A New Circuit for Generating Chaos and Complexity: Analysis of the Beats Phenomenon

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***Abstract**:* – In this paper the attention is focused on the chaotic dynamics of a new circuit driven by two sinusoidal signals. In particular, it is illustrated that the application of signals with slightly different frequencies enables the novel phenomenon of chaotic beats to be generated. Moreover, the paper shows that the proposed circuit can be considered the simplest nonautonomous circuit able to generate chaotic beats. Finally, sinusoidal signals with equal frequencies are considered, with the aim of analyzing in detail the formation of beats in the proposed circuit.

Key-Words: - Nonlinear circuit, chaos and complexity, beats phenomenon, time-domain analysis.

1 Introduction

In recent years several researchers have focused their attention on the study of chaos and complexity in nonlinear systems [1]-[2]. In particular, the observations of different complex behaviors such as period-adding sequences, torus breakdown to chaos, coexistence of multiple attractors and presence of periodic windows have been described in dynamic systems [3]-[6]. Furthermore, synchronization properties, generation of multi-scroll attractors, quasi-periodicity and intermittent transitions have been illustrated in [7]-[10]. Recently, a new interesting phenomenon in nonlinear systems has been reported in [11], where the behavior of nonautonomous systems under two sinusoidal inputs characterized by slightly different frequencies has been investigated. Such phenomenon has been widely studied in linear systems and has been called "beats" [12]. Namely, when two waves characterized by slightly different frequencies interfere, the frequency of the resulting waveform is the average of the frequencies of the two waves, whereas its amplitude is modulated by an envelope, the frequency of which is the difference between the frequencies of the two waves [12]. By generalizing this concept, in [11] the generation of chaotic beats in coupled *nonlinear systems* with very small nonlinearities has been studied. However, at the best of our knowledge, the phenomenon of beats has not yet been investigated in *nonlinear circuits*. Therefore, the aim of this paper is to introduce a new circuit for generating *chaotic beats*. In particular, herein the simplest nonautonomous circuit able to generate chaotic beats is proposed.

The paper is organized as follows. In Section 2 the equations of the proposed second-order nonautonomous chaotic circuit, which includes the well-known Chua's diode, are reported. In Section 3 it is shown that the application of two sinusoidal signals, characterized by large equal amplitudes and slightly different frequencies, enable chaotic beats to be generated. In particular, a time-domain analysis of chaotic beats and corresponding envelopes is carried out. Moreover, in order to confirm the chaotic nature of the phenomenon, the power spectral density and the Lyapunov exponents are reported. Finally, the behaviour of the proposed circuit driven by two sinusoidal signals with equal frequencies is discussed.

2 A New Chaotic Circuit

By exploiting the design approach illustrated in [13], in this Section a new circuit is proposed with the aim of generating chaotic beats. The suggested circuit (Fig.1) contains two external periodic excitations, a capacitor, an inductor, a linear resistor and a nonlinear element, namely, the Chua's diode. By applying Kirchhoff's laws, the state equations for the voltage v_C across the capacitor C and the current i_L through the inductor L are represented by the following set of differential equations:

$$\begin{bmatrix} \frac{dv_C}{dt} \\ \frac{di_L}{dt} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{C} \\ -\frac{1}{L} & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} v_C \\ i_L \end{bmatrix} + \begin{bmatrix} -\frac{1}{C} \\ 0 \end{bmatrix} g(v_C) + \begin{bmatrix} \frac{1}{C} & 0 \\ 0 & \frac{1}{L} \end{bmatrix} \begin{bmatrix} f_2 \sin(\Omega_2 t) \\ f_1 \sin(\Omega_1 t) \end{bmatrix} \quad (1)$$

where f_1 and f_2 are the amplitudes of the periodic excitations whereas Ω_1 and Ω_2 are their angular frequencies. The term

$$g(v_C) = G_b v_C + (G_a - G_b)(|v_C + B_p| - |v_C - B_p|)/2 \quad (2)$$

is the mathematical representation of the piece-wise linear characteristic of the Chua's diode, as in [13].

