On the Nature of the Logarithmic Term in Resistivity of One-Dimensional and Quasi-One-Dimensional Conductors

A. SERGEEV[1], V. MITIN[2], M. Yu. REIZER[3]
[1]Department of ECE
Wayne State University
Detroit, Michigan 48202
[2] Electrical Engineering Department
University at Buffalo
Buffalo, New York 14260
[3]5614 Naiche Rd. Columbus, Ohio 43213
THE UNITED STATES OF AMERICA

We investigate many-body corrections to the conductivity due to the interference of electron-electron scattering and elastic electron scattering from impurities and defects in weakly disordered conductors. In quasi-one-dimensional conductors with 3D and 2D electron spectra (a wire with radius $r < L_T$ and a strip with width $b < L_T$, where $L_T = v_F/T$, v_F is the Fermi velocity) as well as in multichannel one-dimensional conductors, the temperature-dependent corrections are proportional to $\ln T$. The value and the sign of the corrections depend on the strength of the electron-electron interaction in the triplet channel. The results can explain the logarithmic term observed in multiwall nanotubes and quantum-dot molecules.

Key-Words: - interference, scattering mechanisms, many-body effects, resistivity

1 Introduction

Electron-electron interaction determines transport properties of low-dimensional conductors at low temperatures. Corresponding many-body corrections to conductivity have various temperature dependencies. The logarithmic term is widely observed and well studied in two-dimensional conductors in the diffusion limit [1]. Surprisingly, the logarithmic temperature dependence has been observed in conductivity of multiwall nanotubes [2] and quantum-dot molecules with high mobility [3].

To understand temperature-dependent conductivity of low-dimensional conductors with high mobility, we investigate effects of the electron-electron interaction in the quasi-ballistic limit, $T\tau \gg 1$, τ is the electron momentum relaxation time. In weakly disordered conductors the interference corrections are always proportional to the Drude conductivity.

The electron-phonon interaction in the quasi-ballistic limit was theoretically studied in our paper [4]. It was found that the corresponding correction to conductivity is quadratic in the electron temperature,

$$\frac{\delta_{eph}\sigma}{\sigma_3} = \left[1 - \frac{\pi^2}{16} - 2\left(\frac{u_l}{u_t}\right)^3\right] \frac{2\pi^2\beta_l T^2}{3\epsilon_F p_F u_l}, \qquad (1)$$

where $\sigma_n = e^2 v_F^2 \tau \nu_n / n$ is the Drude conductivity in corresponding dimensionality n, ϵ_F and p_F are the Fermi energy and momentum, u_l and u_t are the longitudinal and transverse sound velocities, β_l is the constant of the electron-phonon interaction, and ν_n is the electron density of states. It is interesting that the longitudinal phonons give rise to a positive correction to conductivity, while transverse phonons result in a negative correction, which dominates in the temperature-dependent conductivity due to stronger coupling of transverse phonons. This T^2-term proportional to the Drude conductivity have been observed in a wide temperature range, from 20K up to 200K, in Nb, Al, Be [5], NbC [6], NbN [7], and W [8] films.

Effects of the electron-electron interaction in the quasi-ballistic limit have been studying for years [9, 10, 11, 12, 13]. After series of improvements, all exchange (Fock) and direct (Hartree) processes have been taken into account in the frame of the Landau Fermi-liquid theory in the paper [13]. In this approach numerous scattering processes are reduced to the effective interaction in the singlet and triplet channels. Both singlet and triplet channels give corrections to conductivity,

$$\delta_{ee}\sigma = \delta^s_{ee}\sigma + \delta^t_{ee}\sigma. \quad (2)$$

In the quasiballistic limit, $T\tau > 1$, the correction to conductivity is

$$\frac{\delta_{ee}\sigma}{\sigma_2} = \left(1 + \frac{3F_0^\sigma}{1+F_0^\sigma}\right)\frac{T}{\epsilon_F}, \quad (3)$$

where F_0^σ is the Fermi-liquid parameter describing interaction in the triplet channel. Results of recent measurements in GaAs/GaAsAl heterostructures [14, 15] and Si MOSFETs [16, 17] have shown good agreement with the theory [13] at subkelvin (GaAs/GaAlAs) and helium (Si) temperatures.

