

Generation of Light String Beam in Fourier-transforming Optical System with Partially Coherent Illumination

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Abstract: The optical technique for generating the light string beam is proposed. The capacity of this technique is demonstrated in physical experiment. The experimental results confirm the physical reliability of the light string beam.

Key-Words: Propagation-invariant field, Fourier-transforming Optical system, Liquid crystal light modulator

1 Introduction

Recently we reported the exact solutions of the Helmholtz equation for coherent modes of propagation-invariant optical field [1]. One of these solutions indicates the existence the peculiar optical field, termed by as *light string beam*, which is characterized by extremely sharply localized energy distribution in its transverse section and propagates in free space without any divergency. It is obvious that such a beam is very attractive from the standpoint of its possible applications in communication, measurements, microtechnology, microsurgery, etc. Nevertheless, the practical value of this beam requires the corroboration of possibility of its physical generation. In this paper we shows how the light string beam can be generated by mean of rather simple optical system with specially organized illumination.

2 Light string beam

The light string beam propagated in the direction z is characterized by the transverse cross-spectral density function given by

$$W(\rho_1, \theta_1, \rho_2, \theta_2; z) = \sum_{n=1}^{\infty} J_0\left(\alpha_{0,n} \frac{\rho_1}{R}\right) \times J_0\left(\alpha_{0,n} \frac{\rho_2}{R}\right), \quad (1)$$

where J_0 denotes the Bessel function of the first kind and of order zero, $\alpha_{0,n}$ is the n th zero of the function J_0 , R is a real positive constant, and (ρ, θ) are the

polar coordinates in the plane $z = 0$. The corresponding intensity distribution of the field is

$$I(\rho, \theta) = W(\rho, \theta, \rho, \theta) = \sum_{n=1}^{\infty} J_0^2\left(\alpha_{0,n} \frac{\rho}{R}\right). \quad (2)$$

The transverse intensity distribution (2), calculated with truncating the summation for different numbers N of terms, is shown in Fig. 1. As one may conclude from this figure, when N tends to infinity, the optical field with intensity distribution (2) represents an infinitely thin light beam which propagates in the direction z without any divergency, the fact that justifies the name given to this optical field pattern.

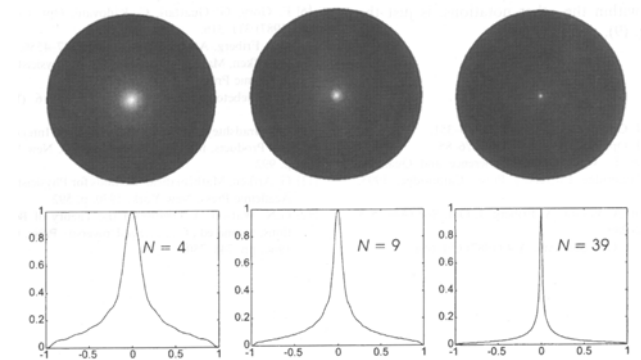


Fig. 1. Normalized intensity distribution, calculated in accordance with Eq. (2) for different values of the truncating parameter of summation N .

To ground the method of generating the light string beam it is useful to take advantage of its representation by the cross-angular spectrum [2],

$$\begin{aligned} \mathcal{A}(r_1, \varphi_1, r_2, \varphi_2) = & \int \int_0^\infty \int_0^{2\pi} \rho_1 \rho_2 \\ & \times W(\rho_1, \theta_1, \rho_2, \theta_2; z) \exp\{-i2\pi[r_2 \rho_2 \\ & \times \cos(\varphi_2 - \theta_2) - r_1 \rho_1 \\ & \times \cos(\varphi_1 - \theta_1)]\} d\rho_1 d\rho_2 dr_1 dr_2, \end{aligned} \quad (3)$$

where (r, φ) are the polar coordinates in the space-frequency domain. On substituting for W from Eq. (1) into Eq. (3) it may be readily shown that the cross-angular spectrum of the light string beam is given by

$$\begin{aligned} \mathcal{A}(r_1, \varphi_1, r_2, \varphi_2) = & \sum_{n=1}^{\infty} \left(\frac{R}{\alpha_{0,n}} \right)^2 \\ & \times \delta\left(\frac{\alpha_{0,n}}{2\pi R} - r_1\right) \delta\left(\frac{\alpha_{0,n}}{2\pi R} - r_2\right), \end{aligned} \quad (4)$$

where $\delta(\cdot)$ is the delta Dirac function.

3 Optical generating the light string beam

As can be seen from Eq. (3), the cross-angular spectrum given by Eq. (4) represents a 4D Fourier transform of the cross-spectral density function of the light string beam defined by Eq. (1). On the other hand, it is well known that the cross-spectral densities of the wave fields in the focal planes of a thin positive spherical lens are also related by a 4D Fourier transform. Both these facts can be used to produce a good approximation of the light string beam by creating a secondary source with a cross-spectral density function $W_s = \mathcal{A}$ in the front focal plane of the Fourier-transforming lens. It may be readily shown that, to create the proper secondary source, one may use a spatial light modulator with a complex amplitude transmittance [3]

$$\begin{aligned} T(\rho', \theta') = & \sum_{n=1}^N \frac{1}{\alpha_{0,n}} \delta(\alpha_{0,n} \rho_0 - \rho') \\ & \times \exp(i\Psi_n), \end{aligned} \quad (5)$$

where (ρ', θ') are the polar coordinates in the front focal plane of the Fourier lens, ρ_0 is a constant, and Ψ_n are the real random time-dependent and mutually uncorrelated variables that are uniformly distributed in the interval $(0, 2\pi)$.