Rescaling Eq.s (1)-(2) as $v_C = x_1 B_p$, $i_L = x_2 B_p / R$, $\omega_1 = \Omega_1 CR$, $\omega_2 = \Omega_2 CR$, $t = \tau CR$ and then redefining τ as t, the following set of dimensionless equations are obtained:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\beta & -\beta \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \end{bmatrix} g(x_1) + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} F_2 \sin(\omega_2 t) \\ F_1 \sin(\omega_1 t) \end{bmatrix} \quad (3)$$

where $\beta = R^2 C / L$, $F_1 = f_1 \beta / B_p$ and $F_2 = f_2 \beta / B_p$.

Furthermore,

$$g(x_1) = bx_1 + (a - b)(|x_1 + 1| - |x_1 - 1|)/2, \quad (4)$$

where $a = G_a R$ and $b = G_b R$.

In the following, it will be shown that the proposed circuit can be considered the simplest nonautonomous circuit able to generate chaotic beats.

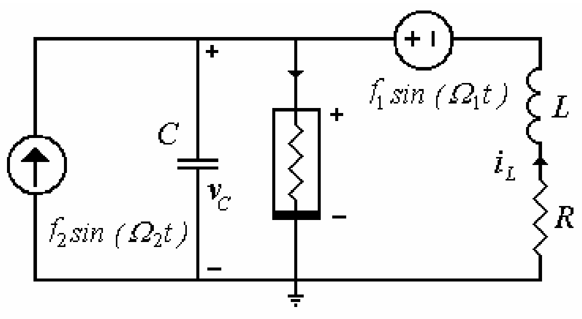


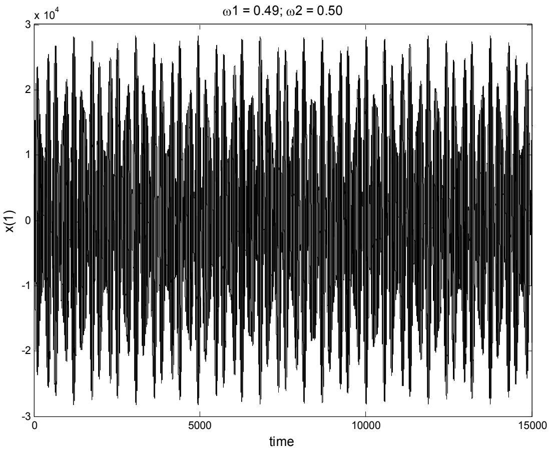
Fig.1 A new circuit for generating chaotic beats.

3 Generation of Chaotic Beats

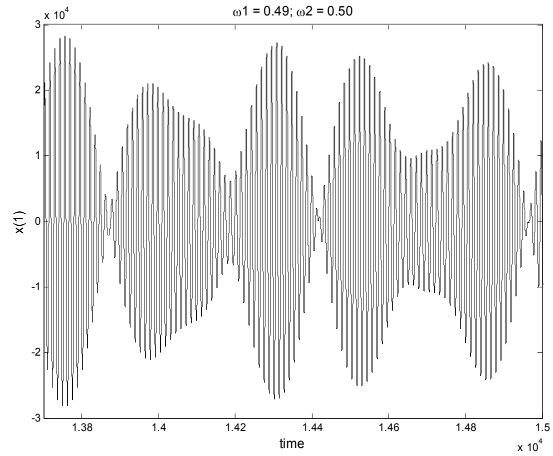
3.1 Time-domain analysis of chaotic beats

The dynamics of Equations (3)-(4) depend on the parameters β, a, b, ω_1, ω_2, F_1 and F_2. For the present analysis the parameters $a = -1.27$ and $b = -0.68$ are fixed, whereas the remaining parameters β, F_1, F_2, ω_1 and ω_2 have to be properly chosen. Since the aim of the paper is to investigate the phenomenon of chaotic beats in the proposed nonlinear circuit, several numerical simulations are carried out for different values of the bifurcation parameters β, F_1, F_2 and for slightly different values of the frequencies ω_1 and ω_2. In particular, it is interesting to analyze the circuit behavior for $\beta = 0.680044$, $F_1 = F_2 = 200$, $\omega_1 = 0.49$ and $\omega_2 = 0.50$. To this purpose, Fig.2 shows the time behaviors of the state variable x_1 for different resolutions of the time scale.