We would like to stress, that the many-body corrections due to the electron-phonon or interelectron interactions (interference corrections) always originate from the *elastic part* of the corresponding collision integral.[1, 4, 13] Therefore, the interference corrections depend on the electron temperature only. Early theoretical papers on the electron-phonon-impurity interference considered inelastic scattering from vibrating impurities and extracted the T^2-term to conductivity from the inelastic part of the collision integral [18]. However, as it is shown in our previous work, [4] such terms cancel out and this is a reason why the T^2-term is independent on the phonon temperature.

In the current paper we calculate many-body corrections to conductivity in one-dimensional and quasi-one-dimensional conductors. In Sec. II we start with the basic equations describing interference phenomena in the electron transport. In Sec. III we calculate the electron-electron corrections to the conductivity in various dimensions with respect to the effective interaction. The cross-over to the lower dimensionality occurs when one of the conductor dimensions becomes smaller than q_c^{-1}, where q_c is the characteristic value of the transferred electron momentum. For the electron-electron interaction in weakly disordered conductors, q_c^{-1} is of the order of $L_T = v_F/T$. At sub-Kelvin and helium temperatures, $L_T \sim 1 - 10\mu m$, and the transition to the quasi-one-dimensional case occurs in wires of radius $r \sim L_T$ and in 2D conducting channels of width $b \sim L_T$. We will show that the interference corrections to the conductivity is mainly determined by the sample dimensionality with respect to the effective interaction. Dimensionality of the electron spectrum just slightly changes numerical coefficients of the interference corrections.

2 Basic Formalism

Effects of interference between scattering mechanisms on the electron transport can be studied by the linear response method as well as by the quantum transport equation. Both methods are based on the digrammatic technique. The linear response method requires many diagrams to be considered, while the transport equation deals only with the electron self-energy diagrams but includes specific terms in the form of Poisson brackets. [4, 10]

In this paper we investigate the interference electron processes, which are characterized by the momentum transfer much smaller than the Fermi momentum. These processes can be described in the frame of the Landau Fermi-liquid theory. The corresponding self-energy diagrams for weakly disordered systems are shown in Fig. 1 and the diagrams of the linear response method are presented in Fig. 2. Results of the papers, [4, 10, 13] show that in the quasiballistic limit the correction to conductivity of one-dimensional and quasi-one-dimensional conductors may be presented as

$$\frac{\delta_{int}\sigma}{\sigma_n} = 2\int \frac{d\omega}{2\pi}\frac{dq}{(2\pi)}\,\frac{f(\omega)}{\omega^2}\,\Im\Upsilon_n^{(k)}(q,\omega), \quad (4)$$

where n is used for the dimensionality of the electron spectrum.

The function $f(\omega)$ is given by

$$f(\omega) = \frac{\partial}{\partial\omega}\left[\omega\coth\left(\frac{\omega}{2T}\right)\right]. \quad (5)$$

The function $\Upsilon(q,\omega)$ is given by

$$\Upsilon_n(q,\omega) = (\omega\tau)^2\ V_n^R(\mathbf{q},\omega)\Phi_n(q,\omega), \quad (6)$$

where $V_n^R(\mathbf{q},\omega)$ is the retarded propagator describing electron-electron interaction, and $\Phi(q,\omega)$ is given by

$$\frac{(v_F\tau)^2}{n}\,\Phi_n = -\left\langle\left\langle\frac{(\mathbf{v}_F\mathbf{e})^2}{(\mathbf{q}\mathbf{v}_F-\omega-i0)^2}\right\rangle_{\mathbf{v}}\right\rangle_{\mathbf{q}} + \left\langle\left\langle\frac{(\mathbf{v}_F\mathbf{e})^2}{\mathbf{q}\mathbf{v}_F-\omega-i0}\right\rangle_{\mathbf{v}}\left\langle\frac{1}{\mathbf{q}\mathbf{v}_F-\omega-i0}\right\rangle_{\mathbf{v}}\right\rangle_{\mathbf{q}} - \left\langle\left\langle\frac{\mathbf{v}_F\mathbf{e}}{\mathbf{q}\mathbf{v}_F-\omega-i0}\right\rangle_{\mathbf{v}}^2\right\rangle_{\mathbf{q}}. \quad (7)$$

