A Geometry Curriculum Featuring the Use of Dynamic Computer Software

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Abstract: - This article is to describe a geometry curriculum featuring the use of dynamic computer software such as The Geometer's Sketchpad (GSP) for preservice mathematics teachers. While emphasizing investigations and discovery learning, this curriculum is unique in the following aspects/focuses: (1) The curriculum will focus on the real world applications of mathematics; (2) The curriculum will use many geometric situations to show how the use of GSP can facilitate mathematical reasoning; 3) The curriculum will discuss intensively three-dimensional geometry; 4) The curriculum will explore dynamic graphing of functions and relations using GSP; and 5) The curriculum will introduce students to the recreation/motivation aspect of using the software.

Key-Words: - Geometry, Technology, Computer, Dynamic software, Preservice mathematics teachers, Mathematical reasoning, Real world application

1. Introduction

This paper is to describe a geometry curriculum that we are developing for prospective 5-12 mathematics teachers. The curriculum will focus on the study of Euclidean Geometry from an advanced standpoint, using technology as a primary tool for investigation and understanding. The curriculum will increase prospective teachers' knowledge of geometry, simultaneously advancing their level of sophistication with Geometer’s Sketchpad and its use in conducting technology supported geometric explorations. While the curriculum is primarily designed to enhance their knowledge of mathematics, pedagogical issues will also be raised indirectly.

2. Who will use this curriculum?

The use of technology in mathematics education has gained considerable attention in recent years. An increasing number of mathematics educators and researchers believe that the use of technology can facilitate learning and teaching at all levels of education. The NCTM Standards (1989, 2000) documents emphasize the effective use of technology as one of the main vehicles of the reform curriculum.

Recent literature on the impact of the use of technology on cognitive development of learners manifests positive messages about their potential for improving learners' problem solving skills. A number of research studies have suggested that the use of interactive software assists learners in making mathematical abstractions and generalizations within algebraic , geometric, and probabilistic domains (Dixon 1997, Kaput & Thompson 1994, Manouchehri 1994, Jiang 1993, Olive 1991, Edwards, 1991). Others have proposed that by providing students with a learning environment enhanced by technology, motivation and interest in mathematics may see tremendous growth (Manouchehri et al. 1998). Only recently, the potential of technology for facilitating the formation of learning communities has been explored (Manouchehri & Pagnucco 2000). There is some indication that the presence of dynamic software with immediate feedback capability engages students in productive and collaborative discourse about mathematical ideas and concepts.

The body of literature is promising. However, technology is only a promising tool if it is effectively used by those critical to reforming education – the teachers. For the widespread and effective use of technology to enhance student learning, greater attention needs to be devoted to preparing teachers in the area of technology use. Without qualified teachers who are both interested and confident in utilizing technology in their instructional repertoire, it is unlikely that students will gain benefits from using technology. In order to be such skilled teachers, preservice teachers must receive adequate technology training in teacher preparation programs and have enough practical experience in using technology with learners in schools. This training needs to be content specific, addressing issues and software of particular use in mathematics. General and theoretical perspectives on how technology should be used to enhance instruction do not enable teachers to modify their mathematical instruction (Manouchehri et al. 1998). Thus, effective use of technology should be a necessary and important component of mathematics teacher preparation programs.

The need to enhance attention to technology is being increasingly recognized by mathematics teacher educators. More and more teacher preparation programs throughout the nation have agreement on two major points: (1) technology should be integrated into all program courses wherever appropriate (especially mathematics content and methods courses) and involve it in students' field experiences; (2) a specific technology course, which focuses on learning mathematics with technology, should be required of all future teachers. In this course prospective teachers develop greater facilities with conducting technology-based explorations, problem solving, and mathematical reasoning. Moreover, they explore ways of teaching mathematics with technology. Many prestigious institutions across the country are currently offering a course of this nature. The institutions include University of Georgia (*Technology and Secondary School Mathematics* at http://jwilson.coe.uga.edu/EMT668), Florida International University, Central Michigan University, Western Michigan University,

University of Illinois, Middle Tennessee State University, University of Missouri-Columbia, University of Texas, Auburn University, Southern Illinois University, University of Chicago, North Carolina State University, and Vanderbilt University. At these institutions, both preservice teachers and practicing teachers who seek graduate degrees in mathematics education) are required to take at least one *Technology in Mathematics Education* course.

