

# Identification of Vehicle Engine at Idle Speed via Neural Networks

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## Abstract

In this paper, dynamic neural networks are used for engine model at idle speed on-line identification. Passivity approach is applied to access several stability properties of the neuro identifier. The conditions for passivity, stability, asymptotic stability and input-to-state stability are established. We conclude that the commonly-used backpropagation algorithm with a modification term which is determined by off-line learning may make the neuro identification algorithm robust stability with respect to any bounded uncertainty.

**Keywords:** system identification, dynamic neural networks, engine model at idle speed

## 1 Introduction

Many applications show that neuro identification has emerged as a effective tool for unknown nonlinear systems. This model-free approach uses the nice features of neural networks, but the lack of model makes it hard to obtain theoretical results on stability and performance of neuro identifiers. It is very important for engineers to assure the stability of neuro identifiers in theory before they apply them to real systems. Two kinds of stability for neuro identifiers have been studied. The stability of neuro identifier may be found in [17] and [20]. The stability of learning algorithms was discussed by [16] and [8]. We will emphasize this paper on deriving novel stable learning algorithms of the multilayer neuro identifier.

The global asymptotic stability (GAS) of dynamic neural networks has been developed during the last decade. Diagonal stability [6] and negative semi-definiteness [7] of the interconnection matrix may make Hopfield-Tank neuro circuit GAS. Multilayer perceptrons (MLP) and recurrent neural networks can be related to the Lur'e systems, the absolute stabilities were developed by [15] and [9]. Lyapunov-like analysis is suitable for dynamic

neural network, signal-layer networks were discussed in [12] and [19], high-order networks and multilayer networks could be found in [8] and [10]. Input-to-state stability (ISS) method [14] is another effective tool for dynamic neural networks. [13] concluded that recurrent neural networks are ISS and GAS with zero input if the weights are small enough.

The stability of learning algorithms can be derived by analyzing the identification or tracking errors of neural networks. [5] studied the stability conditions of the updating laws when multilayer perceptrons are used to identify and control a nonlinear system. In [16] the dynamic backpropagation was modified with NLq stability constraints. Since neural networks cannot match the unknown nonlinear systems exactly, some robust modifications [4] should be applied on normal gradient or backpropagation algorithm [5], [12], [15], [19].

Passivity approach may deal with the poorly defined nonlinear systems, usually by means of sector bounds, and offers elegant solutions for the proof of absolute stability. It can lead to general conclusion on the stability using only input-output characteristics. The passivity properties of static multilayer neural networks were examined in [2]. By means of analyzing the interconnected of error models, they derived the relationship between passivity and closed-loop stable. Passivity technique can be also applied on dynamic neural networks. Passivity properties of dynamic neural networks may be found in [20]. We concluded that the commonly-used learning algorithms with robust modifications such as dead-zone [5] and  $\sigma$ -modification [12] are not necessary.

In this paper, we will extend our prior results of single layer dynamic neural networks [20][?] to the multilayer case. To the best of our knowledge, open loop analysis based on the passivity method for multilayer dynamic neural networks has not yet been established in the literatures.

## 2 Neuro Identification via Passivity Technique

Consider a class of nonlinear systems described by

$$\begin{aligned} \dot{x}_t &= f(x_t, u_t) \\ y_t &= h(x_t, u_t) \end{aligned} \quad (1)$$

where  $x_t \in \mathfrak{R}^n$  is the state,  $u_t \in \mathfrak{R}^m$  is the input vector,  $y_t \in \mathfrak{R}^m$  is the output vector.  $f : \mathfrak{R}^n \times \mathfrak{R}^m \rightarrow \mathfrak{R}^n$  is locally Lipschitz,  $h : \mathfrak{R}^n \times \mathfrak{R}^m \rightarrow \mathfrak{R}^m$  is continuous. Following to [1], let us now recall some passivity properties as well as some stability properties of passive systems.

**Definition 1** A system (1) is said to be passive if there exists a  $C^r$  nonnegative function  $S(x_t) : \mathfrak{R}^n \rightarrow \mathfrak{R}$ , called storage function, such that, for all  $u_t$ , all initial conditions  $x^0$  and all  $t \geq 0$  the following inequality holds

$$\dot{S}(x_t) \leq u_t^T y_t - \varepsilon u_t^T u_t - \delta y_t^T y_t - \rho \psi(x_t)$$

where  $\varepsilon$ ,  $\delta$  and  $\rho$  are nonnegative constants,  $\psi(x_t)$  is positive semidefinite function of  $x_t$  such that  $\psi(0) = 0$ .  $\rho \psi(x_t)$  is called state dissipation rate.