In Eq. 7, $\mathbf{e}=\mathbf{E}/E$ is the unit vector in the direction of the electric field, and $\langle\rangle_{\mathbf{v}(\mathbf{q})}$ stands for the averaging over the directions of $\mathbf{v}_F$ and $\mathbf{q}$. Note, that the averaging over the angle ϕ, the angle between $\mathbf{p}$ and $\mathbf{q}$, is given by

$$\langle\psi(\mathbf{q}\mathbf{v}_F)\rangle_\phi = \begin{cases} \int_{-1}^{1}\psi(qv_Fx)\,\frac{dx}{2}, & \text{3D;}\\ \int_0^{2\pi}\psi(qv_F\cos\phi)\,\frac{d\phi}{2\pi} & \text{2D;}\\ \frac{1}{2}\sum_{x=\pm1}\psi(qv_Fx) & \text{1D,}\end{cases} \quad (8)$$

where $x=\cos(\phi)$.

In the next sections the above equations are used to calculate the quantum corrections to conductivity due to electron-electron interaction in weakly disordered one-dimensional and quasi-one-dimensional conductors with 2D and 3D spectrum.

3 Corrections to Conductivity

As we discussed in the introduction, the effective electron-electron interaction in weakly disordered conductors is characterized by the momentum transfer of the order of T/v_F, which is much smaller than the Fermi momentum (see also calculations below). Therefore, the electron transport can be described in the frame of the Landau Fermi-liquid theory.[13] In the singlet channel the bare interaction is given the sum of the Coulomb potential,

$$V_0(q) = \begin{cases} \frac{4\pi e^2}{q^2}, & \text{3D;}\\ \frac{2\pi e^2}{|q|}, & \text{2D;}\\ e^2\ln\frac{1}{q^2r^2}, & \text{1D,}\end{cases} \quad (9)$$

and the Fermi-liquid interaction,

$$V_F \approx -F_0^p/\nu_n. \quad (10)$$

The screened interaction in the random phase approximation, which is justified for small momentum transfers, is given by

$$V_n^{R(A)}(q,\omega) = \frac{\nu_nV_0(q)-F_0^p}{\nu_n-\left[\nu_nV_0(q)-F_0^p\right]P_n^{R(A)}(q,\omega)}, \quad (11)$$

where $P^{R(A)}(q,\omega)$ is the polarization operator.

In the absence of the magnetic field and spin-orbit scattering the screened propagator in the triplet-channel may be taken in the form [13]

$$V_n^{R(A)}(\mathbf{q},\omega) = -\frac{3F_0^\sigma}{\nu_n-F_0^\sigma P_n^{R(A)}(\mathbf{q},\omega)}, \quad (12)$$

where F_0^σ is the Fermi-liquid constant. The above equation assumes that the Fermi-liquid coupling is independent on electron momenta. Restrictions of this approximation were discussed in the paper[13].

3.1 Singlet Channel

First we consider a conductor with three-dimensional electron spectrum. For $1/\tau \ll \omega \le qv_F \ll \epsilon_F$, the polarization operator is given by

$$P_3^R(q,\omega) = -\nu_3\left[1-\frac{\omega}{qv_F}\,\text{arctanh}\left(\frac{qv_F}{\omega+i0}\right)\right], \quad (13)$$

where the branch of $\text{arctanh}(y)$ is chosen as

$$\text{arctanh}(y) = -\frac{\pi i}{2}+\frac{1}{2}\,\ln\frac{y+1}{y-1}, \quad y>1. \quad (14)$$

Thus, the screened Coulomb potential may be presented as

$$V_3^R(q,\omega) = \frac{\frac{1}{\nu_3}\left(1-F_0^p\frac{q^2}{\kappa_3^2}\right)}{\frac{q^2}{\kappa_3^2}+\left(1-F_0^p\frac{q^2}{\kappa_3^2}\right)\left[1-\frac{\text{arctanh}(qv_F/\omega)}{qv_F/\omega}\right]}, \quad (15)$$

where $\kappa_3^2=4\pi e^2\nu_3$, and $\nu_3=mp_F/\pi^2$. In the limit of strong screening, $\kappa_3 \gg q$, the screened potential is independent on the form of the bare potential (the unitary limit).

