Complex Dynamic Phenomena in Power Converters: Bifurcation Analysis and Chaotic Behavior

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Abstract: – In recent years it has been observed that some power converters can exhibit deterministic chaos. Since converters have wide industrial applications, it is useful to study their bifurcation phenomena in order to understand the change of behaviors as circuit parameters are varied. Therefore this paper aims to investigate some complex dynamic phenomena that can occur in current-programmed DC-DC boost converters. To this purpose, the paper illustrates bifurcation analyses as well as new possible pathways through which the converter may enter chaos. In particular, based on PSpice design, it is shown that variations of supply voltage and inductance generate interesting bifurcations and novel routes to chaos.

Key-Words: - Bifurcation, chaos, DC-DC converter, PSpice design of switching circuit.

1 Introduction

In nonlinear circuits and systems a variety of strange effects have been observed, including subharmonics, quasi-periodic oscillation, intermittency, multi-scroll attractors and chaotic behavior. These phenomena have been intensively studied in the cross-disciplinary science of chaos [1]-[5]. In particular, in recent years it has been observed that some power electronic circuits can exhibit deterministic chaos [6]-[10]. Referring to power converters, it has been demonstrated that current-mode controlled buck and boost converters are prone to subharmonic behavior and chaos [6]-[8]. Even though the approaches in [7]-[8] are very interesting, further analysis is required on the parameter domains in which chaotic behavior may occur. Namely, since nowadays these DC-DC converters have wide industrial applications, it is useful to study their bifurcation phenomena in order to understand the change of behaviors as circuit parameters are varied.

Based on these considerations, the aim of this paper is to investigate some complex dynamic phenomena that can occur in current-programmed DC-DC boost converters. In particular, the paper illustrates a detailed bifurcation analysis and shows new possible pathways through which the boost converter may enter chaos. The paper is organized as follows. In Section 2 the state equations of the current-programmed boost converter are reported. In Section 3 the PSpice design of the boost converter along with its control circuitry is illustrated in detail. In Section 4 it is shown that the variations of the supply voltage and inductance lead to new bifurcation paths and routes to chaos. These results are illustrated in detail by means of time waveforms of the inductor current, proper phase portraits and bifurcation diagrams.

2 State Equations of the Boost Converter

The current-programmed boost converter is a second order circuit, which includes an inductor L, a diode D, a DC source V_{in}, a switch S, a resistance R connected in parallel with a capacitor C and a feedback path that consists of a flip-flop and a comparator (Fig.1). The converter is assumed to operate in continuous mode [11]. Namely, the inductance L and the switching period T are chosen so that the inductor current never falls to zero. Hence, there are two switch states (labeled with (i) and (ii), respectively), according to whether S is closed or open. In particular:

i) switch S on and diode D off;
ii) switch S off and diode D on.

The two switch states toggle periodically. In particular, the converter takes:

state i) for $nT \le t < (n + d)T;$
state ii) for $(n + d)T \le t < (n + 1)T;$

where n is an integer and d is the duty cycle. Therefore, the state equations of the boost converter are [8]:

$$\begin{bmatrix} \frac{dv}{dt} \\ \frac{di}{dt} \end{bmatrix} = \begin{bmatrix} -1/RC & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} v \\ i \end{bmatrix} + \begin{bmatrix} 0 \\ 1/L \end{bmatrix} V_{in} \qquad (1)$$

for $nT \le t < (n + d)T$;

$$\begin{bmatrix} \frac{dv}{dt} \\ \frac{di}{dt} \end{bmatrix} = \begin{bmatrix} -1/RC & 1/C \\ -1/L & 0 \end{bmatrix} \begin{bmatrix} v \\ i \end{bmatrix} + \begin{bmatrix} 0 \\ 1/L \end{bmatrix} V_{in} \qquad (2)$$

for $(n + d)T \le t < (n + 1)T$.

The inductance current $i(t)$ is chosen as the programming variable, which generates the on-off driving signal for the switch S after the comparison with a reference current I_{ref}. More precisely, the switch S is turned on at $t = nT$, i.e. at the beginning of the cycle. While the switch S is on, the inductance current increases until reaches the value of I_{ref}. Then, the switch S is turned off, and remains off until the next cycle begins.