Furthermore, the system is said to be *strictly passive* if there exists a positive definite function  $V(x_t) : \mathfrak{R}^n \rightarrow \mathfrak{R}$  such that  $\dot{S}(x_t) \leq u_t^T y_t - V(x_t)$

*Property 1.* If the storage function  $S(x_t)$  is differentiable and the dynamic system is passive, storage function  $S(x_t)$  satisfies  $\dot{S}(x_t) \leq u_t^T y_t$ .

**Definition 2** A system (1) is said to be globally input-to-state stability if there exists a  $\mathcal{K}$ -function  $\gamma(s)$  (continuous and strictly increasing  $\gamma(0) = 0$ ) and  $\mathcal{KL}$ -function  $\beta(s, t)$  ( $\mathcal{K}$ -function and for each fixed  $s_0 \geq 0$ ,  $\lim_{t \rightarrow \infty} \beta(s_0, t) = 0$ ), such that, for each  $u_t \in L_\infty$  ( $\|u(t)\|_\infty < \infty$ ) and each initial state  $x^0 \in R^n$ , it holds that

$$\|x(t, x^0, u_t)\| \leq \beta(\|x^0\|, t) + \gamma(\|u_t\|_\infty)$$

for each  $t \geq 0$ .

*Property 2.* If a system is input-to-state stability, the behavior of the system should remain bounded when its inputs are bounded.

We construct the following dynamic neural network:

$$\dot{\hat{x}}_t = A\hat{x}_t + W_{1,t}\sigma(V_{1,t}\hat{x}_t) + W_{2,t}\phi(V_{2,t}\hat{x}_t)\pi(u_t) \quad (2)$$

where  $\hat{x}_t \in \mathfrak{R}^n$  is the state of the neural network,  $A \in \mathfrak{R}^{n \times n}$  is a stable matrix.  $W_{1,t} \in \mathfrak{R}^{n \times m}$ ,  $W_{2,t} \in \mathfrak{R}^{n \times m}$  are weight matrices of output layers,  $V_{1,t} \in \mathfrak{R}^{m \times n}$ ,

$V_2 \in \mathfrak{R}^{m \times n}$  are weight matrices of hidden layers. The vector field  $\sigma(x_t) : \mathfrak{R}^n \rightarrow \mathfrak{R}^m$  is assumed to have the elements increasing monotonically. The function  $\phi(\cdot)$  is a map from  $\mathfrak{R}^m \rightarrow \mathfrak{R}^{m \times m}$ .  $\pi(u_t) \in \mathfrak{R}^m$ , is selected as saturation function:  $\|\pi(u_t)\|^2 \leq \bar{u}$ . The typical presentation of the elements  $\sigma_i(\cdot)$  and  $\phi_{ii}(\cdot)$  are as sigmoid functions

$$\sigma_i(x_{i,t}) = a_i / (1 + e^{-b_i x_{i,t}}) - c_i$$

**Remark 1** The neural networks have been discussed by many authors, for example [12], [8], [10] and [19]. It can be seen that Hopfield model [3] is the special case of this networks with  $A = \text{diag}\{a_i\}$ ,  $a_i := -1/R_i C_i$ ,  $R_i > 0$  and  $C_i > 0$ .  $R_i$  and  $C_i$  are the resistance and capacitance at the  $i$ th node of the network respectively.