A review of the content contained in these courses reveals that Geometry is a major curricular focus for such courses. This is for several reasons. First, Geometry has always been a neglected area in mathematics education (NCTM 1991). Many teachers are uncomfortable with teaching Geometry because their own knowledge of the subject is inadequate. A majority of the teachers have never experienced Geometry beyond the high school course they took as students where the focus was on generating two column proofs. Thus, they lack both geometric intuition and knowledge necessary for teaching Geometry in ways that are constructive and conceptual. On the other hand, schools have a strong demand for qualified Geometry teachers. Second, prospective teachers often have little exposure to the type of investigative mathematics that lies at the heart of NCTM's Standards (2000). Geometry represents an excellent arena for problem solving, problem posing, mathematical reasoning, and mathematical communicating. Finally, dynamic geometry in general, and the Geometer's Sketchpad (GSP) in particular, is one of the best examples of technology that can be used to support innovative methods of mathematics teaching. Thus, teacher education programs have recognized the need to increase future teachers' knowledge of Geometry, help them develop their own mathematical power, and expose them to innovative methods of teaching it through the use of dynamic geometry software, especially GSP. Currently, curriculum materials or textbooks that address these needs in a systematic manner are lacking. The curriculum we are developing will speak to this need.

Note also that while some mathematics teacher education programs do not require a *Technology in Mathematics Education* course, even those programs incorporate a significant amount of time revisiting school geometry, as well as addressing technology use. Courses such as *Teaching Secondary (or Middle) School Geometry* or *Methods of Teaching Mathematics* address the content and introduce the teachers with interactive software for teaching and learning Geometry even though the instructors tend to have a smaller focus due to the amount of time available to them. Our proposed curriculum will be a useful resource for the instructors of such courses to identify and implement technology-based, meaningful learning activities to meet their instructional objectives.

3. The needs that this curriculum will address

In planning the foci of the curriculum, we had to first consider the areas of Geometry in which our preservice teachers need the greatest exposure, practice, and improvement. The NCTM *Principles and Standards for School Mathematics* (2000) and our own experience working with the preservice teachers, highlighted five major areas: 1) problem posing and solving in real-world situations, 2) mathematical reasoning and proof, 3) three-dimensional visualization and spatial thinking, 4) connections among different representations, and 5) the ability of motivating their future students in mathematics learning.

3.1. Problem posing and solving in real-world situations

Solving application problems using geometry has not been a focus of American Geometry curriculum (NCTM 2000). Thus, the preservice teachers whose education in geometry consisted only of what they learned in high school have had very little, if any, experience in solving real-world problems using geometry concepts. In fact, in a recent survey, Manouchehri (in preparation) asked nearly 500 undergraduate 4-12 mathematics education majors to describe a real life situation in which mathematics could be used to arrive at a solution. She found that only one student stated a problem whose solution required the use of geometry. This is a critical and yet devastating finding. It is widely agreed that "Geometry offers a means of describing, analyzing, and understanding the world and seeing beauty in its structures, geometric ideas can be useful both in other areas of mathematics and in applied settings. For example, symmetry can be useful in looking at functions; it also figures heavily in the arts, in design, and in the sciences." (NCTM, 2000, p. 309). Future teachers, however, have little to no insight into the power of geometry. Thus, it is necessary to equip them with ideas and techniques that will enable them to formulate, approach, and solve problems in applied settings, so that they can do the same with their future students. Our curriculum will present various real-world situations in which the future teachers will have opportunities to formulate and refine problems (if the problems do not arrive neatly packaged), to investigate problems from multiple perspectives to gain further insights, and to articulate problems clearly enough to arrive at solutions. Specifically, when the preservice teachers experience difficulties in the problem posing and solving processes, our curriculum will have constructed GSP sketches or scripts to help them develop ideas and strategies to approach solutions. These sketches or scripts are usually difficult for the future teaches who lack sound understandings of the problems to construct by themselves in the first place. As an example, consider the following problem:

