A MULTIFUNCTION ANALOG NON LINEAR FILTER.

ARTURO PRIETO F., GUSTAVO RODRIGUEZ, JOEL GARCIA D., ANA MARIA PATRICIA ORTIZ M., SOTERO FUENTES G.*, LUIS ABRAHAM SÁNCHEZ G., HECTOR JUÁREZ S.
Facultad de Ciencias de la Electrónica,
Benemérita Universidad Autónoma de Puebla.
*Escuela de Electrónica.
Instituto Tecnológico de Puebla.
MÉXICO.

Abstract: - This article presents a non linear multifunction filter design; this design, with minor changes performs as a Media, Median Minimum or Maximum Filter. The analog design block system is based on an ideal macromodel, consisting of VCCS elements, resistances and diodes. The analysis, control variables and limitations are presented. Simulated results in SPICE are presented.

Key-Words: - Non linear filter, Media filter, Median filter, Minimum, Maximum filter.

1 Introduction

In many signal processing applications the suppression of unwanted components can be achieved by linear filters, but in signals with sharp edges corrupted by noise, linear filters also smoothes out signal edges (information) and in addition, impulsive noise cannot be sufficiently suppressed. Another type of filters, non linear or adaptive were proposed, filters that preserves edges while suppressing impulsive noise. The non linear Median Filter has had a good performance in such cases, it replaces the input signal value at each point by the median of the signal value in a neighborhood around that point [1,2,3]. These filters are used in Speech processing, video signals and other applications like Neural Networks [4]. Some digitally [3,5,6] and analog implementations [4,6,7,8] are reported. The performance of the filter depends on how well it can suppress the undesired part of the signal while retaining the desired part. Because median filters are non linear, it is very difficult or perhaps impossible to derive general results that would accurately describe the statistical behavior for a wide range of random signals as in the case for linear filters [10]. In this article a novel and versatile design is presented. An analog filter or processor, whose performance can be programmed with minor changes. It can act as a linear Media filter or as a non linear Median , Minimum or Maximum filter.

2 Problem Formulation

Lee and Kassam, based on a Maximun Likelihood Estimators theory, proposed an algorithm to get the median [2, 10]. Suppose we have the set of observations:

$$x_i = \theta s_i + \omega_i, \qquad i = 1, ..., N \qquad (1)$$

where $s_i = \{ s_1 ,\ldots s_N \}$, is a known signal waveform, θ is an unknown parameter to be estimated, and $\omega_i = \{\omega_1, \ldots \omega_N \}$ is a sequence of independent and identically distributed random variables (i.i.d.) with a common distribution function. Assume that N is an odd number of observations, all s_i are one, and that ω_i has a Laplace or biexponential distribution with probability density function:

$$f(x) = \frac{\alpha}{2} e^{-\alpha|x|}, \qquad \alpha > 0 \qquad (2)$$

The set of observations forms a simple random sample from a population having density:

$$f(x) = \frac{\alpha}{2} e^{-\alpha|x-\theta|}, \qquad \alpha > 0 \qquad (3)$$

The maximum likelihood principle states that if there are N independent observations $x_1, x_2, .., x_N$, the joint density function is:

$$L(\theta) = f(x_1,\theta) f(x_2,\theta) \ldots f(x_n,\theta) \qquad (4)$$

The maximum likelihood is obtained when

$$\frac{\partial L(\theta)}{\partial \theta} = 0 \qquad (5)$$

or equivalently

$$\frac{1}{f(x_1,\theta)} \frac{\partial f(x_1,\theta)}{\partial \theta} + \ldots \frac{1}{f(x_N,\theta)} \frac{\partial f(x_N,\theta)}{\partial \theta} = 0 \qquad (6)$$

ergo,

$$f(x_i,\theta) = \left(\frac{\alpha}{2}\right)^N e^{-\alpha \sum_{i=1}^{N} |x_i - \theta|} \qquad (7)$$

The maximum is equivalent minimize

$$L(\theta) = \sum_{i=1}^{N} |x_i - \theta| \qquad (8)$$

The value of θ minimizing L(θ),

$$\arg\{\min_{\theta} \sum_{i=1}^{N} |x_i - \theta|\} \qquad (9)$$

is exactly the median of $x_1, x_2, .., x_N$

$$\hat{\theta} = MED\{x_1, x_2, \ldots x_N\} \qquad (10)$$

The median filter is equivalent of finding the maximum likelihood estimate of the amplitude of a constant signal assuming that noise is independent and identically Laplace distributed.

If a Gaussian i.i.d. distribution is used:

$$f(x) = \frac{1}{\sqrt{2}\sigma} e^{-\frac{x^2}{2\sigma^2}} \qquad (11)$$

Using the same procedure, the media is obtained:

$$\hat{\theta} = \frac{1}{N} \sum_{i=1}^{N} x_i \qquad (12)$$

The output x_i of the median filter M, instead of θ, is defined as the solution of the equation :

$$\sum_{i=1}^{N} f(x_i - \theta) = 0 \qquad (13)$$

If f(x) is a linear function f(x) = ax, the media is obtained. If f(x) is a non linear function like the one shown at Fig.1 and approaching the sign function, the median is obtained.