Generally, the nonlinear system (1) may be represented as following

$$\dot{x}_t = Ax_t + W_1^* \sigma(V_1^0 x_t) + W_2^* \phi(V_2^0 x_t) \pi(u_t) - \tilde{f}_t(V_1^0, V_2^0) \quad (3)$$

where  $W_1^*$  and  $W_2^*$  are optimal matrix which may minimize modelling error  $\tilde{f}_t$ , they are bounded as

$$W_1^* \Lambda_1^{-1} W_1^{*T} \leq \bar{W}_1, \quad W_2^* \Lambda_2^{-1} W_2^{*T} \leq \bar{W}_2 \quad (4)$$

$V_1^0$  and  $V_2^0$  are prior given matrices which are obtained from off-line learning. Let us define identification error as  $\Delta_t = \hat{x}_t - x_t$ ,  $\tilde{\sigma}_t = \sigma(V_1^0 \hat{x}_t) - \sigma(V_1^0 x_t)$ ,  $\tilde{\phi}_t = \phi(V_2^0 \hat{x}_t) \pi(u_t) - \phi(V_2^0 x_t) \pi(u_t)$ ,  $\tilde{\sigma}'_t = \sigma(V_{1,t} \hat{x}_t) - \sigma(V_1^0 \hat{x}_t)$ ,  $\tilde{\phi}'_t = \phi(V_{2,t} \hat{x}_t) \pi(u_t) - \phi(V_2^0 \hat{x}_t) \pi(u_t)$ ,  $\tilde{V}_{1,t} = V_{1,t} - V_1^0$ ,  $\tilde{V}_{2,t} = V_{2,t} - V_2^0$ ,  $\tilde{W}_{1,t} = W_{1,t} - W_1^*$ ,  $\tilde{W}_{2,t} = W_{2,t} - W_2^*$ . Because  $\sigma(\cdot)$  and  $\phi(\cdot)$  are chosen as sigmoid functions, clearly they satisfy Lipschitz condition

$$\begin{aligned} \tilde{\sigma}_t^T \Lambda_1 \tilde{\sigma}_t &\leq \Delta_t^T \Lambda_\sigma \Delta_t, & \tilde{\phi}_t^T \Lambda_\phi \tilde{\phi}_t &\leq \bar{u} \Delta_t^T \Lambda_\phi \Delta_t \\ \tilde{\sigma}'_t &= D_\sigma \tilde{V}_{1,t} \hat{x}_t + \nu_\sigma, & \tilde{\phi}'_t &= D_\phi \tilde{V}_{2,t} \hat{x}_t + \nu_\phi \end{aligned} \quad (5)$$

where

$$\begin{aligned} D_\sigma &= \frac{\partial \sigma^T(Z)}{\partial Z} \Big|_{Z=V_{1,t} \hat{x}_t}, & \|\nu_\sigma\|_{\Lambda_1}^2 &\leq l_1 \left\| \tilde{V}_{1,t} \hat{x}_t \right\|_{\Lambda_1}^2 \\ D_\phi &= \frac{\partial [\phi(Z) \pi(u_t)]^T}{\partial Z} \Big|_{Z=V_{2,t} \hat{x}_t}, & \|\nu_\phi\|_{\Lambda_2}^2 &\leq l_2 \left\| \tilde{V}_{2,t} \hat{x}_t \right\|_{\Lambda_2}^2 \end{aligned}$$

$l_1 > 0$ ,  $l_2 > 0$ ,  $\Lambda_1$ ,  $\Lambda_2$ ,  $\Lambda_\sigma$  and  $\Lambda_\phi$  are positive definite matrices. The dynamic of identification error is obtained from (2) and (3)

$$\begin{aligned} \dot{\Delta}_t &= A\Delta_t + \tilde{W}_{1,t} \sigma(V_{1,t} \hat{x}_t) + \tilde{W}_{2,t} \phi(V_{2,t} \hat{x}_t) \pi(u_t) \\ &\quad + W_1^* \tilde{\sigma}_t + W_1^* \tilde{\sigma}'_t + W_2^* \tilde{\phi}_t + W_2^* \tilde{\phi}'_t + \tilde{f}_t(V_1^0, V_2^0) \end{aligned} \quad (6)$$

From [18] we know if matrices  $A$ ,  $R$  and  $Q$  satisfy the following conditions

(a) the pair  $(A, R^{1/2})$  is controllable, the pair  $(Q^{1/2}, A)$  is observable,

(b) local frequency condition

$$A^T R^{-1} A - Q \geq \frac{1}{4} [A^T R^{-1} - R^{-1} A] R [A^T R^{-1} - R^{-1} A]^T \quad (7)$$

the matrix Riccati equation

$$A^T P + PA + PRP + Q = 0 \quad (8)$$

has a positive solution  $P = P^T > 0$ .