STREET PARKING. You are on the planning commission for Algebraville, and plans are being made for the downtown shopping district revitalization. The streets are 60 feet wide, and an allowance must be made for both on-street parking and two-way traffic. Fifteen feet of roadway is needed for each lane of traffic. Parking spaces are to be 16 feet long and 10 feet wide, including the lines. You job is to determine which method of parking – parallel or angle – will allow the most room for the parking of cars and still allow a two-way traffic flow. (You may design parking for one city block (0.1 mile) and use that design for the entire shopping district.)

When exploring this problem, a considerable number of students (i.e., preservice teachers, the same hereafter if not specified) may intuitively conjecture that angle parking is better than parallel parking (because they mostly experience angle parking). The parallel parking situation is quite simple, and students are able to quickly determine how many cars can be parked in one city block. However, most of the students would have difficulty formulating a mathematical solution for the angle-parking situation, other than the intuitive conjecture. The curriculum will present a constructed GSP sketch (see Figure 1) for students to investigate the situation. By working with the sketch, the students can clearly see that in order not to block the traffic on the right side of the street, the left side of each parking space has to be "dragged down" so that the yellow rectangle does not intersect the lane of traffic (or at most "touches" the traffic lane at one point only). Doing so will make the curb space very long, resulting in the fact that much more space is wasted and fewer vehicles can be parked in one city block. This visual feedback will help students correct their misconceptions. Moreover, their experience with the situation enhances the change that they will be able to identify key factors in the situation. Using similar-triangle relationships and the

Pythagorean Theorem, students can finally arrive at a mathematical solution, and understand why parallel parking is often used in downtown areas where streets may be narrow.

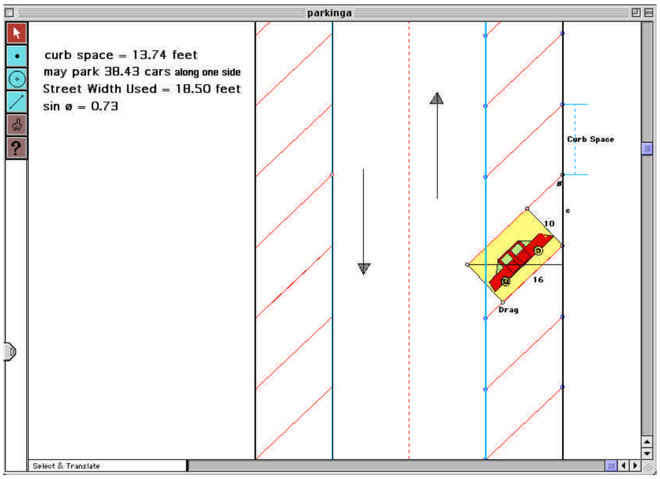

Fig. 1. A GSP sketch to help students understand the angle-parking situation.

3.2. Technology-facilitated mathematical reasoning and proof

> "The problem that students have with perceiving a need for proof is well-known to all high school teachers and is identified without exception in all educational research as a major problem in the teaching of proof" (de Villiers, 1999, p.3).

Several factors contribute to this problem/difficulty. First, in order for students to attempt to prove an assertion, they must first be convinced of its validity. This convincing cannot occur in the abstract and should be grounded in their own examination of various cases. As students construct and examine multiple cases for which the assertion may or may not prove valid, they recognize those central features and relationships that are needed in formulating deductive arguments. These exploratory moves are missing from the geometry curriculum. Thus, students are expected to "prove" statements without having any sense of what is central or peripheral in the context. This missing link then results in low student performance on tasks that require mathematical reasoning using both inductive and deductive means. This problem is not restricted to middle/high school students; preservice teachers share the same difficulty.

