Main Properties Study of the Time-Optimal System Design Algorithm

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Abstract: **-** Some principal characteristics of the time-optimal system design algorithm were studied. An additional acceleration effect of the design process serves as the basis of the time-optimal algorithm construction. The acceleration effect and the special selection of the start point of the design process were defined as the principal ideas to construct of the optimal algorithm. The process of the optimal trajectory construction can be obtained on the basis of the control functions optimal selection. The optimal positions of the control function switching points for the time-optimal algorithm construction were found on the basis of the special Lyapunov function of the design process.

Key-Words: - Time-optimal design algorithm, control theory formulation, acceleration effect, Lyapunov function.

1 Introduction

The problem of the computer time reduction of a large system design is one of the essential problems of the total quality design improvement. Besides the traditionally used ideas of sparse matrix techniques and decomposition techniques some another ways were determine to reduce the total computer design time. The reformulation of the optimization process on heuristic level was proposed decades ago [1]. This process was named as generalized optimization and it consists of the Kirchhoff law ignoring for some parts of the system model. The special cost function is minimized instead of the circuit equation solve. This idea was developed in practical aspect for the microwave circuit optimization [2] and for the synthesis of high-performance analog circuits [3] in extremely case, when the total system model was eliminated. The last paper deals with homotopy how to tighten the dc constraints during the optimization process. Nevertheless all these ideas can be generalized to reduce the total computer design time for the system design. This generalization can be done on the basis of the control theory approach and includes the special control function to control the design process. This approach consists of the reformulation of the total design problem and generalization of it to obtain a set of different design strategies inside the same optimization procedure [4]. An additional acceleration effect [5] serves as the first principal component of the optimal algorithm construction. The second principal can be defined as the special start point selection [6] for design algorithm initialization. Nevertheless, the main problem of the time-optimal algorithm construction is the problem of the optimal switching point position for the control functions switching. This problem is discussed below on the basis of the ideas of the optimal control theory.

2 Problem Formulation

The design process for any analog system design can be defined [4] as the problem of the generalized objective function $F(X,U)$ minimization by means of the vector equation:

$$X^{s+1} = X^{s} + t_s \cdot H^{s} \qquad (1)$$

with the constraints:

$$\left(1-u_j\right) g_j(X) = 0, \qquad j = 1,2,\dots,M \qquad (2)$$

where $X \in R^N$, $X = (X', X'')$, $X' \in R^K$ is the vector of independent variables and the vector $X'' \in R^M$ is the vector of dependent variables ($N = K + M$), $g_j(X)$ for all j is the system model, s is the iterations number, t_s is the iteration parameter, $t_s \in R^1$, $H \equiv H(X,U)$ is the direction of the generalized objective function $F(X,U)$ decreasing, U is the vector of the special control functions $U = (u_1, u_2, ..., u_m)$, where $u_j \in \Omega$; $\Omega = \{0;1\}$. The generalized objective function $F(X,U)$ is defined as: $F(X,U) = C(X) + y(X,U)$ where $C(X)$ is the ordinary design process cost function, which achieves all design objects and $y(X,U)$ is the additional penalty function: $y(X,U) = \frac{1}{e} \sum_{j=1}^{M} u_j \cdot g_j^2(X)$. This problem formulation permits to redistribute the computer time expense between the problem (2) solve and the optimization procedure (1) for the function $F(X,U)$. The control vector *U* is the main tool for the redistribution process in this case. Practically an infinite number of the different design strategies are produced because the vector *U* depends on the optimization current step. The problem of the optimal design strategy search is formulated now as the typical problem for the functional minimization of the control theory. The functional that needs to minimize is the total CPU time *T* of the design process. This functional depends directly on the operation number and more generally on the design trajectory that has been realized. The main difficulty of this problem definition is unknown optimal dependencies of all control functions u_j. This problem is the central for such type of the design process definition.

3 Design Trajectory Subsets

The idea of the system design problem definition as the problem of the control theory does not have dependency from the optimization method (the function *H* form) and can be embedded into any optimization procedure. The numerical results for the different electronic circuits show [4] that the optimal control vector U_{opt} and the optimal trajectory X_{opt} exist and allow reducing the total computer time significantly. This optimal trajectory is differed from the traditional design strategy ($u_j = 0$, $\forall_{j=1,2,..M}$) and differed from the modified traditional design strategy ($u_j = 1$, $\forall_{j=1,2,..M}$), i.e. the idea, which was realized in [2] and [3] is not optimal from the computer time point of view. The main problem is to construct the optimal algorithm, which permits to realize all advantage of the optimal strategy. The analysis of the different electronic systems gives the possibility to conclude that the potential computer time gain of the time-optimal design strategy relatively the traditional strategy increases when the size and complexity of the system increase.

