On Sliding Mode Control
For Nonlinear Electrical Systems

RUBÉN ROJAS, OSCAR CAMACHO, RAMÓN CÁCERES, ALFREDO CASTELLANO
Grupo de Investigación en Nuevas Estrategias de Control Aplicadas (GINECA).
Universidad de Los Andes. Mérida
VENEZUELA

Abstract: This article describes the synthesis of a sliding mode controller SMCr based on a second order linear model using an integral-differential surface of the tracking-error. Different from similar strategies, the tuning parameters keep a close relationship with the system dynamics in terms of conventional specifications of transient response. The proposed controller only needs the output feedback of system and could be satisfactorily used in control of single input-single output nonlinear electric systems such as electronic power converters with pulse width modulation.

Key-Words: Sliding mode, Nonlinear Control, Power Converter, Pulse Width Modulation.

1 Introduction

Numerous physical processes exhibit a behavior whose dynamics can be represented by a single input-single output (SISO), second order model. However, such approach discards process inherent non-linearities that sometimes would cause degradation of control when the conventional PID strategy is used. This becomes more evident when it is working in extreme conditions or in zones where the parametric uncertainty of the model becomes more accented.

In this framework, robustness proper of the sliding mode control strategy (SMC) should provide better performance than conventional control strategies. Properties such as order reduction and invariant dynamics of the system in the sliding mode have stimulated development of multiple procedures for the synthesis of controllers in a wide spectrum of applications. Such are the cases of processes with multi-input / multi-output configuration [1], with strong non linearities, with variable dead time or non minimum phase [2]. In addition, excellent results have been reported in control of electric motors and electronic power converters [3-4] where discontinuous action in variable structure control (VSC) is compliant with the nature of their elements.

The main drawback in VSC is the chattering generally associated with a high control activity that sometimes could not be tolerated by the system. It could excite high frequency unmodeled dynamics or decrease their efficiency [3,5]. This last aspect highlights the convenience of synthesizing the SMCr under an approximately continuous control law.

Based on the SMC robustness, this article shows the synthesis of a controller based on integral-differential surface of error and a continuous approximation of nonlinear part of the controller. The design is pointed to obtain a simple structure that only needs the output feedback making it attractive for control of nonlinear electric systems such as power converters with schemes of pulse width modulation (PWM).

2 System Modeling

In a sliding mode control strategy, the system dynamics is forced by the controller to stay confined in a subset of the state space denominated sliding surface, $\sigma(t)$. The system is directed toward and reaches the sliding surface at a finite time due to the control action. Finally, once the system dynamics reach the user-chosen sliding surface, it will behave according to that surface, which is of a lower order than that of the system and independent of the model parametric uncertainties [6].

Let M be the second order model of system (lower order approximation of the real system Q), with an output variable $\theta_1(t)$ that tracks the reference input $\theta_r(t)$ with a tracking error $e_1(t)$ under control law $U(t)$ of the SMCr . The dynamics of the controlled system M can be described by the following state space equation:

$$\begin{aligned} \dot{\theta}_1(t) &= \theta_2(t) \\ \dot{\theta}_2(t) &= -a_1\theta_1(t) - a_2\theta_2(t) + m(t) + bU(t) \\ y(t) &= \theta_1(t) \end{aligned} \tag{1}$$

where a_1, a_2, b are the parameters of the second order model approximation of the system and $m(t)$ represents the external disturbances. This equation can be rewritten in terms of the tracking error as follows:

$$\begin{aligned} \dot{e}_1(t) &= e_2(t) \\ \dot{e}_2(t) &= -a_1 e_1(t) - a_2 e_2(t) + f(t) - bU(t) \\ y(t) &= e_1(t) \end{aligned} \tag{2}$$

here, $f(t)$ contains the external disturbances $m(t)$, the reference signal $\theta_r(t)$ and its derivatives. Whenever the external disturbance acts in the same state space of the control $U(t)$, a control $U_f(t)$ will exist such that:

$$b\,U_f(t) = f(t) \tag{3}$$

and the sliding mode existence will be ensured if a known upper bound, the supreme $F(t)$, for $f(t)$ exists such that for any instant t:

$$f(t) < F(t) \tag{4}$$

3 Controller Synthesis

The desired performance of the system is usually described in terms of the transient response specifications. This performance must be satisfied in presence of disturbances and set point changes. The transient response specifications are thoroughly used in conjunction with PID control to regulate systems with dominant second order dynamics. This approach is used in this article to evaluate the system response.