The importance of mathematical reasoning and proof is apparent – NCTM (2000) argued that "Judging, constructing, and communicating mathematically appropriate arguments, however, remain central to the study of geometry" (p. 309). Conceptualizing alternative ways of assisting students to develop the necessary skills in developing logical proofs is a central focus in mathematics teacher education. Experiences with GSP can facilitate both prospective teachers' understanding of proof and their ability to effectively motivate and teach proof to their students in the future.

Our curriculum will address this need by posing problems and explorations that facilitate engagement in mathematical reasoning and construction of proofs. Some of the proof-oriented problems might be quite challenging to students when they first experience the problems. However, activities will be designed to provide them with an opportunity to investigate various aspects and/or components of the problems using GSP. Through these technology-based explorations, they will discover new relationships and findings. These new findings may suggest insights for constructing valid proofs. This fact can help correct the common view that GSP is a powerful tool for discovery learning but has little to do with proofs. An example of the problem type and the related learning activities is as follows:

Problem: ABC is an arbitrary triangle. BD = **Error!***BC, CE =* **Error!***CA, and AF =* **Error!** *AB. PQR is formed by the construction of line segments AD, BE, and CF. What is the* *relationship between PQR and ABC? (see Figure 2)*

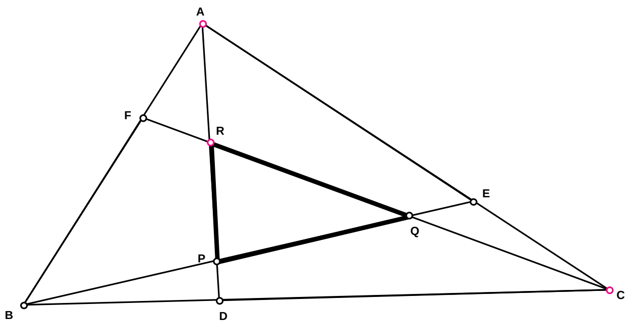

Fig. 2. What is the relationship between PQR and ABC?

With GSP's dynamic measurement feature, it is relatively easy for students to find that Area(PQR) : Area(ABC) = 1 : 7 for any triangle ABC. However, they may have difficulty proving this relationship. To proceed, they will be asked to continue their investigations with GSP. Through further investigation with GSP measurements, students can find that Area(BDP) = Area(CEQ) = Area(AFR) = **Error!**Area(PQR) = **Error!** Area(ABC); Area(Quadrilateral DCQP) = Area(Quadrilateral EARQ) = Area(Quadrilateral FBPR) = **Error!**Area(PQR) = **Error!** Area(ABC); BP : PQ : QE = 3 : 3 : 1; CQ : QR : RF = 3 : 3 : 1; and AR : RP : PD = 3 : 3 : 1. Stimulated by these new findings, the students may be able to come up with ideas for the proof. As a matter of fact, we used this activity in one of our classes of preservice teachers. A student, after having these new findings, decided to try to prove the relationship between the smallest triangles (such as BDP) and ABC, and he succeeded. Figure 3 gives his proof. Two other students found similar proofs. All three students believed it was the investigations supported by GSP that stimulated their insight for proof. They shared their ideas with the whole class.

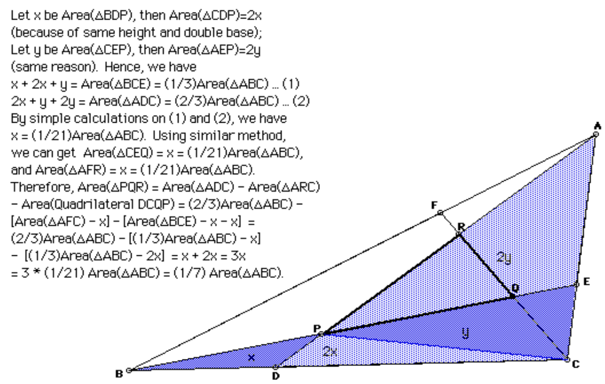

Figure 3. A student's proof.