If  $A$  is a stable diagonal matrix,  $R$  and  $Q$  are positive defined matrices, the conditions (a) and (b) are easy to be satisfied. Now we define

$$R = 2\overline{W}_1 + 2\overline{W}_2, \quad Q = \Lambda_\sigma + \overline{u}\Lambda_\phi + Q_0 \quad (9)$$

We assume the following assumption can be established.

**A1:** There exist a stable matrix  $A$  and a strictly positive defined matrix  $Q_0$  such that (8) has a positive solution.

Next theorem give a stable learning procedure of neuro identifier.

**Theorem 1** *If the weights  $W_{1,t}$ ,  $W_{2,t}$ ,  $V_{1,t}$  and  $V_{2,t}$  are updated as*

$$\begin{aligned} \dot{W}_{1,t} &= -K_1 P \sigma(V_{1,t} \hat{x}_t) \Delta_t^T + K_1 P D_\sigma \tilde{V}_{1,t} \hat{x}_t \Delta_t^T \\ \dot{W}_{2,t} &= -K_2 P \phi(V_{2,t} \hat{x}_t) \pi(u_t) \Delta_t^T + K_2 P D_\phi \tilde{V}_{2,t} \hat{x}_t \pi(u_t) \Delta_t^T \\ \dot{V}_{1,t} &= -K_3 P W_{1,t} D_\sigma \Delta_t \hat{x}_t^T - \frac{1}{2} K_3 \Lambda_1 \tilde{V}_{1,t} \hat{x}_t \hat{x}_t^T \\ \dot{V}_{2,t} &= -K_4 P W_{2,t} D_\phi \Delta_t \hat{x}_t^T - \frac{1}{2} K_4 \Lambda_2 \tilde{V}_{2,t} \hat{x}_t \hat{x}_t^T \end{aligned} \quad (10)$$

where  $P$  is the solution of Riccati equation (8),  $\tilde{V}_{2,t} = V_{2,t} - V_2^0$ , then the dynamic of identification error (6) is strictly passive from  $\tilde{f}_t (V_1^0, V_2^0)$  to the identification error  $2P\Delta_t$

**Remark 2** *Since the updating gain is  $K_i P$  ( $i = 1 \dots 4$ ) and  $K_i$  can be any positive matrix, the learning process of dynamic neural network (10) dose not depend on the solution  $P$  of Riccati equation (8). So the assumption **A1** is to select  $A$  such that (8) has positive solution.  $R$  is related to the upper bounds of the unknown optimal matrices  $W_1^*$  and  $W_2^*$ , we assume we know the upper bounds.  $Q$  is free to chose because of  $Q_0$ . For matrix equation (8) we may change  $A$ ,  $R$  and  $Q$  such that  $P$  is positive, so it is almost always possible to satisfy **A1**.*

**Remark 3**  $W_{1,t} D_\sigma \Delta_t$  is the error backpropagation for the hidden layer,  $\hat{x}_t^T$  is the input to the hidden layer;  $\sigma(V_{1,t} \hat{x}_t)$  is the input for the output layer, so the first parts  $K_1 P \sigma(V_{1,t} \hat{x}_t) \Delta_t^T$  and  $-K_3 P W_{1,t} D_\sigma \Delta_t \hat{x}_t^T$  are the same as the backpropagation scheme of multilayer perceptrons. The second parts are used to assure the passivate properties of identification error.

**Corollary 1** *If neural networks (2) can match nonlinear plant (1) exactly, i.e., only parameters uncertainty present ( $\tilde{f}_t = 0$ ), then the updating law as (10) can make the identification error asymptotic stable,*

$$\lim_{t \rightarrow \infty} \Delta_t = 0 \quad (11)$$

**Theorem 2** *Using the updating law as (10), the dynamic of neuro identifier (6) is input-to-state stability (ISS).*

**Proof:** In view of the matrix inequality

$$X^T Y + (X^T Y)^T \leq X^T \Lambda^{-1} X + Y^T \Lambda Y \quad (12)$$

$$2\Delta_t^T P \tilde{f}_t \leq \Delta_t^T P \Lambda_f P \Delta_t + \tilde{f}_t^T \Lambda_f^{-1} \tilde{f}_t$$