The design problem consists first on choosing the sliding surface so that the system exhibits the desired dynamics defined by the given specifications; and second to guarantee the conditions so that the system reaches that surface at a finite time. There are many options to choose $\sigma(t)$; the Sliding Surface selected in this work is an integral-differential equation acting on the tracking-error [7], which is represented by the following expression:

$$\sigma(t) = \left(\frac{d}{dt} + \lambda\right)^n \int_0^t e(t)dt \tag{5}$$

The advantage of this surface is that it contains an integral term, which ensures the annulment of the steady state error. Then $\sigma(t)$ is a function of the error between the reference, $\theta_r(t)$, and the output, $\theta_l(t)$, values as was described in (2), such that:

$$\sigma(t) = \frac{de(t)}{dt} + 2\lambda_1\lambda_0 e(t) + \lambda_0^2 \int_0^t e(t)dt \tag{6}$$

represents the system dynamics in the sliding mode. The satisfaction of expression $\sigma(t) = 0$ means that in steady state the tracking-error of the system should decrease to zero with a dynamics that depends on the selection of the positive parameters λ_0 and λ_1.

For all SISO systems, the condition for existence of a sliding mode is satisfied if [6]:

$$\sigma(t)\,\dot{\sigma}(t) < 0\,. \tag{7}$$

Geometrically, this inequality means that the time derivatives of the state error vector always point toward the sliding surface when system is in reaching mode, and therefore, the system dynamics will approach to the surface dynamics in a finite time. To satisfy the given specifications and (6), an augmented equivalent control law [1] was used:

$$U(t) = Ueq(t) + U_N(t) \tag{8}$$

$Ueq(t)$, denominated equivalent control, represents the continuous part of the controller that maintains the output of the system restricted to the sliding surface. $U_N(t)$ is the nonlinear part of the controller that ensure the reach of the sliding mode and therefore, it should satisfy the inequality given in (7).

The continuous part $Ueq(t)$ could be determined supposing that in the instant t_0, the state trajectory of the system intercepts the surface and enters the sliding mode. The existence of sliding mode implies that (6) is satisfied, and also:

$$\dot{\sigma}(t) = \dot{e}_2(t) + 2\lambda_1\lambda_0 e_2(t) + \lambda_0^2 e_1(t) = 0 \;;\; \forall \;\; t \geq t_0 \tag{9}$$

Then the equivalent control $Ueq(t)$ is obtained from (9), after substituting the error derivatives by the state equations (2), when the system is not disturbed ($f(t) = 0$):

$$Ueq(t) = \frac{1}{b}\left[\left(\lambda_0^2 - a_1\right)e_1(t) + \left(2\lambda_0\lambda_1 - a_2\right)e_2(t)\right] \tag{10}$$

As can be appreciated in (10), $Ueq(t)$ evidence a proportional-derivative nature, attenuated by the static gain of the model.

Finally, the chattering problem could be solved satisfactorily if the control $U_N(t)$ is designed according to [8]:

$$U_N(t) = Kd\,\frac{\sigma(t)}{|\sigma(t)| + \delta} \quad ; \quad Kd, \delta > 0 \tag{11}$$

where *Kd* is a tuning parameter to assure the reach of the sliding surface and δ is set to obtain suppression of chattering. An estimate of the *Kd* parameter could be obtained from (3) and (4). The overall control law structure is shown in Fig. 1.

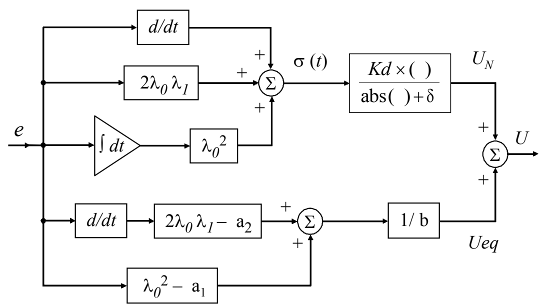

Fig. 1 Structure of the proposed SMCr

4 Controller Tuning

In sliding mode, system dynamics depend on the sliding surface σ(*t*), and therefore the desired transient response will depend on λ_0 and λ_1 parameters selection. Another interesting aspect is the strong relationship between the controller parameters and the system natural response. In (1), the system approximated by the second order model should contain the dominant poles of *Q*, so that an alternative representation of *M* could be obtained in terms of natural frequency ω_n and damping radio ξ from the original system at any operation point, such that:

$$a_1 = \omega_n^2 \tag{12}$$

$$a_2 = 2\xi\omega_n \tag{13}$$

From (10) it is evident that λ_0 is related with ω_n and λ_1 with ξ. As usual, the controlled system must respond faster than the open-loop system and therefore, it is desirable that $\lambda_0 > \omega_n$.