Such a modulator may be realized as a combination of the static pure-amplitude modulator and the dynamic phase modulator with appropriate transmittance functions. The static amplitude modulator may be realized, to a good approximation, as a binary mask in the form of transparent rings with the required radii and widths that are proportional to the weight coefficients attached to the delta functions in Eq. (5) (Fig.2).

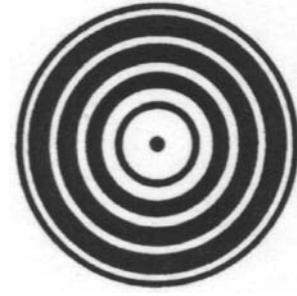


Fig. 2. The binary mask for generating the light string beam.

The dynamic phase modulator must represent a transparent plane screen that introduces the required azimuthal phase delays in the annular zones corresponding to the transparent rings of a binary mask that are changed randomly and independently in discrete moments of time (Fig. 3).

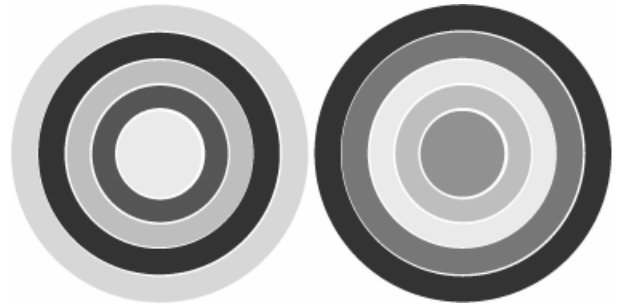


Fig. 3. Two different states of a dynamic phase modulator (the phase delay is presented by grey level).

As it follows from Eq. (5), the dynamic phase modulator realizes the complete destruction of the spatial coherence of illumination, in the radial direction conserving it in the azimuthal direction.

4 Experiments and results

To verify the capacity of the proposed technique of generating the light string beam, we carried out the corresponding physical simulation experiment. In this experiment we used a linearly polarized laser beam ($\lambda = 0.63 \mu\text{m}$) He-Ne laser) as a primary source. To

produce the required secondary source, we used the binary mask with a finite number of transparent rings of variable width that were specially manufactured using a photolithographic technique. Taking into account a very fast decrease of the value $1/\alpha_{0,n}^2$ with n , to simplify our experiments we limited the number of transparent rings to $N = 5$. As a dynamic phase modulator we used the commercial computer-controlled liquid crystal spatial light modulator HoloEye LC2002, which works in transmissive mode and has a 33 mm diagonal active display area with 800×600 pixels. To provide phase-only modulation we placed the modulator between two polarizers with appropriately chosen mutual orientation of their main axes. To control the modulator, we used periodically changing (60 Hz) random video patterns generated by PC in accordance with function (5). The transverse intensity distribution of the generated beam was registered at different distances z behind the back focal plane of the Fourier-transforming lens using a CCD camera SONY-SSC-M374 (768×494 pixels). To provide a solid statistical averaging of the registered data, we used an exposure time of several seconds.

The experimental setup is sketched schematically in Fig. 4. The results of the experiment are shown in Fig. 5. As one can see, the generated optical field may be identified approximately as the light string beam. The finite transverse dimensions of the generated beam can be explained by the finite sum approximation of Eq. (1). The observed expansion of the generated beam, beginning from a certain value of z , is explained by the finite pupil of the Fourier-transforming lens and the finite size of the secondary source.

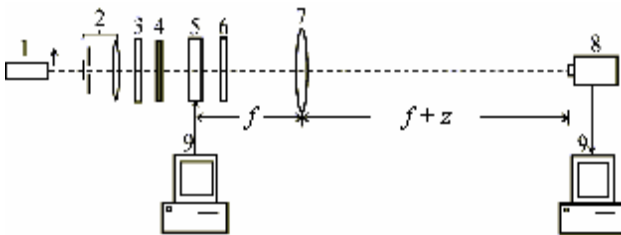


Fig. 4. Experimental setup: 1 – laser, 2 – beam expander, 3 – polarizer, 4 – binary mask, 5 – liquid crystal modulator, 6 – analyzer, 7 – Fourier-transforming lens, 8 – digital camera, 9 – PC.

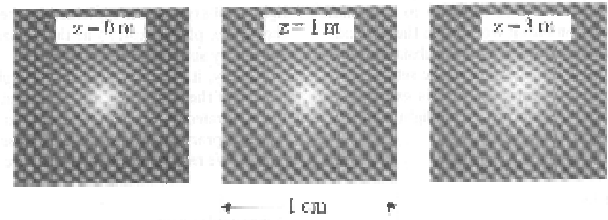


Fig. 5. The intensity distributions registered in experiment.

5 Conclusions

The optical technique for generating the light string beam has been proposed. The capacity of this technique has been demonstrated in physical experiment. The experimental results have confirmed the physical reliability of the light string beam.

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