For quasi-one-dimensional conductors, such as wires with radius r, which is much smaller than L_T, the vectors $\mathbf{q}$ and $\mathbf{e}$ are parallel. Averaging Eq. 7 over the angles of $\mathbf{q}$ we get

$$\Phi_3(q,\omega) = -\frac{3}{\tau^2}\Bigg[\left\langle\frac{x^2}{(qv_F x-\omega-i0)^2}\right\rangle_\phi - \left\langle\frac{x^2}{qv_F x-\omega-i/\tau}\right\rangle_\phi \cdot \left\langle\frac{1}{qv_F x-\omega-i0}\right\rangle_\phi\Bigg]. \quad (16)$$

In the limit of strong screening, the function $\Upsilon(qv_F/\omega)$ (Eq. 6), is given by

$$\nu_3\Upsilon_3^{(1)}(y) = \frac{3}{y^2}\left(1-\frac{\text{arctanh } y}{y}\right)^{-1} \times\left[\frac{2-2y^2}{y^2-1}+\frac{\text{arctanh } y}{y}+\left(\frac{\text{arctanh } y}{y}\right)^2\right]. \quad (17)$$

Then, the correction to the conductivity (Eq. 4) is

$$\frac{\delta_{ee}^s\sigma}{\sigma_3} = -\frac{1}{\pi^3 r^2 v_F\nu_3}\,C_1\int_0^{\epsilon_F} d\omega\,\frac{f(\omega)}{\omega}, \quad (18)$$

where

$$C_1 = -\int_1^\infty dy\; y\;\nu_3\;\Im\Upsilon_3^{(1)}(y)\approx 4.3 \quad (19)$$

Taking into account that

$$\int_0^{\epsilon_F} d\omega\,\frac{f(\omega)}{\omega} = \ln\left(\frac{\epsilon_F}{2T}\right), \quad (20)$$

finally we get

$$\frac{\delta_{ee}^s\sigma}{\sigma_3} = \frac{C_1}{\pi(rp_F)^2}\;\ln\left(\frac{2T}{\epsilon_F}\right). \quad (21)$$

Next we investigate the many-body correction in conductors with two-dimensional electron spectra. For 2D electron gas the polarization operator in the quasi-ballistic limit is

$$P_2^R(q,\omega) = = -\nu_2\left(1-\frac{\omega}{\sqrt{(\omega+i0)^2-(qv_F)^2}}\right), \quad (22)$$

where $\nu_2 = m/\pi$.

We calculate conductivity in the quasi-one- dimensional conductor, such as a narrow channel with width $b < L_T$. Taking into account that in the quasi-one-dimensional case the vectors $\mathbf{q}$ and $\mathbf{E}$ are parallel and averaging Eq. 7 over the angles of $\mathbf{q}$, we get

$$\tau^2\Phi_2(q,\omega)/2 = \left\langle\frac{(\cos\phi)^2}{(qv_F\cos\phi-\omega-i0)^2}\right\rangle_\phi - \left\langle\frac{(\cos\phi)^2}{qv_F\cos\phi-\omega-i/\tau}\right\rangle_\phi \times\left\langle\frac{1}{qv_F\cos\phi-\omega-i0}\right\rangle_\phi. \quad (23)$$

After averaging over the angle ϕ, we find

$$\Phi_2(q,\omega) = \frac{2}{\tau^2}\frac{1}{(\omega+i0)^2-(qv)^2} \times \left[1-\frac{\omega}{\sqrt{(\omega+i0)^2-(qv)^2}}\right]. \quad (24)$$

Therefore, in the unitary limit, the function $\Upsilon(qv_F/\omega)$ (Eq. 6), is given by

$$\nu_2\Upsilon_2(y) = \frac{2}{1-(y-i0)^2}. \quad (25)$$

Calculating the correction to the conductivity in the singlet channel,

$$\frac{\delta_{ee}^s\sigma}{\sigma_2} = \frac{1}{\pi^2\nu_2 v_F b}\int_0^{\epsilon_F} d\omega\,\frac{f(\omega)}{\omega}\int_0^\infty dy\;\Im\Upsilon_2^{(1)}(y),$$

we get

$$\frac{\delta_{ee}^s\sigma}{\sigma_2} = \frac{1}{p_F b}\;\ln\left(\frac{2T}{\epsilon_F}\right). \quad (26)$$

Finally, we study a multichannel conductor consisting of few one-dimensional wires. In this model, electrons can scatter from one wire to another due impurities and defects. For 1D electrons the polarization operator in the quasi-ballistic limit is

$$P_1^R(q,\omega) = \nu_1\,\frac{(qv_F)^2}{(\omega+i0)^2-(qv_F)^2}, \quad (27)$$

where $\nu_1 = 2/(\pi v_F)$. Averaging Eq. 7 in the one-dimensional geometry we find

$$\tau^2\Phi_1(q,\omega) = -\frac{2(qv_F)^2}{(\omega+i0)^2-(qv_F)^2}. \quad (28)$$