3 PSpice Design

This Section illustrates the proposed PSpice design (see Fig.2) of the current-programmed boost converter reported in Fig.1.

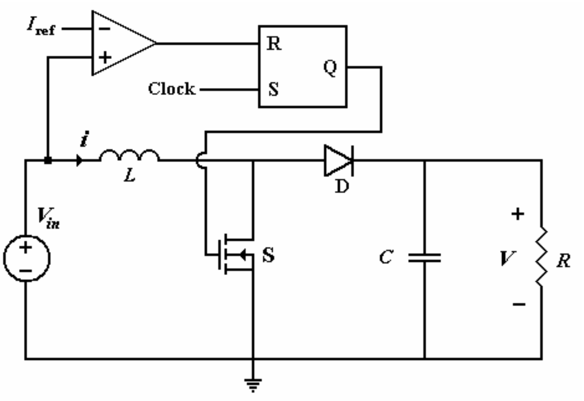


Fig.1 Current-programmed boost converter.

The switch S is implemented using a MOSFET. Its control circuitry is based on the OpAmp LM339 used as a comparator. In particular, the LM339 compares the reference voltage V_{ref} with the voltage across the resistance R_3 in series with the drain of the MOSFET. Note that this voltage is proportional to the current $i(t)$ through the inductor L when the MOSFET is turned on. Therefore, the output of the comparator is high when the inductor current reaches the value $I_{\mathrm{ref}} = V_{\mathrm{ref}} / R_3$, whereas it is low when the inductor current is less than I_{ref}.

Now the generation of the clock signal is described. At first, the integrated device NE555C is considered in order to generate a square wave with duty cycle $d = 0.9$.

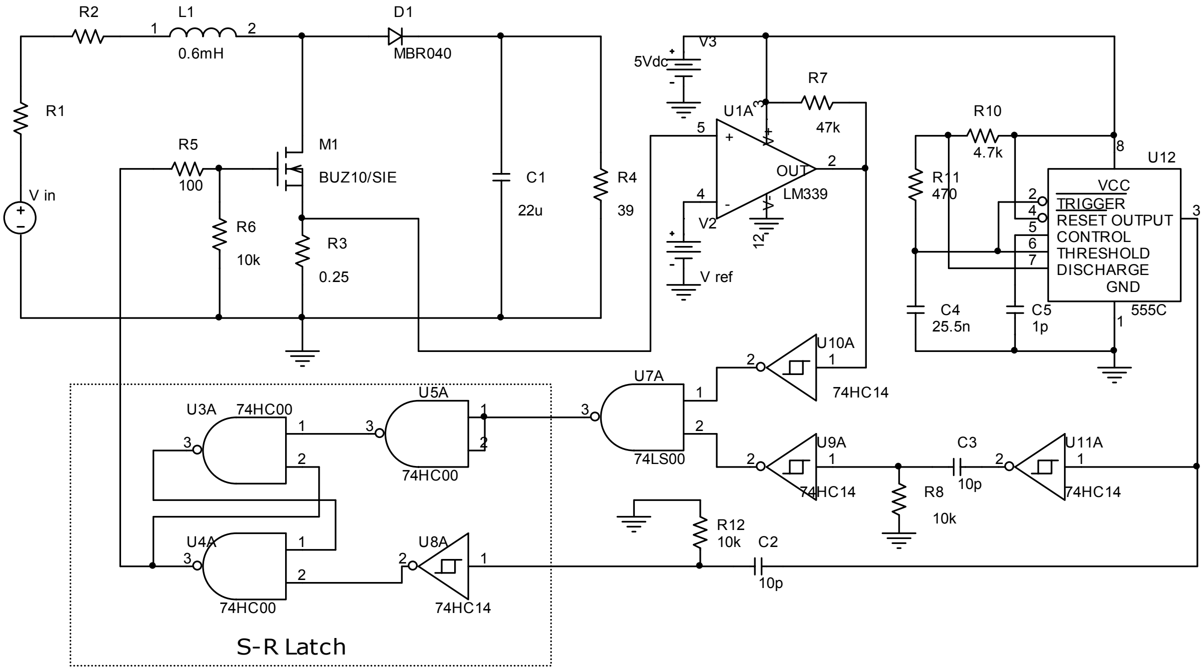


Fig.2 Proposed PSpice design of the current-programmed boost converter.