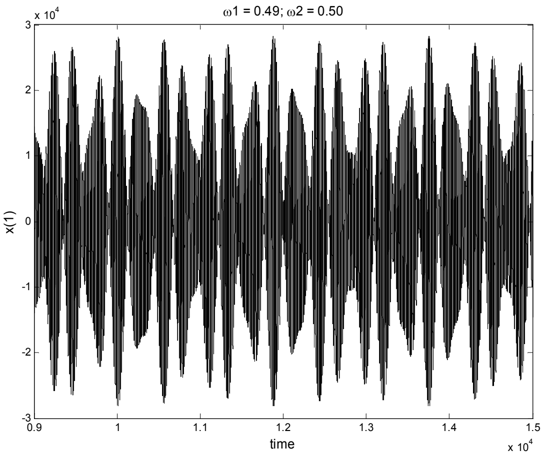
More precisely, Fig.2(a) makes perceive the chaotic nature of the signal x_1, whereas Fig.2(b) highlights the occurrence of chaotic beats generated by its envelope (Fig.2(c)). Moreover, Fig.2(d)-(e) reveal in the signal x_1 both the presence of a fundamental frequency and an amplitude modulation due to the chaotic envelope.



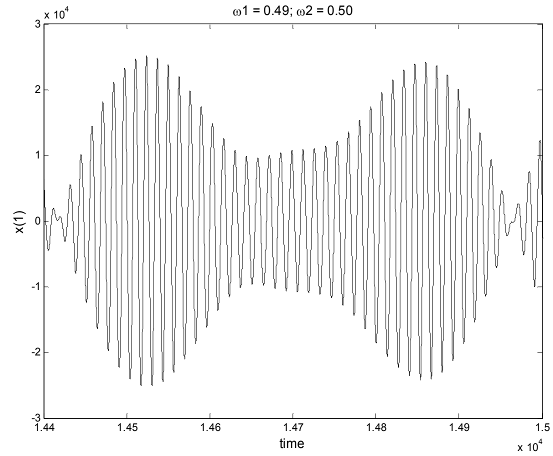
(a)



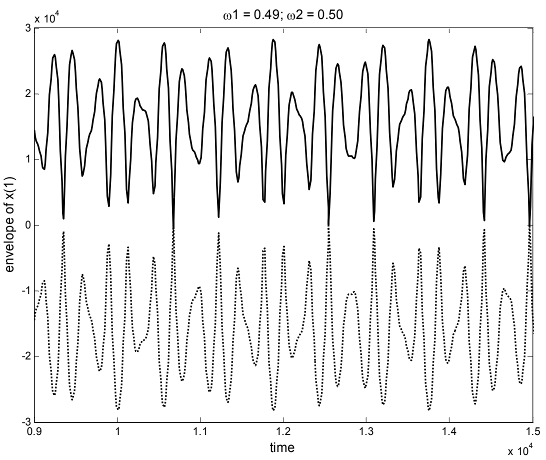
(d)



(b)



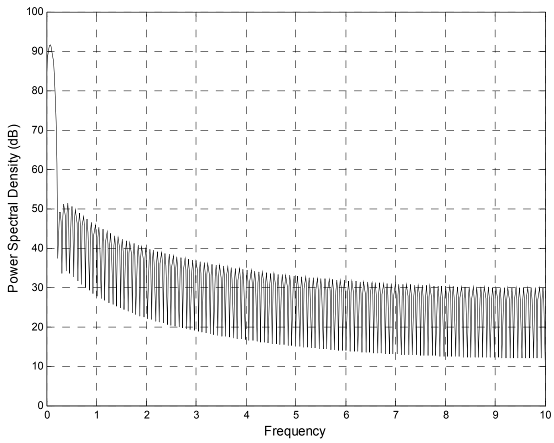
(e)