$$S_t = \Delta_t^T P \Delta_t + tr \left\{ \tilde{W}_{1,t}^T K_1^{-1} \tilde{W}_{1,t} \right\} + tr \left\{ \tilde{W}_{2,t}^T K_2^{-1} \tilde{W}_{2,t} \right\} + tr \left\{ \tilde{V}_{1,t}^T K_3^{-1} \tilde{V}_{1,t} \right\} + tr \left\{ \tilde{V}_{2,t}^T K_4^{-1} \tilde{V}_{2,t} \right\} \quad (13)$$

$$\begin{aligned} \dot{S}_t &= -\Delta_t^T Q_0 \Delta_t + 2\Delta_t^T P \tilde{f}_t \leq -\lambda_{\min}(Q_0) \|\Delta_t\|^2 \\ &+ \Delta_t^T P \Lambda_f P \Delta_t + \tilde{f}_t^T \Lambda_f^{-1} \tilde{f}_t \\ &\leq -\alpha_{\|\Delta_t\|} \|\Delta_t\| + \beta_{\|\tilde{f}_t\|} \|\tilde{f}_t\| \end{aligned}$$

where  $\alpha_{\|\Delta_t\|} := [\lambda_{\min}(Q_0) - \lambda_{\max}(P \Lambda_f P)] \|\Delta_t\|$ ,  $\beta_{\|\tilde{f}_t\|} := \lambda_{\max}(\Lambda_f^{-1}) \|\tilde{f}_t\|$ . We can select a positive defined matrix  $\Lambda_f$  such that

$$\lambda_{\max}(P \Lambda_f P) \leq \lambda_{\min}(Q_0) \quad (14)$$

So  $\alpha$  and  $\beta$  are  $\mathcal{K}_\infty$  functions,  $S_t$  is an ISS-Lyapunov function. Using Theorem 1 of [14], the dynamic of identification error (6) is input to state stable. ■

**Corollary 2** *If the modelling error  $\tilde{f}_t$  is bounded, then the updating law as (10) can make the identification procedure stable*

$$\Delta_t \in L_\infty, \quad W_{1,t} \in L_\infty, \quad W_{2,t} \in L_\infty$$

**Remark 4** *It is well known that structure uncertainties will cause parameters drift for adaptive control, so one has to use robust modification to make identification stable [4]. Robust adaptive methods may be extended to neuro identification directly [5][10] [12]. But neuro identification is a kind of "black-box" method, nobody needs structure information and all of uncertainties are inside the black-box. Although robust adaptive algorithms are suitable for neuro identification, they are not the simplest one. By means of passivity technique, we success to prove our conclusion: the backpropagation-like algorithm (10) is robust with respect to all kinds of bounded uncertainties for multilayer neuro identification.*

The condition (14) can be established if  $\Lambda_f$  is selected as a small matrix. Since the state and output variables are physically bounded, the modelling error  $\tilde{f}_t$  can be assumed to be bounded too ( see, for example [5][10][12]).

For model matching case, Lyapunov-like analysis [19] can reach the same result as Corollary 1. But in the case of modeling error ( $\tilde{f}_t \neq 0$ ), robust modification terms have to be added in the updating law in order to assure stability [5][10] [12]. The robust modification usually depends on the upper bound of modeling error  $\tilde{f}_t$ . Unlike robust adaptive laws, such as dead-zone [10] and  $\sigma$ -modification [8], the updating law does not need the upper bound of uncertainties.

**Theorem 3** *If the modelling error  $\tilde{f}_t(V_1^0, V_2^0)$  is bounded as  $\tilde{f}_t^T \Lambda_f \tilde{f}_t \leq \bar{\eta}(V_1^0, V_2^0)$ ,  $P$  is the solution of the Riccati equation (8) with*

$$R = 2\bar{W}_1 + 2\bar{W}_2 + \Lambda_f, \quad Q = D_\sigma + \bar{u}D_\phi + Q_0 \quad (15)$$

then the updating law (10) may make the identification error converge to

$$\limsup_{T \rightarrow \infty} \frac{1}{T} \int_0^T \|\Delta_t\|_{Q_0}^2 dt \leq \bar{\eta}(V_1^0, V_2^0) \quad (16)$$