By making the derivative of the rising edge of the square wave, it is possible to obtain an impulsive signal that represents the SET input of the S-R latch. Additionally, by making the derivative of the falling edge, the signal able to control the duty cycle is obtained. Referring to the latch, its output signal is high (i.e., the MOSFET is ON) when an impulsive signal arrives at the SET input. On the other hand, its output signal is low (i.e., the MOSFET is OFF) when a proper impulsive signal arrives at the RESET input. Such RESET signal, by means of an OR gate, can be either the output of the comparator or the signal able to control the duty cycle. Note that the OR gate has been realized using a NAND gate where the two inputs have been inverted.

4 Bifurcation Analysis and Chaotic Behavior

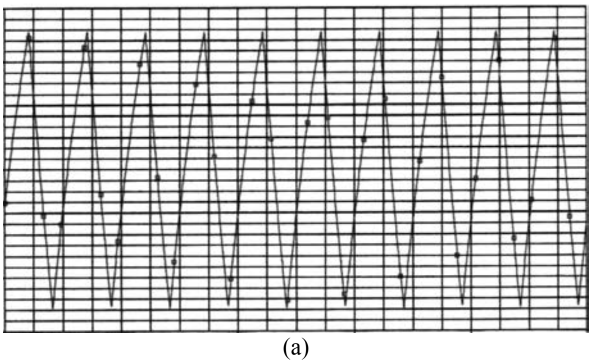
An interesting study on the bifurcations in current-programmed DC-DC boost converters has been illustrated in [8], where the current I_{ref} has been chosen as a primary bifurcation parameter. Differently from [8], in this Section the way the boost converter changes its qualitative behavior is analyzed by varying other meaningful circuit parameters, while keeping fixed the value of the current I_{ref}.

4.1 Route to chaos by varying parameter V_{in}

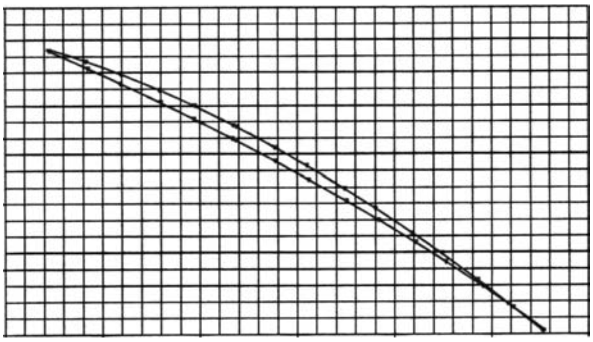
Herein the behavior of the boost converter is analyzed by varying the supply voltage V_{in}, whereas the following circuit parameter values have been fixed:

$$R = 39\Omega,\ C = 22\mu\mathrm{F},\ L = 0.6\mathrm{mH},$$
$$I_{\mathrm{ref}} = 1.05\mathrm{A},\ f = 1/\mathrm{T} = 10\mathrm{KHz}.$$

At first the value of the supply voltage is chosen as V_{in} = 10V. Fig.3(a) shows the inductance current waveform of a typical current-programmed converter under a fundamental periodic operation, whereas the corresponding phase portrait is shown in Fig.3(b).



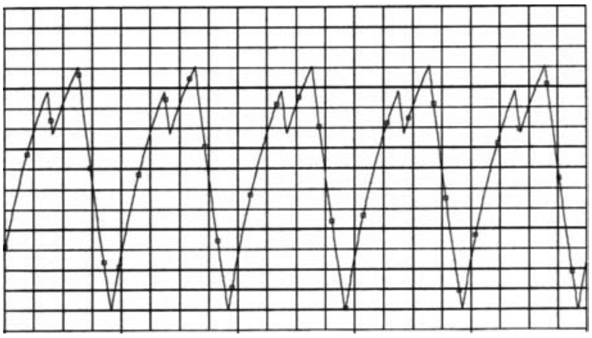
(a)