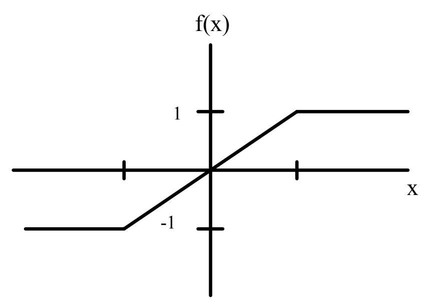

Fig.1. Function to get the analog median filter.

Based on this mathematical basis, an extrapolation was made and a Min or Max filter were also obtained. This article proposes an analog system that performs a Media, Median, Minimum or Maximum filter. Results are simulated in SPICE .

2 Problem solution and results.

A macromodel to solve the Lee and Kassam algorithm is shown in the next figure (fig.2) [11], it should be noted that this diagram corresponds to the conceptual implementation in OTA current mode, three inputs were taken, and as stated in equation (1), an odd number of inputs should be considered.

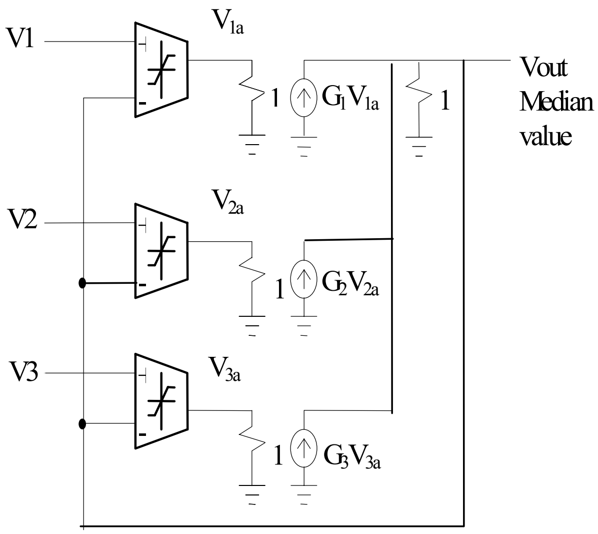

Fig. 2. Block Diagram to implement the median value of three inputs.

The principal block, the hard limiter function macromodel is shown in Fig. 3, in the case of the Median filter, it consists of a linear amplifier followed by a clipping section as shown in fig.3.

It should be mentioned that three parameters can be adjusted, the gain of the amplifier; and limiting voltages V1 and V2.

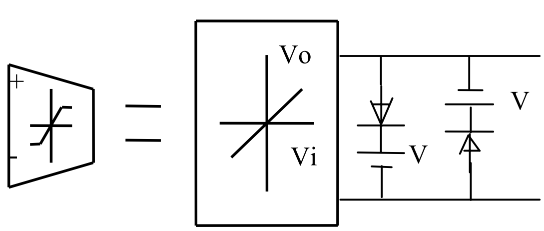

Fig.3 Hard limiter function.

In Fig. 4 three input signals are shown, similar results were obtained for sinusoidal signals, and irregular waveforms.

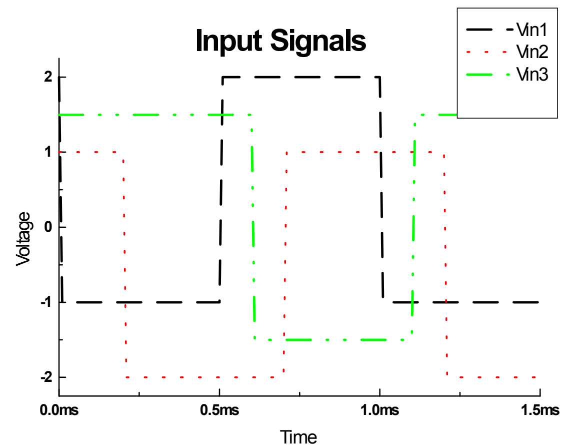

Fig 4. Three input signals

When the gain is high enough, V1=1 and V2=-1, the sign function is approximated and equation (8) is implemented, a Median Filter is obtained, as shown in Fig. 5. A CMOS version of this Filter has been already reported by the authors [11].

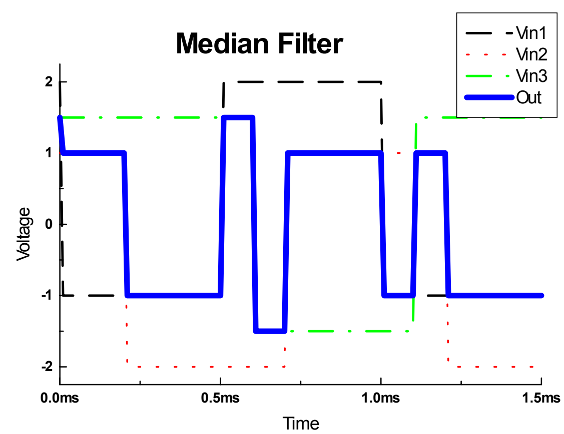

Fig. 5. The median Filter.