Then, in the unitary limit, the function $\Upsilon(qv_F/\omega)$ (Eq. 6), is given by

$$\Upsilon_1(y) = \frac{2y^2}{\nu_1}\,\frac{1}{1-(y-i0)^2}. \tag{29}$$

Using Eq. 4, we get

$$\frac{\delta^s_{ee}\sigma}{\sigma_1} = \frac{1}{4}\,\ln\left(\frac{2T}{\epsilon_F}\right). \tag{30}$$

Thus, in quasi-one-dimensional conductors with 3D electron spectra (Eq. 21) and 2D spectra (Eq. 26) as well as in multichannel one-dimensional conductors (Eq. 30), the corrections have the logarithmic temperature dependence.

3.2 Triplet Channel

Conductivity corrections in the triplet channel are calculated in the same way as the singlet-channel corrections. First, we consider the quasi-one- dimensional conductor carved from the two-dimensional structure. With the triplet channel interaction (Eq. 12), the function $\Upsilon(y)$ is given by

$$\Upsilon_2^{(1)}(y) = -\frac{1}{\nu_2}\,\frac{3F_0^\sigma}{1+F_0^\sigma}\,\frac{2}{1-(y-i0)^2} \times\left(1-\frac{1}{(1+F_0^\sigma)\sqrt{1-y^2-F_0^\sigma}}\right). \tag{31}$$

Integrating Eq. 26, we get

$$\frac{\delta^t_{ee}\sigma^{(1)}}{\sigma_2} = \frac{3\Big(F_0^\sigma+\pi G(F_0^\sigma)\Big)}{1+F_0^\sigma}\,\frac{1}{p_F b}\,\ln\left(\frac{2T}{\epsilon_F}\right), \tag{32}$$

where

$$G(F_0^\sigma) = -1-\frac{2}{\pi}\frac{|\,1+F_0^\sigma\,|}{\sqrt{|\,1+2F_0^\sigma\,|}} \times\begin{cases}\operatorname{arctanh}\dfrac{\sqrt{-1-2F_0^\sigma}}{F_0^\sigma}, & F_0^\sigma<-\frac{1}{2},\\ \arctan\dfrac{\sqrt{1+2F_0^\sigma}}{F_0^\sigma}, & -\frac{1}{2}<F_0^\sigma<0,\\ \arctan\dfrac{\sqrt{1+2F_0^\sigma}}{F_0^\sigma}-\pi, & F_0^\sigma>0.\end{cases}$$

Next we consider the multichannel one-dimensional conductor. Using Eqs. 7 and 12 we find the function $\Upsilon(y)$,

$$\Upsilon_1(y) = \frac{6y^2}{\nu_1}\,\left[\frac{1}{1-(y-i0)^2} -\frac{1}{1-(y-i0)^2(1-F_0^\sigma)}\right]. \tag{33}$$

Finally, substituting $\Upsilon_1(y)$ in Eq. 4 and integrating it, we get

$$\frac{\delta^t_{ee}\sigma}{\sigma_1} = \frac{3}{4}\left[1-\frac{1}{\sqrt{1-F_0^\sigma}}\right]\ln\left(\frac{2T}{\epsilon_F}\right). \tag{34}$$

Thus, the temperature dependence of the conductivity corrections in the triplet channel (Eqs. 32 and 34) is the same as in the singlet channel, but the value and sign depend on the parameter F_0^σ.

4 Conclusions

In this work we investigated the interference of electron-electron scattering and elastic electron scattering from impurities and defects in weakly disordered one-dimensional and quasi-one-dimensional conductors. We have calculated the many-body corrections to the conductivity and demonstrated that even weak disorder significantly modifies its temperature dependence.