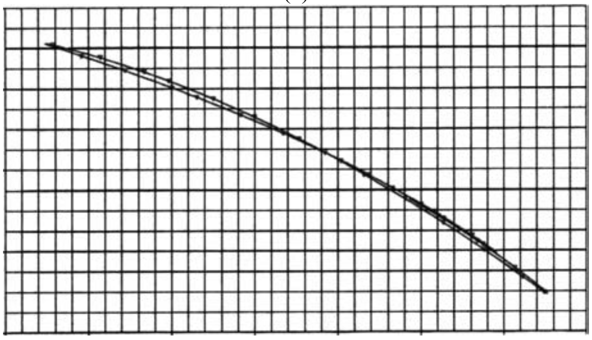
(b)

Fig.3 Fundamental periodic operation: (a) time-domain waveform of the inductor current (time-scale: 9ms–10ms; current-scale: 0.5A-1.1A); (b) (i, v)-phase portrait (current-scale: 0.5A-1.1A; voltage-scale: 12V-13V).

These figures demonstrate the stable and periodic nature of the system. Moreover, when the value of the voltage V_{in} is decreased, many other operating regimes are possible. For example Fig.4(a) shows the time waveform of the current $i(t)$ for a period-two subharmonic operation (V_{in} = 8V). The corresponding phase portrait, shown in Fig.4(b), confirms such period-two behavior.



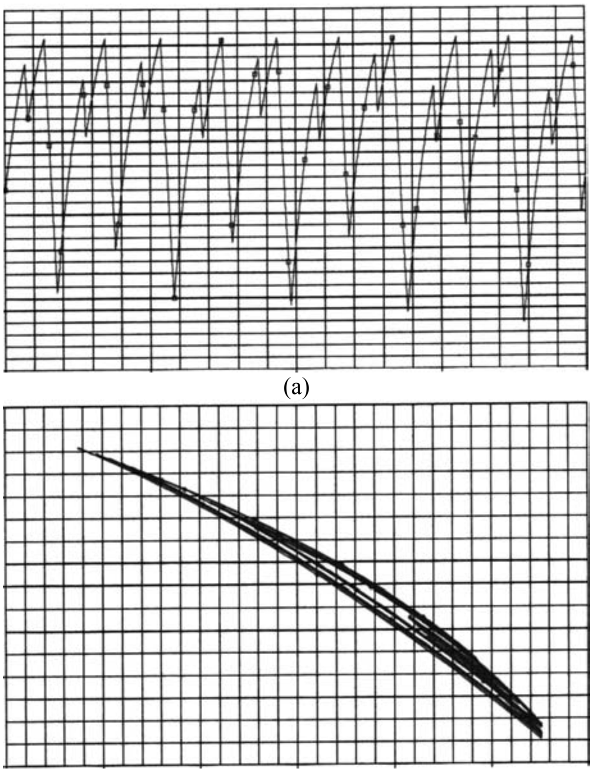
(a)



(b)

Fig.4 2T subharmonic operation: (a) time-domain waveform of the inductor current (time-scale: 9ms–10ms; current-scale: 0.4A-1.2A); (b) (i, v)-phase portrait (current-scale: 0.4A-1.1A; voltage-scale: 8.8V-10.4V).

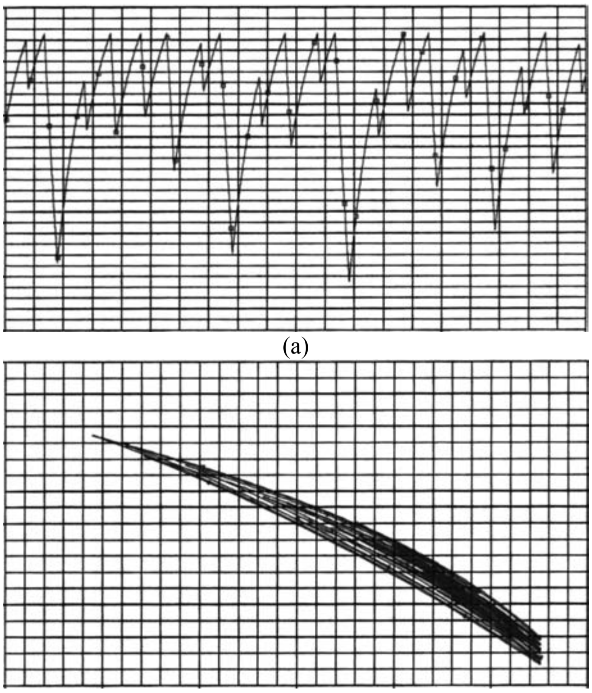
Additionally, by taking V_{in} = 7.2V, it is possible to obtain a quasi-periodic operation. In particular, Fig.5(a) shows the quasi-4T periodic waveform of the inductance current, whereas Fig.5(b) shows the corresponding phase portrait.



