Evaluation of ZCZ Sets on Inter-Cell Interference for Cellular AS-CDMA Systems

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Abstract: – ZCZ sets are families of sequences, whose periodic auto/cross-correlation functions have zero correlation zone at the both side of the zero-shift. They can provide cellular CDMA systems, which can remove intra-cell interference and decrease influence of multipath. This paper evaluates four types of ZCZ sets, designed in consideration of facilitation of hardware construction, on inter-cell interference for a basic model of cellular CDMA systems, so that synchronization among cells can not be controlled and different ZCZ sets are assigned to adjacent cells.

Key words: – sequence design, CDMA, cellular mobile communication, interference

1 Introduction

ZCZ sets are families of sequences having zero-correlation zone (Zcz), that periodic auto-correlation functions take zero for continuous shifts at the both side of the zero-shift, and cross-ones do zero for them and the zero-shift [1]-[10].

For cellular mobile communications, the application of ZCZ sets to approximately synchronized CDMA (AS-CDMA) systems, that synchronization among users in a cell can be controlled within permissible time difference, is possible to cancel intra-cell interference and decrease influence of multipath. Therefore we expect that efficient cellular CDMA systems in the view of frequency usage can be constructed successfully. However influence of inter-cell interference from adjacent cells can not be ignored, since we hear that it is about a half of that of intra-cell interference, on the data error.

In this paper we evaluate four types of ZCZ sets, which are designed in consideration of facilitation of hardware construction, on inter-cell interference for a basic model of cellular CDMA systems, so that synchronization among cells can not be controlled, different ZCZ sets are severally assigned to adjacent cells, and a user in a cell employs a sequence within a ZCZ set assigned to it. They include (a) binary ZCZ sets [1]-[6] (b) ternary ZCZ sets [8][9] (c) paired ZCZ sets [10] (d) ternary complementary ZCZ sets[10].

In section 2, the four types of ZCZ sets are introduced. In particular we explain that binary ZCZ sets can not provide good cellular CDMA systems in the view of efficient frequency usage.

In section 3 we explain the basic model of cellular AS-CDMA systems, and introduce the average interference parameter [12], which can investigate inter-cell interference for it. In section 4 we evaluate the four types of ZCZ sets on inter-cell interference.

2 ZCZ Sets

Let x be a complex sequence of length N expressed by

$$x = (x_0, \cdots, x_i, \cdots, x_{N-1}), x_i \in C.$$

In particular x is called a binary sequence if $x_i \in \{1, -1\}$, ternary one if $x_i \in \{\pm 1, 0\}$, and real one if $x_i \in R$. The periodic correlation function between sequences of length N, x and y, at a shift τ, is defined by

$$R_{xy}(\tau) = \sum_{i=0}^{N-1} x_i y^*_{(i+\tau) \bmod N},$$

where $*$ denotes complex conjugate. It is called the periodic auto-correlation function, if $x_i = y_i$ for any i, and periodic cross-one otherwise. And also, if $x_i = y_{i+\tau}$ for any i, x and y are called the periodically same.

We uniformly explain four types of ZCZ sets, which have been presented. Let Z be a set of M sequences z^j's or M sequence pairs $z^j = [x^j, y^j]$, expressed by

$$Z = \{z^1, \cdots, z^j, \cdots, z^k, \cdots, z^M\}. \qquad (1)$$

If an appropriately defined periodic correlation function between z^j and z^k, $G_{z^j z^k}(\tau)$, satisfies

$$G_{z^j z^k}(\tau) = \begin{cases} K\eta N & , \tau = 0, j = k, \\ 0 & , \tau = 0, j \neq k, \\ 0 & , 1 \leq |\tau| \leq T, \end{cases} \qquad (2)$$

the set Z is called a ZCZ set with zero correlation zone ($Zcz = T$), where K takes 1 or 2 in this paper, and

$$\eta = \frac{1}{N} \sum_{i=0}^{N-1} |z_i^j|^2 \leq 1.$$

Note that $\eta = 1$ for a binary sequence.