When the gain is high enough but we do not limit the function i.e. the diodes and voltage sources are disabled, the media of the three input signals is obtained, as shown in Fig. 6. This implements equation (12).

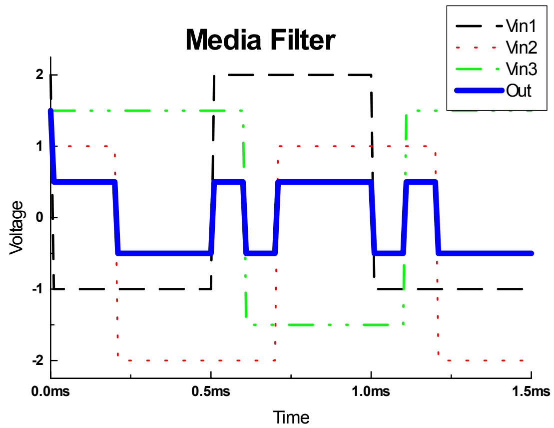

Fig. 6. The Media Filter.

The cases that follows, are obtained by looking at equation (9), Minimum Minimorum values are obtained when no limit is imposed to the positive parts of the signal, this is shown in Fig. 7, Maximum Maximorum values are the opposite case, results are shown in Fig.8. Spice files of these simulations can be acquired with the authors.

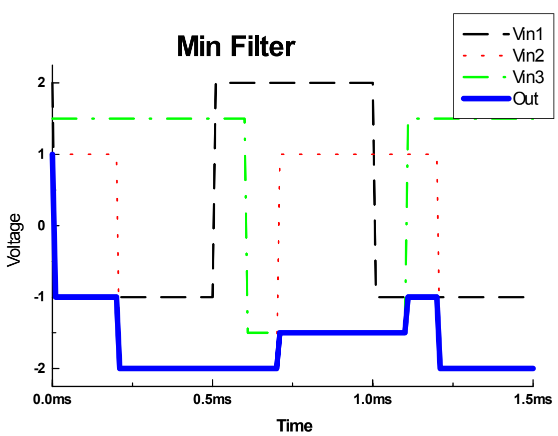

Fig.7 The Minimum Filter.

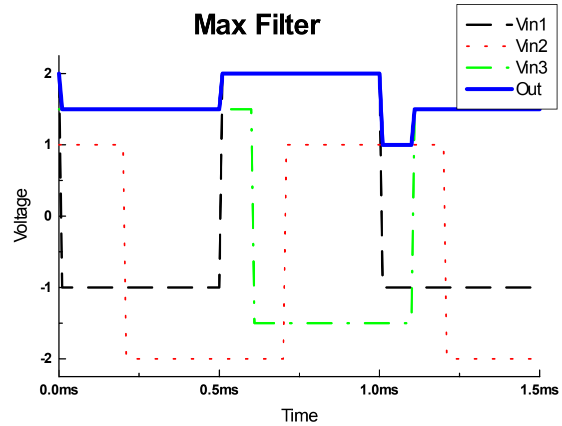

Fig.8 The Maximum Filter.

This ideal macromodel is the basis for a design implementation [11], it is advisable for other applications to handle this macromodel and its variables prior to the implementation process. Although the ideas are general, because of the non linear essence of the design it is recommended an iterative process between this macromodel and real design parameters. Other aspects must be considered, but they can be macro modeled in each block, i.e. finite gain of each OTA; parasitic and associated capacitances in order to check the frequency response; load effects etc.

Some comments and comparisons of this design and reported ones are presented. Digital Median filters get the Median value using numerical algorithms. Three different algorithms to get the Median are discussed in reference [3], a recursive algorithm is discussed in reference [5], although they are software solutions, implicit associated hardware is needed. They need memory, an ALU, A/D converters for the input signal and D/A converters at the output.
Analog reported ones in general do not make differences between the media or median value [6,8], in [7] they get the Mean value and only in the limit the Median, in [11] a current mode is presented. There are no reported articles with a multifunction non linear filter function, The mathematical justification presented in this article, simple and concrete was not found in any of the reviewed articles.

3 CONCLUSIONS

A multifunction analog non linear filter was designed, based on Lee and Kassam's algorithm. An ideal macromodel simulates the algorithm. It performs a Median, Media, Minimum or Maximum Filter. From the model the control variables were determined and tradeoffs between them have to be made to optimize these non linear filters. The value of the three variables, loco citato, must be readapted for each case. Spice simulated results are presented. This analog processor can be used in quality control applications, because of its statistical properties and real time performance.

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