(a)

(b)

Fig.5 Quasi-4T subharmonic operation: (a) time-domain waveform of the inductor current (time-scale: 8ms–10ms; current-scale: 0.5A-1.1A); (b) (i, v)-phase portrait (current-scale: 0.5A-1.1A; voltage-scale: 7.2V-8.8V).



(a)

(b)

Fig.6 Chaotic operation: (a) time-domain waveform of the inductor current (time-scale: 8ms–10ms; current-scale: 0.5A-1.1A); (b) (i, v)-phase portrait (current-scale: 0.5A-1.1A; voltage-scale: 6.5V-8.5V).

Finally, when the value of the supply voltage V_{in} is further decreased, the chaotic operating regime appears. The current waveform and the phase portrait for the circuit operating in the chaotic regime (V_{in} = 7V) are shown in Fig.6(a) and Fig.6(b), respectively. All the above mentioned dynamic behaviors are confirmed by the bifurcation diagram reported in Fig.7.

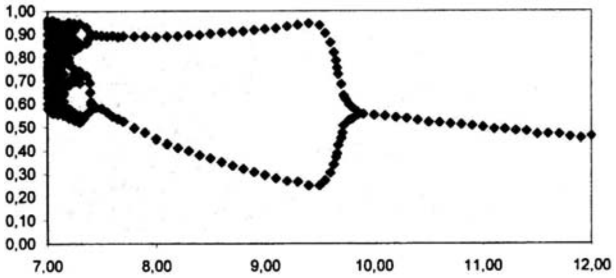


Fig.7 Bifurcation diagram of the inductor current $i(t)$: the bifurcation parameter is the supply voltage V_{in}.

4.2 Route to chaos by varying parameter *L*

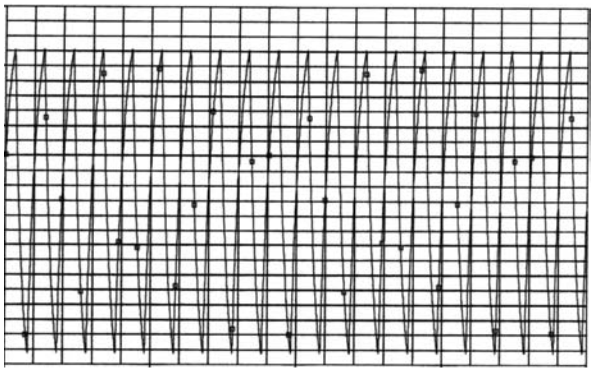
Herein the behavior of the boost converter is analyzed by varying the inductance L, whereas the following circuit parameter values have been fixed:

$$R = 39\Omega,\ C = 22\mu F,\ V_{in} = 7V,$$

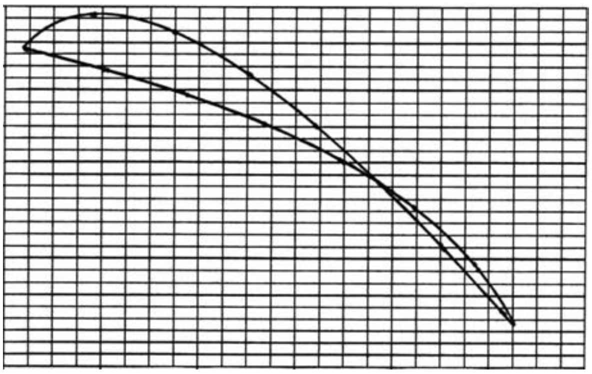
$$I_{ref} = 1.05A,\ f = 1/T = 10KHz.$$

At first the value of the inductance is chosen as L = 0.15mH. Fig.8(a) and Fig.8(b) show the inductance current waveform and the corresponding phase portrait, respectively, under periodic operation of period T.