For some types of ZCZ sets, the upper bound of set size M for zero correlation zone Zcz and length N can be written as

$$M \leq \frac{KN}{Zcz + 1}$$

[7][10][11]. If $M = \frac{KN}{Zcz+1}$, we simply say "Z satisfies the bound ".

Here we explain the synthesis of sequences, which can produce three types of ZCZ sets except binary ones.

Let v and w be respectively sequences of length N_1 and length N_2. In this paper we assume that N_1 and N_2 are relatively prime, i.e., $gcd(N_1, N_2) = 1$. Let $x = v \odot w$ be a sequence of length $N = N_1 N_2$ synthesized by v and w, which is written as

$$x_i = v_{i \bmod N_1} w_{i \bmod N_2}.$$

The periodic correlation function between synthesized sequences of length N, $x = v \odot w$ and $x' = v' \odot w'$, can be expressed by the product of ones for original sequences, i.e.,

$$R_{xx'}(\tau) = R_{vv'}(\tau \bmod N_1) R_{ww'}(\tau \bmod N_2).$$

Note that if either of periodic correlation functions for original sequences takes zero, i.e., $R_{vv'}(\tau \bmod N_1) = 0$ or $R_{ww'}(\tau \bmod N_2) = 0$, that for the synthesized sequences does zero, i.e., $R_{xx'}(\tau) = 0$ at a shift τ.

(a) Binary ZCZ sets

If Z of (1) consists of a binary sequences and satisfies (2) with $K = 1$ for

$$G_{z^j z^k}(\tau) \equiv R_{z^j z^k}(\tau), \qquad (3)$$

it is called a binary ZCZ set.

We can easily produce binary ZCZ sets by $N = 2^n$, $Zcz = 2^m$, and $M = \frac{N}{2Zcz}$ [2]-[5], which can can facilitate hardware construction. However they can not satisfy the bound except $Zcz = 1$, and include the periodically same sequences. The former means the decrease of the user number or that of the system capacity. The latter does the increase of influence of long multipath. Therefore it seems that binary ZCZ sets can not construct effective cellular CDMA systems in the view of frequency usage.

(b) Ternary ZCZ sets

If Z of (1) consists of ternary sequences, and satisfies (2) with $K = 1$ for (3), it is called a ternary ZCZ set [8][9].

Let v be a perfect sequence of length N_1, whose periodic auto-correlation function is impulsive, i.e, $R_{vv}(\tau) = 0 (\tau \neq 0)$. There are some different perfect sequences for length $N_1 = 7, 13, 21, 28, 31,$ and so on. On the other hand, there is only binary one, $(1, 1, 1, -1)$. Let C be an orthogonal code of N_2 codewords c^j's written as

$$\begin{aligned} C &= \{c^1, \cdots, c^j, \cdots, c^{N_2}\}, \\ c^j &= (c_0^j, \cdots, c_i^j, \cdots, c_{N_2-1}^j), \end{aligned}$$

which satisfies $R_{c^j c^k}(0) = 0$ $(j \neq k)$. The codewords correspond to the rows of a Hadamard matrix.

We obtain a ternary ZCZ set with $N = N_1 N_2$, $M = N_2$ and $Zcz = N_1 - 1$, if

$$z^j = v \odot c^j.$$

Note that it satisfies the bound, and facilitates hardware construction as well as a binary ZCZ set.

(c) Paired ZCZ sets

If Z of (1) consists of real sequence pairs $z^j = [x^j, y^j]$'s, and satisfies (2) with $K = 1$ for

$$G_{z^j z^k}(\tau) \equiv R_{x^j y^k}(\tau),$$

it is called a paired ZCZ set.