3.3. Three-dimensional visualization and spatial thinking

Three-dimensional geometry has been a blank slate in high school geometry teaching and learning for decades. Even though some textbooks have a chapter or so on three-dimensional geometry, teachers rarely show interest in covering its content. The result is very few students (including our preservice teachers) have the knowledge of solid geometry. Research shows that only a small number of preservice teachers know the Platonic solids and their geometric properties. The consequences are apparent. If teachers do not know the concepts, it is likely that they skip teaching them in their own classrooms. The NCTM *Principles and Standards for School Mathematics* (2000) recognizes the challenge and emphasizes that instructional programs from pre-kindergarten through grade 12 should enable all students to analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships. Three-dimensional geometry is equally as (if not more) important as two-dimensional geometry. Therefore, the discussion on three-dimensional visualization and spatial thinking will be a major component of our curriculum. To address this goal, the students will be asked to observe and manipulate a variety of three-dimensional objects such as the Platonic and Archimedean solids, construct two-dimensional representations of these objects, and solve problems related to these objects/representations. Although we still concentrate on the two-dimensional representations of three-dimensional objects, these representations are dynamic in the GSP environment. This makes a difference, as seen in the following example.

Problem: Are the two red segments (see Figure 4a) congruent in the real model (of the icosahedron)? Why?

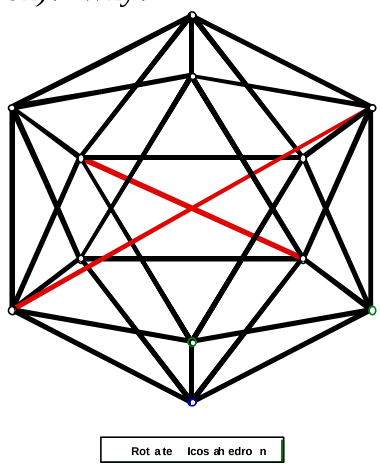

Fig. 4a. The two red segments do not look congruent.

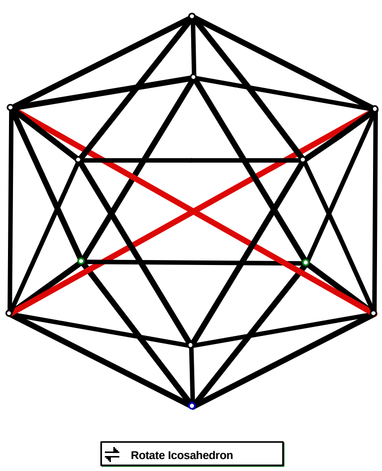

Fig. 4b. The two red segments look congruent in another orientation.

If a student is not familiar with the geometric properties of an icosahedron, he (or she) may conjecture that the two red segments are not congruent because they appear to have different lengths. Making incorrect conjectures is not a bad thing. In fact, it is a natural component of constructive learning process. From the constructivist point of view, substantive learning takes place over a long period of time and occurs during periods of confusion and conflict. Once this conflict is resolved, assimilation of new understanding occurs.

In this activity, students have the autonomy to explore and make conjectures. Following the conjecturing phase, they will be allowed to manipulate the GSP dynamic representation shown in Figure 4 to test their conjectures. Students may easily click the button under the two-dimensional model (representation) to view different results when the model changes its orientation. One of the orientations (Figure 4b) shows that the two red segments are indeed congruent. The immediate visual feedback provided by the model will stimulate the student's strong desire to struggle against and clear his (or her) mental confusion and conflict, thus achieving improved conceptual understanding. They will realize that for a three-dimensional object, even though we construct its two-dimensional representation using the perspective approach, the visual properties of the two-dimensional representation are usually not consistent with the logical properties of the geometrical object (e.g., perpendicularity and congruency of segments and angles). Furthermore, if the student looks more closely at the situation shown in Figure 4b, he (or she) will find that these two red segments are actually two diagonals of a rectangle. This is a critical finding for him (or her) to arrive at the explanation or proof for the "Why" question. Thus, this example also illustrates how the use of GSP can facilitate mathematical reasoning (spatial reasoning in particular) and proof.