When the value of the inductance L is increased, many other operating regimes are possible. For example Fig.9(a) shows the time waveform of the inductance current for a period-two subharmonic operation (L = 0.3mH). The corresponding phase portrait, shown in Fig.9(b), confirms such 2T-periodic behavior. Additionally, for the value L = 0.53mH, a quasi-periodic operation is obtained.

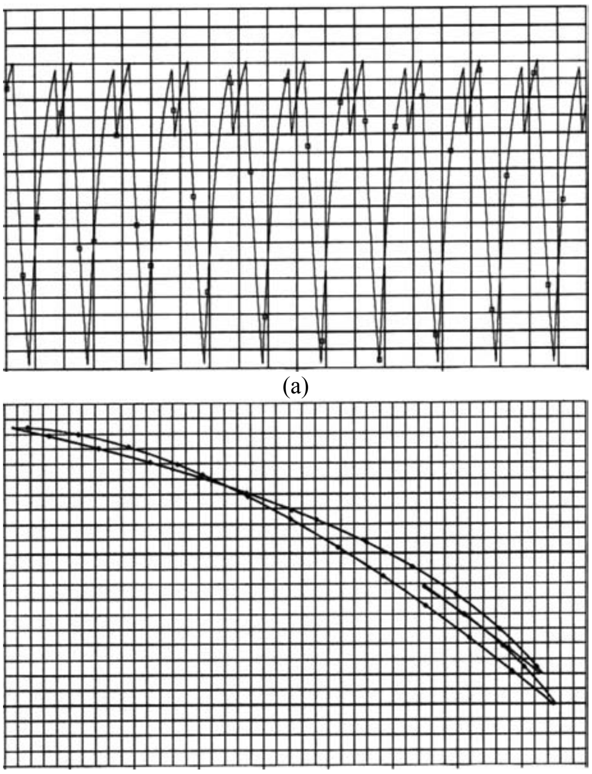


(a)



(b)

Fig.8 Fundamental periodic operation: (a) time-domain waveform of the inductor current (time-scale: 8ms–10ms; current-scale: 0.0A-1.2A); (b) (*i*, *v*)-phase portrait (current-scale: 0.0A-1.2A; voltage-scale: 7.0V-7.6V).

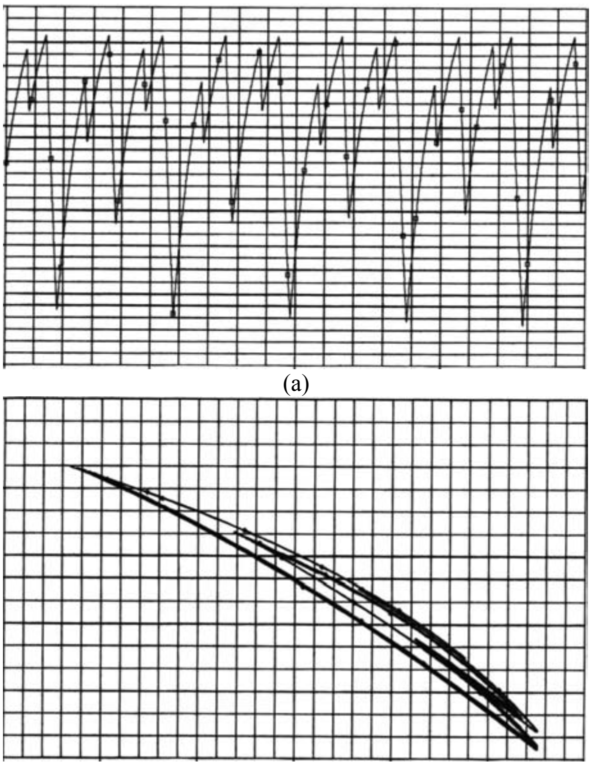


(a)

(b)

Fig.9 2*T* subharmonic operation: (a) time-domain waveform of the inductor current (time-scale: 8ms–10ms; current-scale: 0.2A-1.2A); (b) (*i*, *v*)-phase portrait (current-scale: 0.2A-1.1A; voltage-scale: 7.0V-8.2V).