On the basis of the described methodology, by means of the start point of the vector *X* variation, an additional acceleration effect of the design process was discovered [5]. This effect appears for all analyzed circuits when al least one coordinate of the start point is negative and gives the possibility to reduce the total computer time additionally. This effect can serves as the basis for the optimal algorithm construction in case when the sequence of the switching points of the control functions u_j is founded. So, the main problem to construct the optimal algorithm is the problem of the optimal switching point of the control functions searching during the design process.

The analysis of some examples gives the possibility to conclude that all the trajectories that appear for the different control vector *U* can be separated in two subsets. In Fig. 1 there is a three-node circuit that has four admittances y_1, y_2, y_3, y_4 (*K*=4) and three nodal voltages V_1, V_2, V_3 (*M*=3). The nonlinear elements of the circuit have been defined by the following dependencies: $y_{n1} = a_{n1} + b_{n1} \cdot (V_1 - V_2)^2$, $y_{n2} = a_{n2} + b_{n2} \cdot (V_2 - V_3)^2$. The mathematical model (2) of this circuit includes now three equations and the optimization procedure (1) includes four equations. The one plane trajectory projections of the different design strategies, which correspond to the different control vector *U* are shown in Fig. 2. This projections correspond to the plane $y_4 - V_3$ and the points *S* and *F* correspond to the start and the final points of the design process. The complete basis of the different design strategies includes eight strategies.

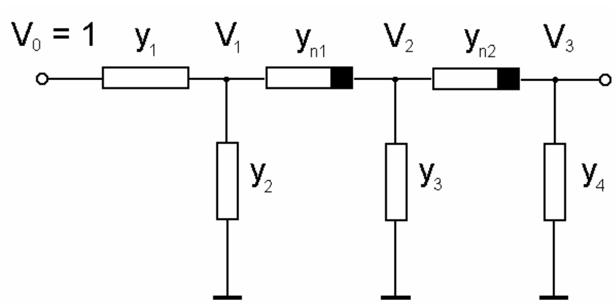


Fig. 1. Circuit with four independent (*K*=4) and three dependent (*M*=3) variables.

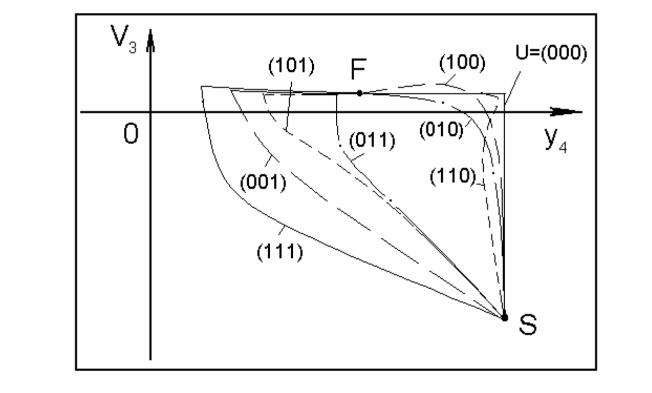


Fig. 2. $y_4 - V_3$ plane trajectory projections for different control vector *U*.

We can define the two subsets of the trajectories: 1) the trajectory projection, which corresponds to the traditional strategy *U*=(000) and the like traditional strategy projections (010), (100), (110) and 2) the trajectory projection, which corresponds to the modified traditional strategy (111) and the like modified traditional strategy projections (001), (011), (101). The main differences between two these groups are the different curve behavior and the different approach to the final point. The curves from two these groups draw to the finish point from the opposite directions. The time-optimal algorithm has includes one or some switching points where the switching is realized from the like modified traditional strategy to like traditional strategy with an additional adjusting. At least one negative component of the start value of the vector *X* is needed to realize the acceleration effect. In this case the optimal trajectory can be constructed.