Let $[v, w]$ be a pair whose periodic cross-correlation function is the same as the periodic auto-one for a perfect sequence, i.e.,

$$R_{vw}(\tau) = 0 \quad \text{for } \tau \neq 0. \tag{4}$$

If v is appropriately given, its mate w can be uniquely derived from the property of (4), i.e.,

$$v^t = W^{-1}(N, 0, \cdots, 0)^t,$$

where v^t denotes the transpose of v, and W^{-1} the inverse of the Hankel matrix of w expressed by

$$W = \begin{bmatrix} w_0 & w_1 & \cdots & w_{N-1} \\ w_1 & w_2 & \cdots & w_0 \\ \vdots & \vdots & & \vdots \\ w_{N-1} & w_0 & \cdots & w_{N-2} \end{bmatrix}.$$

We now call it an orthogonal pair. Note that that the mate w of a binary sequence v is a real sequence.

We have a paired ZCZ set with $N = N_1 N_2$, $M = N_2$ and $Zcz = N_1 - 1$, if

$$x^j = v \odot c^j, \quad y^j = w \odot c^j.$$

Note that a paired ZCZ set satisfying the bound for any length can be produced, since an orthogonal pair $[v, w]$ exists for any length. For CDMA systems, when a user uses a binary sequence x^j, he will looks as if a binary ZCZ set satisfying the bound is used. Note that a data bit multiplied by x^j can be decoded by correlation for y^j.

(d) Ternary complementary ZCZ sets

If Z of (1) consists of sequence pairs $z^j = [x^j, y^j]$'s, and satisfies (2) with $K = 2$ for

$$G_{z^j z^k}(\tau) \equiv R_{x^j x^k}(\tau) + R_{y^j y^k}(\tau),$$

it is called a complementary ZCZ set.

Since it is considered as a set with twice length, the bound of size can be expressed as $M = \frac{2N}{Zcz+1}$.

We consider a ternary perfect complementary pair of length N_1, $\{[v, w], [v', w']\}$, satisfying

$$\begin{aligned} C_{vv'}(\tau) + C_{ww'}(\tau) &= 0 \text{ for any } \tau, \\ C_{vv}(\tau) + C_{ww}(\tau) &= 0 \text{ for } \tau \neq 0 \\ C_{v'v'}(\tau) + C_{w'w'}(\tau) &= 0 \text{ for } \tau \neq 0, \end{aligned}$$

where $C_{vv'}(\tau)$ denotes the aperiodic correlation function defined by

$$C_{vv'}(\tau) = \begin{cases} \displaystyle\sum_{i=0}^{N_1-1-\tau} v_i v'^*_{i+\tau} & , 0 \leq \tau \leq N_1 - 1, \\ \displaystyle\sum_{i=0}^{N_1-1+\tau} v_{i-\tau} v'^*_i & , 1 - N_1 \leq \tau < 0, \\ 0 & , |\tau| \geq N_1. \end{cases}$$

It has been known that a perfect complementary pair of length $L2^n (L = 2, 10, 26)$ can be easily constructed [13][15]. If zeros are added to it, before or behind, a ternary perfect complementary pair is given easily.

We have a ternary complementary ZCZ set with $N = N_1 N_2$, $M = 2N_2$ and $Zcz = N_1 - 1$, if

$$x^j = v \odot c^j, y^j = w \odot c^j, x^{2j} = v' \odot c^j, y^{2j} = w' \odot c^j.$$

We note that the ternary complementary ZCZ set satisfies the bound, and its length is changeable free. CDMA systems using those may be complicated, since two signals multiplied a data bit by x^j and y^j must be separately transmitted by FDMA (frequency division multiple access) or TDMA (time division multiple access).

(e) Others

Quadriphase ZCZ sets, whose elements take values in $\{\pm 1, \pm\sqrt{-1}\}$, can be produced, but they do not satisfy the bound, generally. Complex and real ZCZ sets satisfying the bound can be easily produced by the same method as (b)-(d), but the hardware cost for CDMA systems increases. In particular dynamic range of the elements sometimes come into trouble question, except ZCZ sets, whose elements take the unit magnitude. Therefore we do not discuss them in this paper.