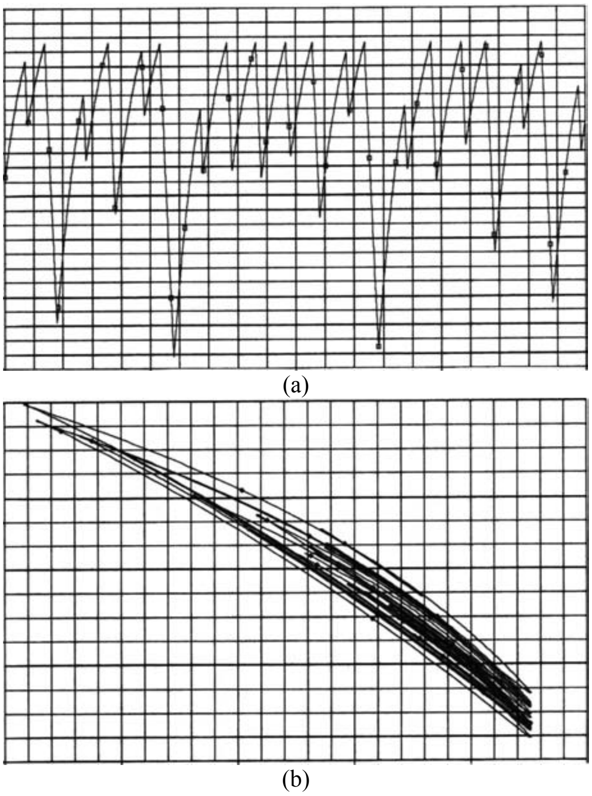
In particular, Fig.10(a) and Fig.10(b) show the waveform of the inductance current and the corresponding phase portrait, respectively, for the quasi-4T periodic operation. Finally, when the value of the inductance L is further increased, the circuit behavior goes toward chaotic operating regimes. For example, for L = 0.6mH, the current waveform and the phase portrait for the chaotic regime are shown in Fig.11(a) and Fig.11(b), respectively.



(a)

(b)

Fig.10 Quasi-4*T* subharmonic operation: (a) time-domain waveform of the inductor current (time-scale: 8ms–10ms; current-scale: 0.5A-1.1A); (b) (*i*, *v*)-phase portrait (current-scale: 0.5A-1.1A; voltage-scale: 6.8V-8.4V).



(a)

(b)

Fig.11 Chaotic operation: (a) time-domain waveform of the inductor current (time-scale: 8ms–10ms; current-scale: 0.6A-1.1A); (b) (*i*, *v*)-phase portrait (current-scale: 0.6A-1.1A; voltage-scale: 6.5V-8.0V).

All the above mentioned dynamic behaviors are confirmed by the bifurcation diagram reported in Fig.12.

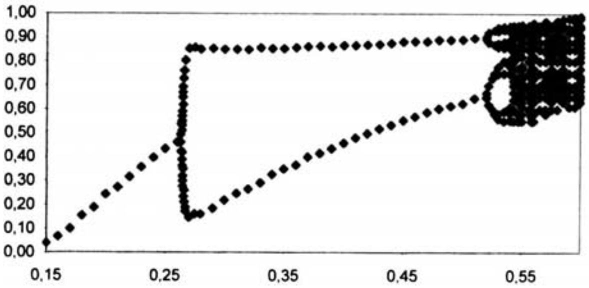


Fig.12 Bifurcation diagram of the inductor current $i(t)$: the bifurcation parameter is the inductance L.

5 Conclusion

This paper has analysed some complex dynamic phenomena that can occur in current-programmed DC-DC boost converters. Namely, bifurcation analyses as well as new possible pathways through which the converter may enter chaos have been shown. In particular, based on the proposed PSpice design, it has been shown that variations of supply voltage and inductance may lead to interesting bifurcation paths and novel routes to chaos.

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