**Proof:** Let define a Lyapunov function as (13), in view of the matrix inequality (12)

$$\begin{aligned} 2\Delta_t^T P \tilde{f}_t &\leq \Delta_t^T P \Lambda_f P \Delta_t + \tilde{f}_t^T \Lambda_f^{-1} \tilde{f}_t \\ &\leq \Delta_t^T P \Lambda_f P \Delta_t + \bar{\eta}(V_1^0, V_2^0) \end{aligned}$$

Using the updating law (10), the derivative of the Lyapunov function (13) is

$$\begin{aligned} \dot{S}_t &\leq \Delta_t^T [PA + A^T P + P(2\bar{W}_1 + 2\bar{W}_2 + \Lambda_f)P \\ &\quad + (D_\sigma + \bar{u}D_\phi + Q_0)] \Delta_t - \Delta_t^T Q_0 \Delta_t + \bar{\eta} \end{aligned}$$

From (15) we have

$$\dot{S}_t \leq -\Delta_t^T Q_0 \Delta_t + \bar{\eta}(V_1^0, V_2^0) \quad (17)$$

Integrating (17) from 0 up to  $T$  yields

$$V_T - V_0 \leq -\int_0^T \Delta_t^T Q_0 \Delta_t dt + \bar{\eta}T$$

So

$$\int_0^T \Delta_t^T Q_0 \Delta_t dt \leq V_0 - V_T + \bar{\eta}T \leq V_0 + \bar{\eta}T$$

(16) is established  $\blacksquare$

**Remark 5** *The identification error will converge to the ball radius the upper bounded of  $\tilde{f}_t$ , and it is influenced by the prior known matrices  $V_1^0$  and  $V_2^0$ . Theorem 2 shows that  $V_1^0$  and  $V_2^0$  do not influence the stability property, we may select any values for  $V_1^0$  and  $V_2^0$  at first. From Theorem 3 we know the algorithm (10) can make the identification error convergent.  $V_1^0$  and  $V_2^0$  may be selected by following off-line steps:*

1. Start from any initial values for  $V_1^0$  and  $V_2^0$
2. Do on-line identification with  $V_1^0$  and  $V_2^0$
3. Let  $V_{1,t}$  and  $V_{2,t}$  as new initial conditon, i.e.,  $V_1^0 = V_{1,t}, V_2^0 = V_{2,t}$
4. If the identification error decreases, repeat the identification process, goto 2. Otherwise, stop off-line identification, now  $V_{1,t}$  and  $V_{2,t}$  are final values for  $V_1^0$  and  $V_2^0$ .

After we get these finial  $V_1^0$  and  $V_2^0$ , we may start on-line identification with them.

**Remark 6** *Since the updating rate in the learning algorithm (10) is  $K_i P$  ( $i = 1 \dots 4$ ), and  $K_i$  can be selected as any positive matrix, the learning process of the dynamic neural network (10) is free of the solutions of the two Riccati equations (8) and (15). These two Riccati equations are proposed in order to prove the stability results. When we use the update law for the weights, we only need to select good updating rates  $k_i = K_i P$ .*

### 3 Simulation

The engine operation at idle is a nonlinear process that is far from its optimal range. We assume that the occurrence of plant disturbances, such as engagement of air conditioner compressor, shift from neutral to drive in automatic transmissions, application and release of electric loads, and power steering lock-up, are not directly measured. The dynamic engine model a two inputs and two outputs system [11]:

$$\begin{aligned} \dot{P} &= k_P (\dot{m}_{ai} - \dot{m}_{ao}), \quad \dot{N} = k_N (T_i - T_L) \\ \dot{m}_{ai} &= (1 + k_{m1}\theta + k_{m2}\theta^2) g(P) \\ \dot{m}_{ao} &= -k_{m3}N - k_{m4}P + k_{m5}NP + k_{m6}NP^2 \end{aligned}$$

The engine model parameters are for a 1.6 liter, 4-cylinder fuel injected engine

$$\begin{aligned} g(P) &= \begin{cases} 1 & P < 50.6625 \\ 0.0197\sqrt{101.325P - P^2} & P \geq 50.6625 \end{cases} \\ T_i &= -39.22 + 325024m_{ao} - 0.0112\delta^2 + 0.635\delta \\ &\quad + \frac{2\pi}{60}(0.0216 + 0.000675\delta)N - \left(\frac{2\pi}{60}\right)^2 0.000102N^2 \\ T_L &= \left(\frac{N}{263.17}\right)^2 + T_d, \quad m_{ao} = \dot{m}_{ao}(t - \tau)/(120N) \end{aligned}$$

$$\begin{aligned}
k_P &= 42.40, & k_N &= 54.26, \\
k_{m1} &= 0.907, & k_{m2} &= 0.0998 \\
k_{m3} &= 0.0005968, & k_{m4} &= 0.0005341, \\
k_{m5} &= 0.000001757, & \tau &= 45/N
\end{aligned}$$