The similar behavior of the complete basis of the different design strategies with the constant control vector is observed for all studied circuits. We can separate all the trajectories to two subsets. The first subset includes the traditional and like traditional design trajectories and the other one includes the modified and like modified traditional trajectories. We can conclude that the second groups trajectories can be serve as the first part for the optimal algorithm trajectory and the first group trajectories serve as the continuation. The next principal problem of the time-optimal algorithm construction is the unknown optimal position of the control function switching points that provide the minimal computer time. This problem is discussed in the next section.

4 Optimal Algorithm Structure Study

To obtain the optimal sequence of the switching points during the design process, we need to define a special criterion that permits to find the optimal control vector *U*. The problem of the minimal time strategy searching is connected with the more general problem of the stability of each trajectory. There is a well known idea to study of any dynamic process stability properties by means of the Lyapunov direct method. We have been defined the system design algorithm as the dynamic controllable process. In this case we can study the stability of each trajectory and the design process transit time properties on the basis of the Lyapunov direct method. We propose now to use a Lyapunov function of the design process for the optimal algorithm structure revelation, in particular for the optimal switching points searching. There is a freedom of the Lyapunov function choice because of a non-unique form of this function. Let us define the Lyapunov function of the design process (1)-(2) by the following expression:

$$V(X) = \sum_i (x_i - a_i)^2 \qquad (3)$$

where a_i is the stationary value of the coordinate x_i, in other words the set of all the coefficients a_i is the one of the objectives of the design process. Let us define other variables $y_i = x_i - a_i$. In this case the formula (3) can be rewritten as:

$$V(Y) = \sum_i y_i^2 \qquad (4)$$

The design process (1)-(2) can be rewritten by means of the variables y_i in the same form. The function (4) satisfies all of the conditions of the standard Lyapunov function definition. In fact the function *V(Y)* is the piecewise continue, and has piecewise-continue first partial derivatives. Besides there are three characteristics of this function: i) *V(Y) >0*, ii) *V(0)=0*, and iii) $V(Y) \to \infty$ when $\|Y\| \to \infty$. In this case we can discuss the stability of the zero point solution. On the other hand, the stability of the point $(a_1, a_2, ..., a_N)$ is analyzed by the definition (3). It is clear that the both problems are identical. Inconvenience of the formula (3) is an unknown point $(a_1, a_2, ..., a_N)$, because this point can be reached at the end of the design process only. We can analyze the stability of all different design strategies on the basis of the formula (3) if we already found the design solution someway. On the other hand, it is very important to control the stability process during the design procedure. In this case we need to construct other form of the Lyapunov function that doesn't depend on the unknown stationary point. Let us define the Lyapunov function by the next formula:

$$V(X,U) = \sum_i \left(\frac{\partial F(X,U)}{\partial x_i} \right)^2 \qquad (5)$$

where *F(X,U)* is the generalized objective function of the optimization procedure. This function has the same properties as the function (3) for the sufficiently large neighborhood of the stationary point. Really, all derivatives $\partial F / \partial x_i$ are equal to zero in the stationary point $a = (a_1, a_2, ..., a_N)$, so *V(a,U)=0*, on the other hand *V(X,U)>0* for all *X* and at last, the function *V(X)* of the formula (5) is the function of the vector *U* too, because all coordinates x_i are the functions of the control vector *U*. The property iii) is not proved only, because nobody know the function *V(X,U)* behavior when $\|X\| \to \infty$. However we can consider, from the practical experience, that the function *V(X,U)* increases in a sufficient large neighborhood of the stationary point. The direct calculation of the Lyapunov function time derivative gives the conditions of the process stability. The design process is stable if the Lyapunov function time derivative is negative. On the other hand, the direct method of Lyapunov gives the sufficient stability conditions but not necessary [7], so the process loses the stability (or not loses) if this derivative becomes positive. The stability of the different design strategies for three-transistor cells amplifier of Fig. 3 was analyzed by the Lyapunov direct method.

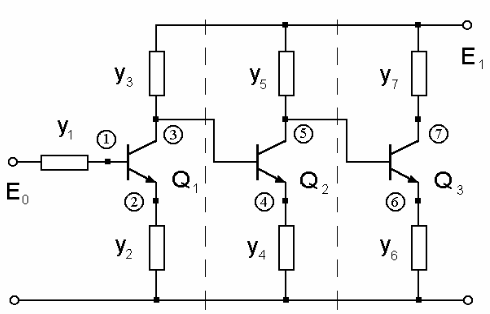


Fig. 3. Circuit topology for three-transistor cell amplifier.