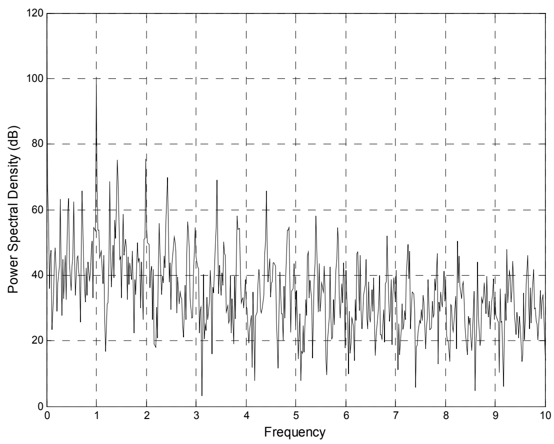
(c)

Fig.2 Time behaviors of the state variable x_1 for different resolutions of the time scale; (a): $t \in [0, 15000]$; (b): $t \in [9000, 15000]$; (c): envelope of signal x_1 for $t \in [9000, 15000]$; (d): $t \in [13700, 15000]$; (e): $t \in [14400, 15000]$.

Referring to the fundamental frequency, the power spectral density of the signal x_1 (Fig.3(a)) shows a high peak at the frequency $f^* = 0.078125$, which corresponds to $\omega^* = 0.49087$. It should be noted that $\omega^* \in [\omega_1, \omega_2]$ but $\omega^* \neq (\omega_1 + \omega_2)/2$. Additionally, the power spectral density of the envelope of the signal x_1 (Fig.3(b)) confirms its chaotic nature.



(a)



(b)

Fig.3 Power spectral densities: (a) of the signal x_1; (b) of the envelope of the signal x_1.

On the other hand, referring to the chaotic amplitude modulation of the signal x_1, the Lyapunov exponents of system (3) are calculated. In particular, the 2^{th}-order time-periodic nonautonomous system (3) is converted into a 3^{th}-order autonomous system by appending an extra state variable [14]. Thus, since sinusoidal forcing terms in Equation (3) are treated as a parameter, a null exponent is obtained. Namely, the Lyapunov exponents are:

$$\lambda_1 = 1.8893\text{e}{-}005, \quad \lambda_2 = 0.00000, \quad \lambda_3 = -2.3767\text{e}{-}005. \tag{5}$$

Notice that the presence of one positive Lyapunov exponent confirms the chaotic dynamics of the proposed circuit. The projection of the chaotic attractor on the (x_1, x_2)-state space is reported in Fig. 4.

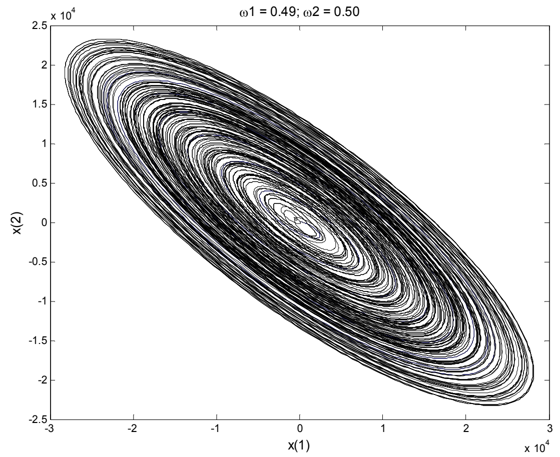


Fig.4 Projection of the chaotic attractor on the (x_1, x_2)-state space.