The system outputs are manifold press  $P$  (kPa) and engine speed  $N$  (rpm). The control inputs are throttle angle  $\theta$  (degree) and the spark advance  $\delta$  (degree). Disturbances act to the engine in the form of unmeasured accessory torque  $T_d$  (N-m). The variable  $\dot{m}_{ai}$  and  $\dot{m}_{ao}$  refer to the mass air flow into and out of the manifold.  $m_{ao}$  is the air mass in the cylinder. The parameter  $\tau$  is a dynamic transport time delay. The function  $g(P)$  is a manifold pressure influence function.  $T_i$  is the engine's internally developed torque,  $T_L$  is the load torque. If we define  $x = (P, N)^T$ ,  $u = (\theta, \delta)^T$ , then the dynamic of vehicle idle speed are  $\dot{x}_t = \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} f_1(x, u) \\ f_2(x, u) \end{pmatrix}$ .  $f_1$  and  $f_2$  are assumed to be unknown and only  $x$  and  $u$  are measurable. In order to do the simulation, we select input as  $\delta = 30 \sin \frac{t}{2}$ ,  $\theta$  is sawtooth wave with amplitude 10, frequency  $\frac{1}{2}$ ,  $T_d$  is square wave with amplitude 20, frequency  $\frac{1}{4}$ ,  $x_0 = [10, 500]^T$ .

Let us select dynamic neural network as

$$\dot{\hat{x}}_t = A\hat{x}_t + W_{1,t}\sigma(V_{1,t}\hat{x}_t) + W_{2,t}\phi(V_{2,t}\hat{x}_t)\pi(u_t)$$

where  $A = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$ ,  $\hat{x}_0 = [0, 0]^T$ ,  $W_{1,t}$  and  $W_{2,t} \in \mathfrak{R}^{2 \times 3}$ ,  $V_{1,t}$  and  $V_{2,t} \in \mathfrak{R}^{3 \times 2}$ . The sigmoid functions are  $\sigma(x_i) = \frac{2}{1+e^{-2x_i}} - 0.5$ ,  $\phi(x_i) = \frac{0.2}{1+e^{-0.2x_i}} - 0.05$ .  $\pi(u_t) = u_t$ .  $D_\sigma = \text{diag}[D_{\sigma_1}, D_{\sigma_2}, D_{\sigma_3}]$ ,  $D_\phi = \text{diag}[D_{\phi_1}, D_{\phi_2}, D_{\phi_3}]$ ,  $u_3 = 0$ ,

$$\begin{aligned}
D_{\sigma_i} &= \frac{4e^{-2Z_{1,i}}}{(1+e^{-2Z_{1,i}})^2}, & Z_{1,i} &= (V_{1,t}\hat{x})_i \\
D_{\phi_i} &= \frac{0.04e^{-0.2Z_{2,i}}}{(1+e^{-0.2Z_{2,i}})^2}u_i, & Z_{2,i} &= (V_{2,t}\hat{x})_i
\end{aligned}$$

We select  $K_1P = K_2P = K_3P = K_4P = 2I$ .

We choose the initial  $V_1^0 = V_2^0 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}$ , using the

learning law (10) we can get  $V_{1,20} = \begin{bmatrix} 0 & 0.94 \\ 0.45 & 0.25 \\ -0.44 & 0.49 \end{bmatrix}$ ,