As it was previously shown [9], the percent overshoot only depends on λ_1, which provides a straight way to achieve the desired percent overshoot in system response. For instance, choosing $\lambda_1 > \xi$ is convenient if $\xi < 0.707$. Then, next step is to adjust λ_0 to provide the desired response in terms of settling time or peak time, which is totally determined according to the λ_1 range.

A reasonable value for *Kd* could be determined assuming that in the meantime of an external disturbance or when step change is introduced, the sliding surface value σ(*t*) >> δ. Therefore, the nonlinear part of the controller $U_N(t)$ approximates *Kd*.sign(σ(*t*)) and its magnitude will be *Kd*. In order to guarantee the reaching condition in any circumstance, from (3) and (4), it is enough that:

$$Kd > \left| \frac{1}{b} F(t) \right| \tag{14}$$

For systems with saturated single input, the supreme estimate *F*(*t*) in (4) is given by the control input bounds:

$$|f(t)| < |b(U_{max} - U_{min})| \tag{15}$$

because it is impossible to allow, without loss of stability, a disturbance in the system that requires a control magnitude larger than the range of values of *U*(*t*) for an indefinite period of time. Finally, the parameter δ (always positive) is adjusted to suppress the chattering.

5 Case Example: Buck-Boost Converter

To show the proposed controller performance and its parameter tuning simplicity an example of a SISO system, with moderated complexity, is presented in this section. Beginning with the original system state variables description and their inherent characteristic dynamics. Because the order of the original system is higher (grater than two) a second order model approximation around the operation point is determined for tuning the controller. Finally, the proposed control strategy is evaluated by simulation using the original state variable description in MATLAB®. The resulting response of the system with the controller set at the previous established tuning parameters is compared and contrasted with the predicted response given by the controller equations.

The DC-AC buck-boost converter is an inverter of commuted mode that works in for quadrants. It is characterized by its capacity to generate sinusoidal voltages that can be grater or lower than the DC supply voltage, depending on the duty cycle. Its main advantage is that it does not need a second power conversion stage [4].

The voltage control on this inverter topology is a troublesome task due to the inherent nonlinear behavior of the system and the sensitivity to abrupt load changes that eventually could drive it to instability, particularly when classical PID control methods are used [10]. Control is even more difficult because of the no minimum phase characteristic of the system.

The design and experimentation of a boost type similar converter show very satisfactory experimental results following a sliding mode control strategy with a frequency modulation scheme [4]. To reach the control goal, nevertheless, two controllers (one for each branch of the inverter) and four state variables feedback (two voltages and two currents for each branch), were needed. In the proposed control strategy, the control input is a continuous signal to be used beside a PWM auxiliary circuit with a fix commutation frequency.

DC-AC conversion is obtained using two DC-DC bidirectional buck-boost converters, modulating the output voltage in a sinusoidal way in counter-phase as it is shown by the basic electrical scheme of Fig. 2. Each converter generates a unipolar sinusoidal voltage with a DC component. This unipolar component of the voltage DC is suppressed by the differential connection of the load.

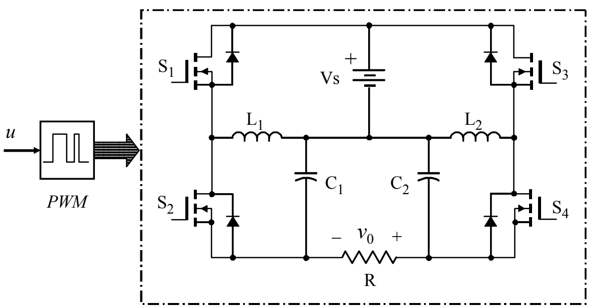

Fig. 2 Buck-boost DC-AC converter

The average state variable representation of the converter is shown in Fig. 3 according with the following relationships [11]:

$$\frac{d\,i_{L1}}{dt} = \frac{1}{L} v_{C1} + \left(\frac{Vs - v_{C1}}{L} \right) u$$

$$\frac{d\,v_{C1}}{dt} = \frac{1}{C} \left(\frac{v_{C2} - v_{C1}}{R} - i_{L1} \right) + \frac{i_{L1}}{C} u \qquad (16)$$

