coherent-mode structure of the field with unknown cross-spectral density function has been formulated as the problem of finding the coherent-mode structure of an alternative field defined in the chosen orthonormal basis. It has been shown that such a problem may be solved by means of optimal approximating the physically measured radiant intensity of the original field according to one calculated in terms of an alternative coherent-mode structure. An efficiency of the proposed technique has been illustrated with the results of the numerical simulation experiment. We consider that the proposed technique reveals the new promising means in practical application of the coherent-mode representation of processes and systems in optics.

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