3.4. Connections among different representations

The NCTM *Principles and Standards for School Mathematics* (2000) indicates that students should recognize connections among different representations, thus enabling them to use these representations flexibly. This includes describing spatial relationships using coordinate and transformation geometry systems. GSP combines its dynamic feature with function/relation graphing in rectangular and polar coordinate systems, thus revealing the natural connection among geometry, trigonometry, and algebra, and especially the natural connection among the graphical, numerical, and symbolic representations of a function or relation. Our curriculum will take advantage of this unique capability and include a series of function/relation graphing activities. These activities allow students to dynamically manipulate graphs and observe the corresponding changes of the numerical and algebraic expressions of the corresponding functions or relations, or to examine the dynamic generating process of the graphs. The process of linking the symbolic and graphical representations in these activities will enable students to deepen their understandings of the related functions and relations, thus developing their mathematical sophistication. An important part of the activities will require that students pay close attention to the parameter effect on various functions/relations. For any function/relation with coefficient parameters, students can create a controlling tool or slider (e.g., a "segment" with a value relative to the unit) for each coefficient parameter, and change the parameter by dragging the controlling tool. Students can then observe how the numerical change of each parameter affects both the graphical and algebraic representations of the function/relation. In the next task, students are asked to give interpretations of the parameter effects. By doing so, the students' conceptual understanding will be significantly improved. Figure 5 shows the graph of a trigonometric function of the form y = a*sin(b*x+c)+d, where a, b, c, and d are coefficient parameters. After dragging a, b, c, and d respectively, the students will clearly see that a determines the amplitude of "vibration" – the maximum and minimum values of the function, b determines the period of the function, c translates the graph horizontally, and d translates the graph vertically (showing the increase or decrease of the value of the function).

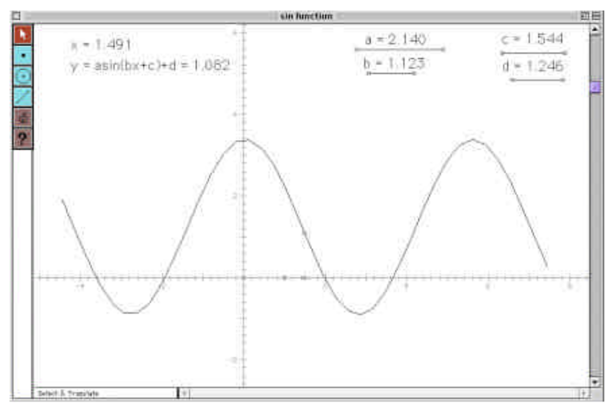

Figure 5. The coefficient effects on the sine function.

3.5. The ability of motivating their future students in learning mathematics

Lack of student interest in mathematics learning has a long history, and is well documented. Research indicates that the problem lies not in the structure of the subject matter, but in how the subject is taught (NCTM 1992, 2000). A majority of the students experience mathematics as a dry and rigid field that is only about right or wrong answers. Teachers' instructional practices should be carefully planned so as to challenge students' perceptions. The teachers need to become aware of multiple instructional tools useful for teaching mathematics along with ways to motivate and engage students in learning. Selecting and using suitable curricular materials is a critical consideration in the process. Using technology as an instructional tool is not sufficient to help overcome the challenge of lack of student interest. The teachers need to also learn how to design environments and contexts that address the affective needs of the classroom. Our curriculum will focus on assisting teachers to develop pedagogical skills in this area using GSP. For instance, the transformations available in GSP and its animation feature, as well as the ease at using buttons make the software a wonderful tool to design and implement various projects that present an engaging environment for doing mathematics. Observing and learning how to construct various designs and projects will enhance the preservice teachers' ability to motivate their future students in learning mathematics. Our curriculum will contain a set of such designs and projects created by our students and ourselves. One example is shown in Figure 6. The "Eiffel Tower" was constructed with transformations such as dilation and reflection. If you click the "Space Ship" button, then the "Space Ship" would move along a curve; if you click the "Jump" button, then the "person" on the top of the tower would jump down to the ground and the angle of jump can be adjusted, …