In weakly disordered conductors, characteristic momentum transfers are of the order of T/v_F, which is significantly smaller than the Fermi momentum. Therefore, the Landau Fermi-liquid theory is applicable and all processes with the large momentum transfer are taken into account by the effective Fermi-liquid constants. Due to the Coulomb potential divergence at small momenta, the singlet-channel interaction corresponds to the unitary limit and corresponding corrections are independent on the Fermi-liquid parameters. Our main results for the singlet channel are presented by Eqs. 21, 26, 30. We found that in quasi-one-dimensional conductors with 3D electron spectra (Eq. 21) and 2D spectra (Eq. 26) as well as in multichannel one-dimensional conductors (Eq. 30), the corrections have the logarithmic temperature dependence. The triplet-channel corrections (Eqs. 32 and 34) have the same temperature dependence as the singlet-channel corrections. Contrary to the singlet channel, the triplet channel corrections are not universal. Therefore, the value and sign of the total correction depend on the Fermi-liquid parameter F_0^σ in the triplet channel. In the weak coupling limit, $|F_0^\sigma| << 1$, the singlet-channel dominates over the triplet one and the corrections to conductivity are positive. Negative values of F_0^σ may result in the negative total correction, which is observed in heterostructures [16].

Note, that at sub-Kelvin temperatures the characteristic length, $d_c = v_F/T$, is of the order of 1 - 10 μm. Therefore, experiments with wires and channels of μm-sizes would allow to observe crossovers to lower dimensions. Note, that the logarithmic term has been recently observed in arrays of open quantum dots of μm-sizes at sub-Kelvin temperatures [3]. The logarithmic temperature dependence is also often observed in multiwall carbon nanotubes [2]. These observations may be relevant to the interference corrections calculated in this paper.

The research was supported by the ONR grant. We would like to thank I. Aleiner, B. Narozhny, and J. Bird for useful discussions.

References

[1] B. L. Altshuler and A. G. Aronov, *Electron-Electron Interaction in Disordered Systems*, edited by A. L. Efros and M. Polak (North-Holand, Amsterdam, 1985).

[2] L.Forro and Ch. Schonenberger, *Carbon Nanotubes*, edited by M.S. Dresselhaus, G. dresselhaus, and Ph. Avouris (Springer, Berlin, NY, 2000).

[3] A. Shailos, J.P. Bird, C. Prasad et al, *Phys. Rev. B* Vol. 63, 2001, p. 241302(R).

[4] M. Yu. Reizer and A. V. Sergeev, *Zh. Eksp. Teor. Fiz.* Vol. 92, 1987, p. 2291 [*Sov. Phys. JETP* Vol. 65, 1987, p. 1291].

[5] N.G. Ptitsina, G.M. Chulkova, K.S. Il'in et. al, *Phys. Rev. B* Vol. 56, 1997, p. 10089.

[6] K.S. Il'in, N.G. Ptitsina, A.V. Sergeev et. al, *Phys. Rev. B* Vol. 57, 1998, p. 15623.

[7] A. Sergeev, B.S. Karasik, N.G. Ptitsina et. al, *Physica B* Vols. 263-264, 1999, p. 190.

[8] A. Stolovits, A. Sherman, T. Avarmaa, O. Meier, and M. Sisti, *Phys. Rev. B* Vol. 58, 1999, p. 11111.

[9] A. Gold and V.T. Dolgopolov, *Phys. Rev. B* Vol. 33, 1986, p. 1076.

[10] M. Reizer, *Phys. Rev. B* Vol. 57, 1998, 12338.

[11] F. Stern and S. Das Sarma, *Solid State Electron.* Vol. 28, 1985, p. 158.

[12] D.V. Khveshchenko and M. Reizer, cond-mat/9609174.

[13] G. Zala, B.N. Narozhny, and I.L. Aleiner, *Phys. Rev. B* Vol. 64, 2001, p. 214204.

[14] Y.Y. Proskuryakov, A.K. Savchenko, S.S. Safonov et al., *Phys. Rev. Lett.* Vol. 89, 2002, p. 076406.

[15] H. Noh, M.P. Lilly, D.C. Tsui, J.A. Simmons, E.H. Hwang, S. Das Sarma, L.N. Pfeifer, and K.W. West, cond-mat/0206519.

[16] V.M. Pudalov, E.M. Gershenson, A. Kojima, G. Brunthaler, A. Prinz, and Bauer, cond-mat/0205449.

[17] S.A. Vitkalov, K. James, B.N. Narozhny, M.P. Sarachik, and T.M. Klapwijk, cond-mat/0204566.

[18] S. Koshino, *Prog. Theor. Phys.* Vol. 24, 1960, p. 484, P.L. Taylor, *Phys. Rev. A* Vol. 135, 1964, p. 1333.