The Lyapunov function time derivative *dV/dt* is negative for all trajectories on the initial part of the design process; i.e. all admissible strategies are stable at the beginning. It is supposed that the integration step is sufficiently small. However, when the current point of the trajectory gets to the e -neighborhood of the stationary point $(a_1, a_2, ..., a_N)$ some strategies can lose the stability because the Lyapunov function time derivative becomes positive. It means that all trajectories of this group do not guarantee the convergence from the e -neighborhood. In fact, each of the trajectory of this group has own critical e -neighborhood, which defines the maximum achievable precision (minimal achievable error). Another consideration is important too: the design process convergence slow down strongly before the e -neighborhood reaching for all strategies of this group. It means that the derivative *dV/dt* is the negative but very small on the absolute value. It is interesting that the traditional design strategy belongs to this group. The critical e values of some design trajectories for the circuit of the Fig. 3 and two types of the optimization procedure are shown in Table 1.

Table 1. Critical value of the e -neighborhood for some design strategies.

N	Control functions vector U (u1, u2, u3, u4, u5, u6, u7)	Critical epsilon neighborhood	
		Gradient method	DFP method
1	(0 0 0 0 0 0 0)	9.85E-11	9.76E-11
2	(0 0 0 0 0 0 1)	5.92E-06	6.25E-07
3	(1 0 0 0 0 0 0)	9.51E-07	9.35E-07
4	(0 1 1 0 0 0 0)	6.88E-12	5.33E-12
5	(0 1 1 0 1 0 0)	7.55E-15	4.17E-15
6	(1 1 1 1 1 0 1)	3.94E-17	3.53E-17
7	(1 1 1 1 1 1 0)	9.15E-16	6.65E-16
8	(1 1 1 1 1 1 1)	8.15E-17	4.74E-17

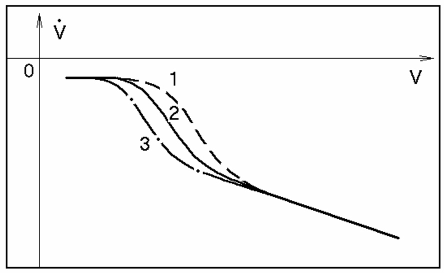
Three last strategies have the critical parameter e practically on the boundary of the reachable computer precision. We used the double length words for all numbers during the computing. At the same time these strategies are characterized of the negative values of the derivative *dV/dt* during the all design process. This property guarantees the process stability. On the other hand, the first five design strategies have the critical e -neighborhood, which depends on the intrinsic properties of the strategy. The derivative *dV/dt* is not negative when the current point approaches to the critical e -neighborhood for all of these strategies. It results to relative instability and slowing down the design process. We can conclude that all strategies of this group, including the traditional one, have the problem with the stability when the high precision is needed and therefore the total design time for these strategies is very large. On the other hand there is a group of the strategies (for example 6,7 and 8 of the Table 1) that don't lose the stability until practically any precision. The strategies of this group are characterized a large number of units in the corresponding control vector *U* and on the contrary, the strategies of the first group are characterized a large number of zeros as shown in Table 1. The time-optimal trajectory consists of the different design strategies in *N*-dimensional case, but it is very important that it includes strategies with the large number of units in the control vector on its final part. Therefore the time-optimal strategy has a very good stability and that's why this strategy is more rapid than any other is.

