WAVELET BASED NOISE DETECTION AND FILTERING

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ABSTRACT : We describe a novel method of removing additive noise of known variance from images . The method is based on a characterization of statistical properties of natural images represented in a complex wavelet decomposition . Specifically , we decompose the noisy image into wavelet sub bands , estimate the correlation of both the nose-free raw coefficients and their magnitudes within each sub band , impose these statistics by projecting onto the space of images having the desired correlations , and reconstruct an image from the modified wavelet coefficients . This process is applied repeatedly , and can be accelerated to produce optimal results in only a few iterations . De-noising results compare favorably to three- point formulae .
The proposed filter has shown extremely good performance in intelligence preservation and noise suppression while eliminating noise .Moreover this filter has shown excellent results with training data and highly corrupted noisy images.

Keywords:-de-noising, thresholding , rejective-filter, shrinkage, discretization.

I.WAVELET TRANSFORM

Wavelet transform is being used to represent a time and frequency varying signal in the appropriate reference frame conforming with Heisenberg`s uncertainty principle. A signal or an image which is varying in time and frequency simultaneously cannot be represented solely as a function of time (Fourier transform) or a function of frequency (laplace transform).
What is needed is a transform that can cater to both time and frequency variations at the same time with good resolution. The inhibition was primarily dealt by Heisenberg in a totally different perspective by formulating the well known uncertainty principle.

$\nabla x * \nabla p \geq h/4\pi$ ------ (1)

where;

∇x --difference in position of particle or signal

∇p --difference in momentum of particle or signal

h ---plank's constant

This leads us to an implicit function of time interval ,

$A = F(t, f)$ -------(2)

Where, $t \rightarrow$ time, $f \rightarrow$ frequency

The continuous wavelet transform is given by

$CWT(u,\tau) = \left(1/\sqrt{a}\right)\int s(t)\psi((t-\tau)/a)dt$ -----(3)

Where, $\psi(t)$ is the basic wavelet or mother wavelet and $\psi((t-\tau)/a)/\sqrt{a}$ is wavelet basis function called baby wavelet. The term wavelet means a small wave .The smallness refers to the condition that this (window) function is of finite length (compactly supported). The wave refers to the condition that this function is oscillatory. The term mother implies that the functions with different region of support that are used in the transformation process are derived from one main function , or the mother wavelet . In other words, the mother wavelet is a prototype for generating the other window functions.

The various wavelet function prototypes are given as below :

1)Modulated Gaussian (Morlet)

$\psi(t) = e^{jwt} e^{-t^2/2}$ -------------(4)

2.Second derivative of a Gaussian

$\psi(t) = (1-t^2)e^{-t^2/2}$ -------------(5)

3.Haar

$$\psi(t) = \begin{cases} 1, & 0 \leq t \leq ½ \\ -1, & ½ \leq t \leq 1 \\ 0, & \text{otherwise} \end{cases} \quad \text{-------------(6)}$$

4.Shannon

$\psi(t) = (\sin(\pi t/2)/(\pi t/2))(\cos 3\pi t/2)$ ----------(7)

The term translation is related to the location of window, as the window is shifted through the signal. This term corresponds to time information in transform domain.

Scaling as a mathematical operation , either dilates or compresses the signal. In terms of mathematical functions, if $f(t)$ is a given function $f(st)$ corresponds to a contracted signal (compressed) version of f(t) if $s \succ 1$ and to an expanded (dilated) version of $f(t)$ if $s \prec 1$.However in the definition of wavelet transform , the scaling term is used in the denominator and therefore, the opposite of the above statements holds, i.e scales $s \succ 1$ dilates the signal whereas $s \prec 1$ compresses the signals.

II.WAVELET BASICS

The discrete wavelet transform (DWT)

$$DWT(m,n) = 2^{-m/2} \sum s(k) \psi(2^{-m} k - n) \text{--------}(8)$$

Where the discrete wavelet $\psi(k)$ can be ,but not necessarily,a sampled version of a continuous counterpart. That is, it is possible that $\psi(k)$ may not have a continuous time version. When $\psi(k)$ is a discretization of a $\psi(t)$,the DWT is identical to the $DTWT$ (Discrete time wavelet transform).