$$\frac{d\,i_{L2}}{dt} = \frac{1}{L} Vs - \left(\frac{Vs - v_{C2}}{L} \right) u$$

$$\frac{d\,v_{C2}}{dt} = \frac{1}{C} \left(\frac{v_{C1} - v_{C2}}{R} \right) - \frac{i_{L2}}{C} u \qquad (17)$$

$$v_0 = v_{C2} - v_{C1}$$

where i_{L1} and i_{L2} are the average currents in L1 and L2 inductors, respectively. v_{C1} are v_{C2} the average voltages through C1 and C2, respectively. Vs is the DC external voltage supply, R is the load connected to the converter. v_0 is the average voltage on the load. u is the average control input.

After replace each pair of commuters S1–S4, S2–S3 by the PWM switch the equivalent diagram of the inverter is shown in Fig. 4. During the time interval, DT, energy is storage on the L1 inductor simultaneously the storage energy L2 is transferred to the RC net. Similarly during the following interval, D'T, energy is transferred from the L1 inductor to the RC net while energy is storage on L2 inductor.

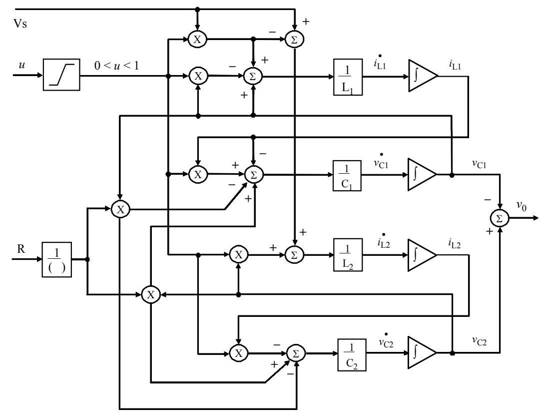

Fig. 3 Average state variable model of the converter

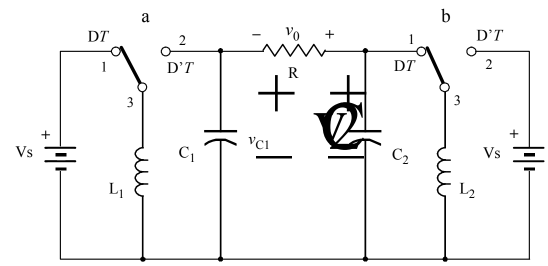

Fig. 4 Buck-boost converter equivalent Diagram

Assuming converter operates on the DC conduction mode during all the commutation period T and this commutation occurs on a sufficiently high frequency, the average model of the PWM switch is used to obtain the steady state and the fundamental frequency models, based to derived an approximated transfer function Q(s), of the BBC around the operation point D, [11]:

$$Q(s) = \frac{v_0(s)}{d(s)} = k \frac{z_3 s^3 + z_2 s^2 + z_1 s + z_0}{p_4 s^4 + p_3 s^3 + p_2 s^2 + p_1 s + p_0} \qquad (18)$$

where: $k = R$

$z_3 = -L^2 C\,(I_a + I_b)$

$z_2 = LC\,((1-D)\,V_a - D\,V_b)$

$z_1 = -L\,(D^2 I_a + (1-D)^2 I_b)$

$z_0 = D\,(1-D)\,(D\,V_a - (1-D)\,V_b)$

$p_4 = R\,(LC)^2$

$p_3 = 2\,L^2 C$

$p_2 = R\,L\,C\,(1-2D + 2D^2)$

$p_1 = L\,(1-2D + 2D^2)$

$p_0 = R\,(1\text{-}D)^2\,D^2$

Because D is cycling between 0.3 and 0.7, from (18) can be observed that the system presents minimum and non-minimum phase responses respect to the output voltage giving to the controller a challenger problem. Furthermore, the no linear characteristic of the static gain, cause the open loop output voltage distortion during a sinusoidal reference tracking as is shown in Fig. 5.