Now the function (5) is used for the analysis of the design trajectory behavior with the different switching points. We can define the system design process as a dynamic transition process that provides the stationary point during some time. The problem of the time-optimal design algorithm construction is the problem of the transition process searching with the minimal transition time. There is a well-known idea [7]-[8] to minimize the transition process time by means of the special choice of the right hand part of the principal system of equations, in our case the form of the vector function $H(X,U)$. By this conception it is necessary to change the functions $H(X,U)$ by means of the control vector *U* selection to obtain the maximum speed of the Lyapunov function decreasing (the maximum of *-dV/dt*) at each point of the process. Unfortunately the direct using of this idea does not serve well for the time-optimal design algorithm construction. It occurs because the change of the design strategy produces not only continuous design trajectories (when we change the strategy $u_j = 0, \forall_j$ to the strategy $u_j = 1, \forall_j$ for instance) but non-continuous trajectories too (in opposite case). Non-continues trajectories had never been appeared in control theory for the objects that are described by differential equations, but this is the ordinary case for the design process on the basis of the described design theory. In this case we need to correct the idea to maximize *-dV/dt* at each point of the design process. We define another principle: it is necessary to obtain the maximum speed of the Lyapunov function decreasing for that trajectory part which lies after the switching point. In this case the trajectories with the different switching points are compared to obtain the maximum value of *-dV/dt*. Technically this idea is realized by comparing some probes with the different switching points and selecting the one of them that provides the maximum of *-dV/dt* after the switching. Numerical results prove this idea. As shown in [6] the optimal design strategy for the circuit in Fig. 3 has the time gain near 600 comparing with the traditional design strategy, providing the optimal algorithm in reality. The behavior of the function *dV/dt* for this circuit for three neighbor switching points 1, 2 and 3 that correspond to the five consecutive integration steps before (a), (b) in (c) and after (d), (e) the optimal point is shown in Fig. 4. The optimal switching point corresponds to the curve 3 of Fig. 4 (b), or curve 2 of Fig. 4 (c), or curve 1 of Fig. 4 (d). It is clear that this point corresponds to the maximum negative value of function *dV/dt* and at the same time corresponds to the minimum value of the total design steps. In this case the optimal switching points are found and serve as the basis to the time-optimal algorithm construction.

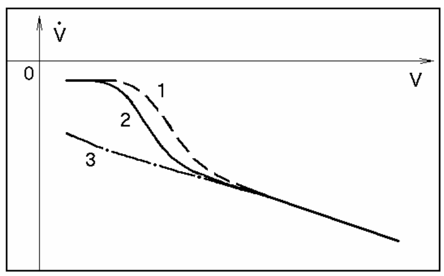
It is clear that we are forced to lose the computer time to do some probes and to look for the optimal position of the switching points. It means that we can never obtain the time gain, which characterizes the optimal strategy. The time loses can have the same order as the optimal algorithm computer time. So, the maximum time gain is equal 250-300 for the circuit in Fig. 3. This result worse than theoretic prediction, but this gain is significant too and the total design time reduction is the sufficient basis for the new design methodology development.

5 Conclusion

The problem of the time-optimal system design algorithm construction is solved more adequately as the functional optimization problem of the control theory. The three main components of the optimal algorithm construction can be picked out now: the additional acceleration effect of the system design process that appears when any quasi modified traditional design strategy is changed to any quasi traditional design strategy; the start point of the design process, which is selected with al least one negative component; and the optimal position of the necessary switching points that is defined by means of the careful current analysis of the time derivative of the special Lyapunov function of the system design process. These three ideas serve as the basis to the realistic time-optimal design algorithm construction.



(a)



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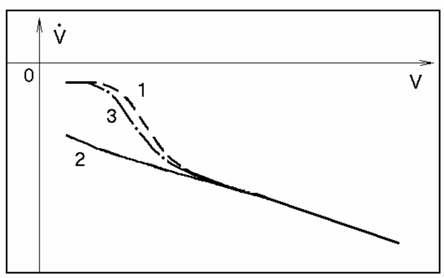

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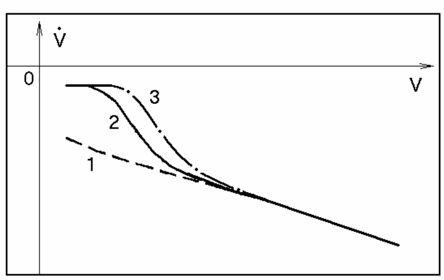
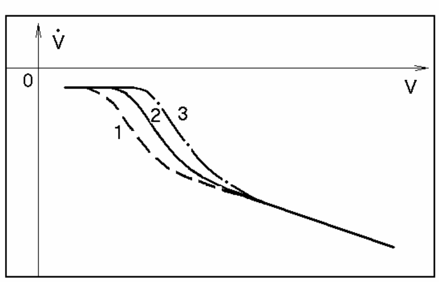


(d)



(e)

Fig. 4. Time derivative of Lyapunov function behavior for three switching points 1,2,3 consecutive integration steps before (a), (b) in (c) and after (d), (e) the optimal point.

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