$$DTWT(m,n) = a_0^{-m/2} \sum s(k) \psi(a_0^{-m} k - n\tau_0) \text{------}(9)$$

III.THE WAVELET SYNTHESIS

The continuous wavelet transform is reversible ,if equation (11) is satisfied , even though the basis functions are in general may not be orthonormal. Fortunately this is very non-restrictive requirement. The reconstruction is possible by using the following reconstruction formula:

$$X(t) = 1/c_x^2 \int_s \psi_x^{\psi}(\tau, s) \frac{1}{s^2} \psi\{(t-\tau)/s\} d\tau ds \text{-------}(10)$$

Where ψ is a constant that depends on the wavelet used. The success of the reconstrution depend upon this constant called,the admissibility constant,to satisfy the following admissibility condition :

$$c_\psi = 2\pi \int_{-\infty} \psi \mid (\xi) \mid^2 / \mid \xi \mid d\xi \prec \infty \quad \text{----------}(11)$$

Where $\psi \mid (\xi) \mid$ at (x_i) is the FT of $\psi(t)$. Equation (11) implies that $\psi \mid (\xi) \mid$ at $x(0) = 0$,

which is $\int \psi(t) dt = 0$ -----------(12)

As stated above, equation(12) is not a very restrictive requirement since many wavelet functions can be found whose integral is zero. For equation (12) to be satisfied ,the wavelet must be oscillatory.

IV.COMPUTATION OF THE CWT

The computation of CWT starts with the choice of mother wavelet is chosen to serve as a prototype for all windows in the process. Once the mother wavelet is chosen the computation starts with $s = 1$ and the continuous wavelet transform is computed for all values of $s \prec\succ 1$,i.e. smaller and larger than"1". However, depending upon the signal, a complete transform is usually not necessary. For all practical purposes, the signals are band-limited, and therefore, computation of the transform for a limited interval of scales is usually adequate. In this study, some finite interval values of $'s'$ are used.

For convenience, the procedure will be started from scale $s = 1$ and will continue for the increasing values of $'s'$, i.e., the analysis will start from high frequencies and proceed towards low frequencies. This first value of s will correspond to the most compressed wavelet. As time the value of $'s'$ increased, the wavelet will dilate.

The wavelet is placed at the beginning of the signal at the point which correspond to time $t = 0$. The wavelet function at scale "1" is multiplied by the signal and then integrated over all times. The result of the integrated is then multiplied by the constant number $1/\sqrt{s}$ .This multiplication is for energy normalization purposes so that the transform signal will have the same energy at every scale .The final result is the value of the transformation, i.e., the value of the continuous wavelet transform at time zero and scale $s = 1$. In other words, it is the value that corresponds to the point $\tau = 0$, $s = 1$ in time-scale plane.

The wavelet at scale $s = 1$ is then shifted towards the right by τ amount to the location $t = \tau$, and the above equation is computed to get the transform value at $t = \tau$, $s = 1$ in the time-frequency plane. This procedure is repeated until the wavelet reaches the end of the signal. One row of points on the time-scale plane for scale $s = 1$ is now completed.

Then, $'s'$ is increased by a small value. Note that, this is a continuous transform, and therefore, both τ and $'s'$ must be incremented continuously. However, if this transform needs to be computed by a computer, then both parameters are increased by a sufficiently small step size. This corresponds to sampling the time-scale plane.

V. PROBLEM DEFINITION

Consider an observed signal modeled as

$$b = L_a + n \text{------------}(13)$$

where n is a random, zero mean , signal dependent noise, and L_a is a liner operator defining the distortion. Let the distorted signal be described in an orthogonal transform domain as

$$\beta_r = \lambda_r \alpha_\eta + v_r \text{--------}(14)$$

where λ_r are representation coefficients of the linear operator L_a in the transform domain , and v_r are zero mean spectral coefficients of the realization of the noise interference , and $'r'$ is the spectral component index. Transform domain filtering basically consists of the following three steps :

1). Computing spectral coefficients $\beta = Tb$ of the observed image fragment b within a window over the chosen orthogonal transform T .

2). Multiplication of the obtained spectral coefficients by the filter coefficients $\{\eta_r\}$ given by

$$\alpha_r = \eta_r \beta_r \qquad \text{----------(15)}$$

3). Inverse transformation T^{-1} of the output signal spectral coefficients $\{\alpha_\eta\}$,

where subscripts $'r'$ are corresponding indices in the transform domain.