3 Evaluation of Inter-Cell Interference

We now explain that application of ZCZ sets to cellular AS-CDMA systems, that synchronization among users in a cell can be controlled within permissible time difference (PTD), can remove inter-cell interference, and decrease influence of multipath. We assume ZCZ sets of (a) or (b) in section 2 to discuss simply. Suppose that the following discussion can adapt to ZCZ sets of (c) and (d) in section 3.

Let $\hat{z}^j$ be a spreading sequence of length $N+2T$ derived from z^j in a ZCZ set of (1), written as

$$\hat{z}^j = (z^j_{N-T}, \cdots, z^j_{N-1}, \underbrace{z^j_0, \cdots, z^j_{N-1}}_{z^j}, z^j_0, \cdots, z^j_{T-1}),$$

where T corresponds to PTD.

We assume that the heads of data frames, $d^j\hat{z}^j$'s, that users in a cell transmit, are received within PTD. The output of a correlator between received data frames and a sequence z^j at $|\tau| \le T$ is always expressed by periodic auto/cross-correlation functions between z^j and each of sequences z^k's even if binary data bits change $1(-1)$ to $-1(1)$. Therefore the ZCZ set can remove inter-cell interference, successfully, and decrease influence of multipath.

Now we consider a basic model of cellular AS-CDMA systems, that synchronization among cells can not be controlled. Let S be a set consisting of U ZCZ sets Z^l's written as

$$\begin{aligned} S &= \{Z^1, \cdots, Z^l, \cdots, Z^U\}, \\ Z^l &= \{z^{l,1}, \cdots, z^{l,j}, \cdots, z^{l,M}\}. \end{aligned}$$

We assume that ZCZ sets Z^l's are severally assigned to adjacent cells. For hexagonal cells, assignment of different ZCZ sets to adjacent cells needs $U=3$ at least. We note that the construction in section 2 can give a lot of periodically different ZCZ sets, if perfect sequences of length N_1 and orthogonal codes of length N_2 are different severally. We also note that influence of inter-cell interference relates to values for cross-correlation functions among ZCZ sets at any shift. The cross-correlation functions include the periodic correlation function (even correlation function) and the odd-correlation function, which are expressed by the sum or difference of two aperiodic cross-correlation functions.

From the above discussion inter-cell interference can be evaluated by the average signal to noise ratio (SNR) or the average interference parameter (AIP) for asynchronous CDMA systems [12]. For decoding of binary data bits of a user having a sequence $z^{l,j} \in Z^l$, SNR can be written as

$$SNR_{lj} = \frac{1}{\frac{1}{6NC^2_{z^{lj}z^{mk}}(0)} \sum D_k \gamma_{lj,mk} + N_0/2E_{lj}},$$

where $N_0/2$ denotes the Gaussian noise power density, E_{mk} the received signal power level per a bit from a user having $z^{mk} \in Z^m$ with $m \ne l$, $D_{mk} = E_{mk}/E_{lj}$, and $\gamma_{lj,mk}$ is AIP expressed by

$$\begin{aligned} \gamma_{lj,mk} &= \sum_{\tau=1-N}^{N-1} \{2C^2_{z^{lj}z^{mk}}(\tau) \\ &\quad + C_{z^{lj}z^{mk}}(\tau) C_{z^{lj}z^{mk}}(\tau+1)\}. \end{aligned}$$

In order to evaluate of a set S of ZCZ sets, we investigate the average of all the AIP's between z^{lj} and z^{mk} of length N,

$$\gamma = \mathrm{E}[\gamma_{lj,mk}], \ \ l \ne m.$$

It seems that we should exactly discuss the average of AIP's between z^{lj} of length N and $\hat{z}^{mk}$ of length $N+2T$, $\hat{\gamma}$. However, since we can assume $2T << N$, we have $\hat{\gamma} \approx \gamma$. Therefore we can use γ on inter-cell intereference for this model.