3.2 Chaotic beats in the simplest circuit

Based on the previous analysis, it can be concluded that for the parameter values given by:

$$\beta = 0.680044,\ a = -1.27,\ b = -0.68, \tag{6}$$

$$F_1 = F_2 = 200, \tag{7}$$

$$\omega_1 = 0.49,\ \omega_2 = 0.50, \tag{8}$$

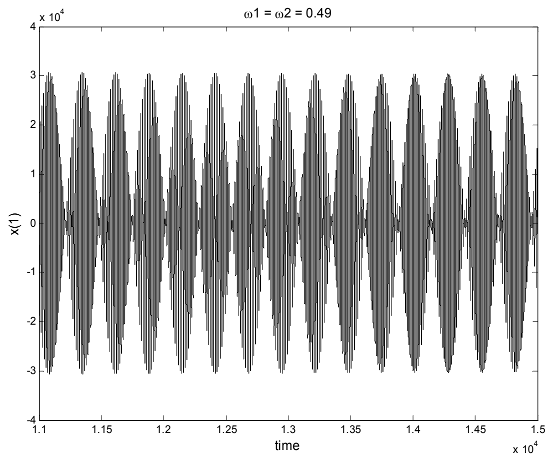
the suggested circuit is able to generate chaotic beats. Note that the proposed circuit can be considered the *simplest* nonautonomous circuit for generating chaotic beats. Namely, it contains the *minimum number of elements* able to produce the beats phenomenon. In particular, the two storage elements (i.e., a capacitor and a real inductor modeled as a series connection of an ideal inductor and a linear resistor) are required for obtaining a *dynamic* circuit. Moreover, the nonlinear element is required for producing *chaos*, whereas the two external excitations (with slightly different frequencies) are necessary for generating *beats*.

3.3 Discussion

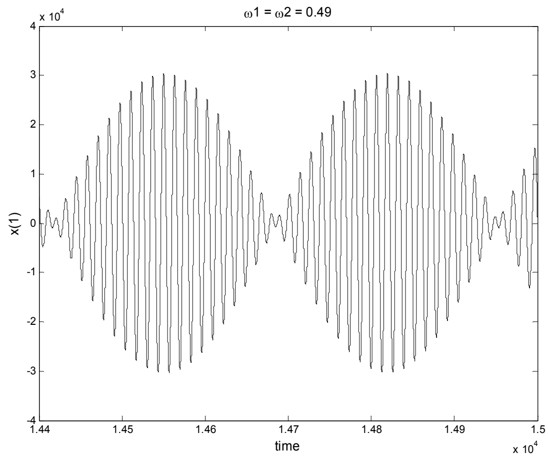
In order to better understand the formation of the chaotic beats in system (3), its dynamics are analyzed for the parameter values (6)-(7), whereas equal frequencies $\omega_1 = \omega_2 = 0.49$ are chosen. The resulting time waveforms of the state variable x_1 are reported in Fig.5(a)-(b) for different resolutions of the time scale. Fig.5 clearly highlights that the expanding and contracting behavior goes on *periodically* for increasing times. More precisely, Fig.5(a) highlights the presence of beats due to a *periodic* envelope, whereas Fig.5(b) reveals in the signal x_1 also the presence of a fundamental frequency. Based on these considerations, it can be argued that for the parameter values (6)-(7) and equal frequencies

$$\omega_1 = \omega_2 = 0.49, \quad (8)$$

the proposed nonautonomous circuit is not able to generate chaotic beats. However, notice that in this case the proposed nonlinear circuit exhibit *periodic* amplitude modulated signals, which are similar to the beats obtained in *linear* systems.



(a)



(b)

Fig.5 Behaviors of the state variable x_1 for different resolutions of the time scale; (a): $t \in [11000, 15000]$; (b): $t \in [14400, 15000]$.

4 Conclusions

In this paper the chaotic dynamics of a new circuit driven by two sinusoidal signals have been studied. In particular, it has been shown that the application of signals with slightly different frequencies leads to the novel phenomenon of chaotic beats. Moreover, the dynamics of chaotic beats and envelopes have been analyzed in time-domain with full particulars. Finally, it is worth noting that the proposed circuit has proved to be the simplest nonautonomous circuit able to generate chaotic beats.

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