VI.WAVELET DENOISING

The idea of shrinkage of transform coefficient that are lower (in magnitude) than a certain threshold recently reappeared and obtained popularity in the form of wavelet shrinkage .

Wavelet transform has the locality , multi-resolution and compression properties which make it a popular analysis tool for several signal processing applications. It compresses a signal into a very small number of large coefficients . Given a signal corrupted with large wavelet coefficients whereas noise is distributed across small wavelet coefficients.

Wavelet de-noising operates in the same three steps introduced earlier , where T is now a wavelet transform . Although wavelet de-noising operates on the overall image rather than on a sliding window , localization property of wavelet transform makes it possible to consider local behavior of the data . The wavelet transform domain filtering is performed either by soft thresholding where the filter coefficients are

$$\eta_x = \max[0, (|\beta_n| - thr)/|\beta_n|] \qquad \text{----------(16)}$$

and the filter can be realized in the form given by zero-order estimation method as

$$AVsys|\beta_r|^2 \cong |\beta_n|^2$$

$$AVsig|\lambda_r|^2|\alpha_r|^2 \cong \left(0, |\beta_r|^2 - |v_r|^2\right) \qquad \text{---------(17)}$$

from which the following filter realization for signal de-noising is formed as

$$\eta_x = \max\left[0, \left\langle |\beta_r|^2 - |v_r|^2 \right\rangle / (\lambda_r |\beta_r|^2)\right], \text{ for } \lambda_r \neq 0.$$

$$= 0, \qquad \text{otherwise -----------(18)}$$

Where $AVsys$ and $AVsig$ denote averaging over realizations of signaling system sensor noise and unknown parameters of the signals respectively and the value of $'thr'$ i.e. threshold is associated with the variance of the additive noise. It is assumed here that the noise spectral density $|v_r|^2$ is known . If noise is assumed to be 'white', $|v_r|^2$ is a constant .

This filter can also further simplified to its 'binary' implementation called as 'rejective filter' and is given by

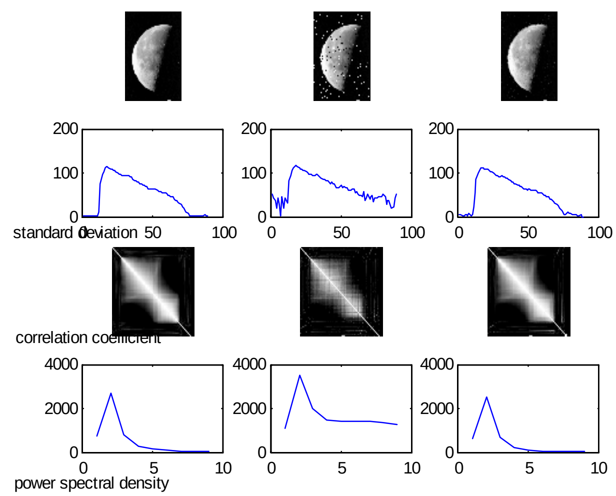
$$\eta_r = 1/\lambda_r, \quad |\beta_r|^2 \geq thr \text{ and } \lambda_r \neq 0.$$

$$= 0, \qquad \text{otherwise} \qquad \text{-----------(19)}$$

i.e. hard thresholding implementation where filter coefficient we defined in equation (19) with $\lambda_r = 1$. In this filter using soft thresholding, those signal spectral components that do not exceed by magnitude, a certain thresholds are zeroed and therefore removed from the signal .

VII.RESULT

We have obtained the desired success working with different with varying noise concentration. We have shown the results of moon and Saturn image for justification of our algorithm.



VIII.CONCLUSION

Local transform domain de-noising and wavelet de-noising are reviewed and their performance is tested over different images . We have introduced a correlation-dependent model for thresholding noisy coefficients of non-orthogonal wavelet transforms . This model is based on the correlation structure of the transform and includes a scale-wavelet dependent threshold that reduces to the famous uniform threshold $\left(\sqrt{2}\log N\right)$ in the case of an orthogonal wavelet transform . The threshold is explicitly calculated for biorthogonal and translation invariant wavelet systems . We compared denoising results for hard and soft thresholding techniques using the new thresholding scheme .

Using hard-thresholding the new correlation model clearly outperforms the uniform model since outliers , that occur in the latter case, vanishes . When applying soft-thresholding a more global smoothness of the estimate is achieved and small oscillations , that disturb the global smoothness after soft-thresholding with the uniform threshold , are removed.

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