4 Numerical Results

It has been known that AIP for a binary set with low periodic correlation properties, e.g., the Gold set [14], or a binary set with good randomness, is almost the same as twice of a square of the peak value of the periodic auto-correlation function, i.e.,

$$\gamma \approx 2R_{z^{lj}z^{lj}}(0) = 2C^2_{z^{lj}z^{lj}}(0).$$

Therefore inter-cell interference for the basic model will be discussed by

$$r = \gamma / 2C^2_{z^{lj}z^{lj}}(0).$$

As shown in Tables 1 and 2 the above fact can be understood by $r \approx 1$.

It means that if $r > 1$, intere-cell interference for the basic model is higher than that or intra-cell interference for asynchronous CDMA systems, on assumption of the same received power. Note that AIP's for paired ZCZ sets and complementary ZCZ sets can be similarly considered.

Tables 3-6 show r's for sets S's consisting of ZCZ sets in section 3. We made ZCZ sets in each set S different, as much as possible.

Table 1: On AIP's for the Gold sets.

N	r	N	r
63	0.931	127	1.044
511	1.027	1023	1.000

Table 2. On AIP's for randamly given sets.

64	1.023052	128	1.010425
256	0.993595	512	0.991014
1024	0.980798		

Table 3. On AIP's for binary ZCZ sets.

N	Zcz	M	r	N	Zcz	M	r
64	8	4	1.115	256	8	16	1.406
64	4	8	1.237	512	64	4	1.123
128	16	4	1.118	512	32	8	1.272
128	8	8	1.248	512	16	16	1.429
256	32	4	1.123	1024	32	16	1.454
256	16	8	1.263				

Table 4. On AIP's for ternary ZCZ sets.

N	Zcz	M	r	N	Zcz	M	r
52	12	4	0.982	248	30	8	1.011
84	20	4	0.864	336	20	16	0.953
104	12	8	0.980	416	12	32	1.013
124	30	4	0.974	496	30	16	1.027
168	20	8	0.982	672	20	32	0.983
208	12	16	1.019	992	30	32	1.002

Table 5. On AIP's for paired ZCZ sets.

N	Zcz	M	r	N	Zcz	M	r
56	6	8	1.269	252	62	4	1.336
60	14	4	1.390	480	14	32	1.283
112	6	16	1.382	496	30	16	1.339
120	14	8	1.238	504	62	8	1.340
124	30	4	1.270	508	126	4	1.398
224	6	32	1.224	992	30	32	1.352
240	14	16	1.404	1008	62	16	1.392
248	30	8	1.303	1016	126	8	1.357

Table 6. On AIP's for complementary ZCZ sets.

N	Zcz	M	r	N	Zcz	M	r
68	16	4	0.961	272	16	16	1.156
72	8	8	1.355	288	8	32	1.466
132	32	4	1.055	528	32	16	1.284
136	16	8	1.272	544	16	32	1.382
144	8	16	1.269	1056	32	32	1.539
264	32	8	1.420				

5 Conclusion

For the basic model of celular AS-CDMA systems, we have examined AIP's for the four types of ZCZ sets, which can inverstigate inter-cell interference. It has been shown that AIP's for ternary ZCZ sets are almost the same as that for the Gold set in asynchronous CDMA systems. However AIP's for the others are higher than them. As a result ternary ZCZ sets are suitable for cellular AS-CDMA systems with asynchronously considered enviroment. We note that binary ZCZ sets can not good cellular AS-CDMA systems, since they can not satisfy the bound, and their AIP is higher than that of ternary ZCZ sets.

We will construct good cellular AS-CDMA systems using ternary ZCZ sets, which can realize high system capacity and realize quality service, so that each user requests for multi-media transmission.

Acknowledgment

This reserch is supported by the Millenium Project Foundation of the Ministry od Education, Culture, Sports, Science and Technology of Japan.

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