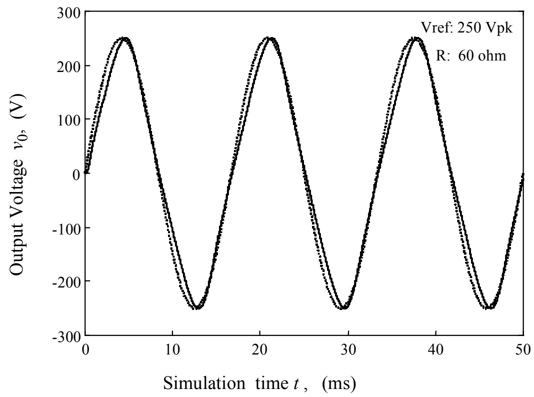

Fig. 5 Static gain effect on the average output voltage

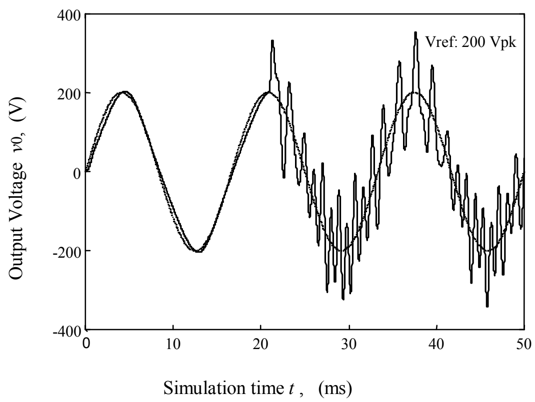

Fig. 6 Step load change converter response (from 60 to 6000 ohm)

Figure 6 shows the converter response against an abrupt change in load. Due to energy storage on the inductor and filters, the converter experiments severe variations in the loop gain when abrupt decrements in load occur. On the other hand, it is less sensitive to overloads.

5.1 Controller Tuning

The control aim is a 60 Hz sinusoidal reference tracking to generate output voltages from 0 to 200 volts approximately, using a supply source of 100 Vdc.

As was described in the controller synthesis procedure, first of all the system is approximated by a second order model that contains the dominant poles of $Q(s)$. The model parameters static gain, b, natural frequency, ω_n, and damping coefficient, ξ, can be estimated using linear systems simplification techniques such as the root locus. The converter design parameters, Q(s) coefficients and the parameters given by the root locus are summarized in Table 1.

Table 1 Converter, system $Q(s)$, and model $M(s)$, parameters.

Converter design parameters				
D	L1, L2	C1, C2	R	Vs
0.5	1 mH	10 μF	60 Ω	100 Vdc

Q(s) coefficients for Vo = 200Vdc (D=0.707)				
k	$z_3 \times 10^{-11}$	$z_2 \times 10^{-6}$	z_1 x 10^{-3}	z_0
60	−6. 67	2.00	−5.29	58.58
$p_4 \times 10^{-15}$	$p_3 \times 10^{-11}$	$p_2 \times 10^{-7}$	$p_1 \times 10^{-4}$	p_0
6.00	2.00	3.51	5.86	2.57

M (s) model parameters obtain by root locus	
Re (p)	895
Im (p)	2898
ξ	0. 295
ω_n	3033
K	1.256×10^{10}
a_1	9.199×10^{6}
a_2	1.789×10^{3}

Using the second order model parameters the tuning parameters range for λ_0 and λ_1 are defined as:

$$\lambda_0 > \omega_n = 3033 \qquad \lambda_1 > \xi = 0.295 \tag{19}$$

Considering the non-minimum phase characteristic (that constrain the system direct loop gain) and the calculated natural frequency, the procedure to follow is to choose initial moderated gains as λ_0 = 1.1(or 1.2)ω_n. Then, increase progressively the λ_1 parameter from a base value, equal to ξ, up to reach a value that guaranties a satisfactory reference tracking.

The gain of the controller nonlinear part, Kd, is obtained from the control input signal bounds (in this case: $0 < u < 1$). The sufficiency condition to reach the sliding surface is satisfied if:

$$Kd \geq 1 \tag{20}$$

6 Results

Using the average model, Fig. 3, the system was simulated using MATLAB® with a sampling period of 5 μs, following a 60 Hz sinusoidal reference of 200 volts.

The tuning parameters were set as follows: λ_0 = 3336 ($1.1\omega_n$); λ_1 = 0.443 (1.5ξ); Kd = 1; δ = 0.35; $\sigma_N = \sigma/\omega_n^2$ (sliding surface normalized respect to ω_n^2)

Figure 7 shows the system tracking response for a 200V amplitude, 60 Hz sinusoidal reference signal against abrupt load (from 60 to 6000 ohm) and DC voltage (from 100 to 90 V) changes at t = 21 ms, when the system is more sensitive to disturbances (on top of the sinusoidal wave approximately).