$V_{2,20} = \begin{bmatrix} 0.25 & 0.83 \\ 0.68 & 0.36 \\ 0.29 & 0.52 \end{bmatrix}$  after 20s. Now we use

them as the new  $V_1^0$  and  $V_2^0$ , training the neural networks 30s with learning law (10), we have  $V_{1,30} = \begin{bmatrix} 0.001 & 0.94 \\ 0.47 & 0.23 \\ -0.45 & 0.43 \end{bmatrix}$ ,  $V_{2,30} = \begin{bmatrix} 0.25 & 0.80 \\ 0.71 & 0.35 \\ 0.28 & 0.48 \end{bmatrix}$ . We can

see that  $V_{1,t}$  and  $V_{2,t}$  do not change a lot, so stop our searching procedure for  $V_1^0$  and  $V_2^0$ , we use  $V_{1,30}$  and

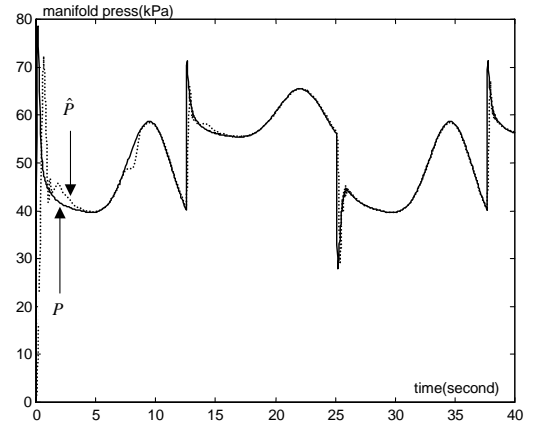


Figure 1: Manifold press

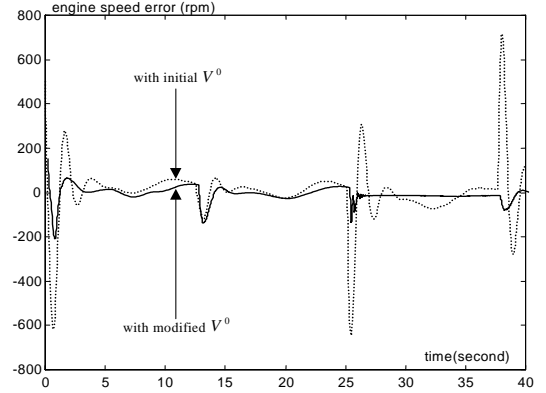


Figure 2: Engine speed error with different  $V_1^0$  and  $V_2^0$

$V_{2,30}$  as the newest  $V_1^0$  and  $V_2^0$ . The identification results are shown in Figure 1.

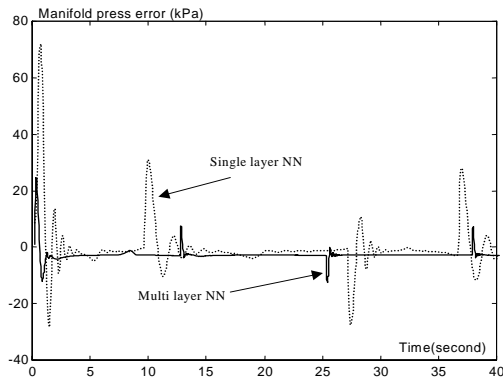
Figure ?? shows how  $V_1^0$  and  $V_2^0$  influence the identification error, the dash lines are corresponded  $V_1^0 =$

$V_2^0 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}$ , the solid lines are the identification

error with  $V_1^0 = V_{1,30}$  and  $V_2^0 = V_{2,30}$ . So we can see that suitable  $V_1^0$  and  $V_2^0$  may be found by the training procedure. Now let compare the multilayer networks with the single layer networks as following

$$\dot{\hat{x}}_t = A\hat{x}_t + W_{1,t}\sigma(\hat{x}_t) + W_{2,t}\phi(\hat{x}_t)\pi(u_t)$$

all of conditions are the same as multilayer neural networks, the learning law is used as in [?]. Figure 3 shows the compensation of the identification error with single layer and multilayer neural networks. One can see that the multilayer dynamic neural networks are more powerful than single-layer dynamic neural networks, and they are robust with respect to bounded uncertainties.



**Figure 3:** Manifold press error with single layer and multilayer networks

#### 4 Conclusion

By means of passivity technique, we give some new results on neuro identification with multilayer dynamic neural networks. Compared with other stability analysis of neuro identifications, our algorithm is more simple because robust modifications are not applied, so the algorithm proposed in this paper is more suitable for engineering application. We success to prove that even the backpropagation learning algorithm may guarantee the identification error robust stable.

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