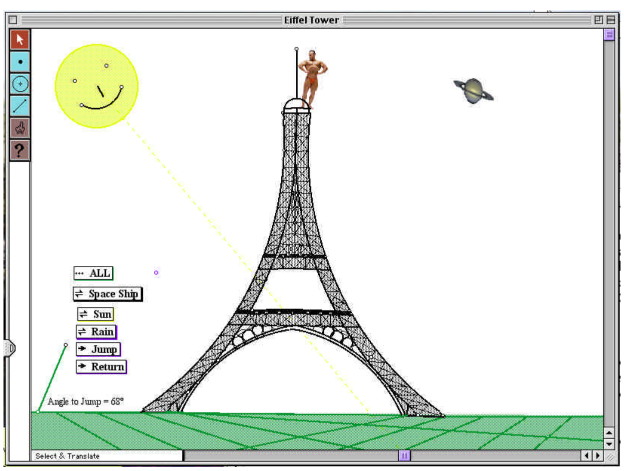

Figure 6. The " Eiffel Tower" and animations

In summary, while emphasizing investigations and discovery learning, our curriculum will be unique in the following aspects/focuses: 1) It will focus on the real world applications of mathematics; 2) It will use many geometric situations to show how the use of GSP can facilitate mathematical reasoning and proof; 3) It will discuss three-dimensional geometry intensively; 4) It will explore dynamic graphing of functions and relations using GSP features; and 5) It will introduce preservice teachers to the recreation/motivation aspect of using the software.

4. A comparison with existing curriculums

To the best of our knowledge, there is currently no other curriculum in the market that shares the same audience, visions, scope, and content as the one we propose. There are, of course, a number of curriculums, which concentrate on the use of GSP. Exploring Geometry with the Geometer’s Sketchpad (Key Curriculum Press, 1993) is an excellent activity curriculum that helps students learn geometric concepts and problem solving using GSP. Its drawback, however, is that it does not address proofs. We believe that the inductive nature of GSP cannot replace proof. In contrast, we propose to look at GSP as a tool to not only stimulate mathematical investigations but also enhance reasoning and constructing logical proofs. Moreover, this curriculum does not discuss three-dimensional geometry at all. Rethinking Proof with the Geometer’s Sketchpad (Key Curriculum Press, 1999) offers a stimulating discussion of multiple functions of proof in GSP environments. However, because it addresses only one aspect of the needs of our preservice teachers, it would not be suitable to use this curriculum as a text for mathematics teacher education. Moreover, our curriculum emphasizes how the use of technology can facilitate deductive reasoning, which has not been addressed by any existing text including the "Rethinking" curriculum. Lastly, no existing curriculum has developed in any depth the notion of utilizing mathematical modeling (solving application problems) and using geometry concepts in mathematics curriculum. In our curriculum we will attend to this need.

5. Conclusion

The ultimate goal of current efforts to reform mathematics education is to improve mathematics teaching and learning. The central importance of the teacher in reform is not open to debate. Ironically, while there has been a great deal of effort invested in designing and developing curriculum materials that capture the spirit of reform for school mathematics, little attention has been devoted to the same agenda at the teacher education level. This is particularly evident in middle and secondary mathematics teacher education. A review of current publications and texts indicate that although a number of curriculum materials aimed at improving pedagogical knowledge of teachers have been developed (appropriate for use in methods courses), there are virtually no curriculum that explicitly addresses the specific mathematical needs of future middle and high school teachers. We believe that our curriculum will set a new benchmark in mathematics teacher education. As mentioned above, the target audience of this curriculum is preservice mathematics teachers. However, it should also be very useful for inservice teachers and as a resource material. Since our work is heavily influenced by current research on learning and teaching mathematics, and well grounded in constructivist learning epistemology, a philosophy heavily emphasized by innovative teacher education programs, we anticipate that our proposed curriculum will be welcomed by both the mathematics education research community and the practitioners involved in mathematics education.

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