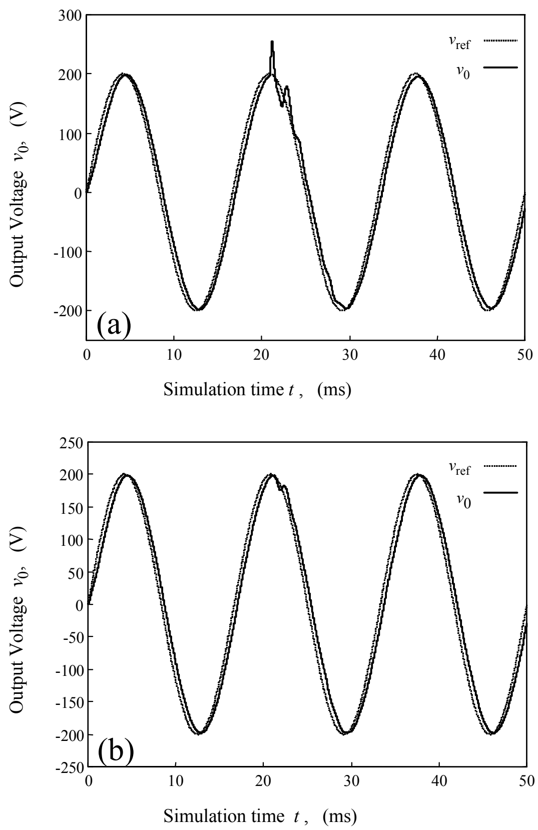

Fig. 7 System tracking response for a 60 Hz sinusoidal, reference signal of amplitude 200 V, when abrupt load and DC voltage changes were applied. (a) Step load change from 60 to 6000 ohm. (b) DC voltage change from 100 to 90 V.

7 Conclusions

The designed SMC controller, based on an integral-differential surface of the tracking-error, performed very well when reference and load step changes were introduced, with zero steady state error and without chattering.

Its main attribute is the close relationship between the controller tuning parameters and the desired closed-loop transient response. The possibility to obtain the desired percent overshoot unilaterally (adjusting λ_1) allows achieving demanding agreements between percent overshoot and settling time.

Deduced from a second order model of the system, the control law has a simple structure and it requires only an output feedback loop. This could be suitable in design of control schemes of minor complexity, especially for power converters with schemes of pulse width modulation (PWM), where the current feedback is usual in the control strategy.

References

[1] DeCarlo R., Zak S. and Matthews G., Variable structure control of nonlinear multivariable systems: A tutorial, *Proceedings of the IEEE*, Vol. 76, Nº 3, 1988, pp. 212-232.

[2] Camacho O. and Rojas R., A General Sliding Mode Controller For Non-Linear Chemical Processes. *ASME J. Dyn. Sys,. Meas., Control*, Vol. 122, 2000, pp. 650-655.

[3] Utkin, V.J., Sliding mode control design principles and applications to electric drives. *IEEE Transactions on Industrial Electronics*, Vol. 40, Nº 1, 1993, pp. 23-36.

[4] Cáceres R. and Barbi I. A boost dc-dc converter: Analysis, design and experimentation. *IEEE Transactions on Power Electronics*, Jan, 1999, pp.134-141.

[5] Escobar G., Ortega R., Sira-Ramirez H., Vilain J.P. and Zein I., An experimental comparison of several non linear controllers for power converters. *IEEE International Symposium on Industrial Electronics*, Portugal, 1997, pp. SS89-SS94.

[6] Utkin V.J., Variable structure systems with sliding modes. *IEEE Transactions on Automatic Control*, Vol. AC-22, 1977, pp. 212-222.

[7] Slotine J. J. and Li W. *Applied Nonlinear Control*. Prentice Hall, New Jersey, 1991.

[8] Zinober, A.S. *Variable Structure and Lyapunov Control*. Spring-Verlag, London, 1994.

[9] Cáceres R., Rojas R., and Camacho O., Robust PID control of a Buck-Boost DC-AC converter, *IEEE 22nd International Telecommunications Energy Conference*, USA, 2000, pp. SS89-SS94.

[10] Tymerski R., Vorpérian V., Lee F. and Baumann W. Nonlinear modelling of the PWM switch. *IEEE Power Electronics Specialist Conference*, Apr., 1988, pp. 968-976.

[11] Rojas R., Camacho O., Cáceres R., and Castellano A., A sliding mode control proposal for nonlinear electrical systems, *IFAC 15th World Congress*, Spain, 2002.