AN ALGEBRAIC ANALYSIS OF LINEAR HYBRID SYSTEMS

Zhenyu Yang

Department of Computer Science and Engineering
Aalborg University Esbjerg
Niels Bohrs Vej 8, DK-6700, Esbjerg
Denmark
yang@cs.aue.auc.dk, http://www.cs.aue.auc.dk/ yang

Abstract: The controllability of a class of hybrid systems, called *linear switched systems*, is analyzed using an algebraic approach. A necessary and sufficient condition is proposed based on the manipulation of system matrices. By applying the obtained results to the analysis of fault-tolerant control, the reconfigurability of the considered systems is also discussed and illustrated by one example.

Keywords: Linear hybrid systems, controllability, fault-tolerance, reconfigurability

1. INTRODUCTION

Hybrid systems (HS) are a kind of complex systems consisting of two distinct components, namely discrete-event driven subsystems and continuous -time evolving subsystems, which interact with each other and operate in real time. HS have been an intensive studied topic in the areas of control engineering and computer science, because of their importance in applications, such as hardware and embedded software verification, mobile communication networks and large scale, multi-agent systems analysis and design ((Antsaklis and Nerode (1998); Alur et al. (1993)).

If we focus on the control problem of hybrid systems, these systems are usually referred to as *Hybrid Control Systems (HCS)* (Branicky et al. (1998); Caines and Wei (1998); Stiver and Antsaklis (1993); Yang and Blanke (2000)). Within the feedback control framework (Rugh (1996)), a fundamental concept - *controllability* - is used to study the effect of the controller to the dynamical system operations. It can be observed that the functionality of the controller in HCS often relates to the dynamical characteristics of the continuous evolution and those of discrete transitions as well (Branicky et al. (1998); Yang et al. (1998); Yang and Blanke (2000)). Then, a challenging topic arises as how to evaluate and analyze this kind of functionality, and it is usually referred to as the controllability problem of hybrid control systems (Bemporad et al. (2000); Ezzine and Haddad (1989); van Schuppen (1998); Tittus and Egardt (1998); Yang et al. (1998); Yang and Blanke (2000)).

An amount of research work can be found for the controllability problem of HCS. Such as the controllable language method based on an abstracted Discrete-Event-System (DES) model of a continuous plant (Stiver and Antsaklis (1993)); the *hybrid controllability* definition and analysis for integral hybrid systems in Tittus and Egardt (1998); Some sufficient and/or necessary conditions in van Schuppen (1998); Yang and Blanke (2000). But, from the complexity point of view, the controllability problem of HCS is NP hard as stated in Bemporad et al. (2000); Blondel and Tsitsiklis (1999), even for simple classes of hybrid systems. However, if we focus on some specific systems, the controllability analysis can still be dealt with in an efficient way. Such as in Ezzine and Haddad (1989) the controllability of a class of linear switched systems was analyzed under assumption that the switching sequences and points were fixed in advance. One sufficient condition and one necessary condition were proposed in Yang et al. (1998) for a class of linear switched systems. Therefore, in the following, we focus on a class of piecewise linear hybrid systems called *linear switched systems.*

A linear switch system is a typical hybrid system with simple formulation about the system structures relating to discrete and continuous dynamics. Many complicated nonlinear systems can be approximated by these kind of systems with switching mechanism (Liberzon and Morse (1999)). In this paper, some sufficient and necessary conditions for the controllability analysis of the linear switched systems are obtained using an algebraic manipulation of system matrices. It can be noted that the existing result for LTI systems and results in Ezzine and Haddad (1989); Yang et al. (1998) become special cases of the results of this paper. Furthermore, By employing the obtained results, the reconfigurability, one fundamental property of fault-tolerant control (Blanke et al. (2001)), can be analyzed directly for the considered systems.

The paper is organized as follows: Section 2 formulates the controllability analysis problem. Section 3 defines some matrix manipulations; Section 4 states the main results; Section 5 discusses the reconfigurability for fault-tolerant control. Finally, we conclude the paper in Section 6.

2. PROBLEM FORMULATION

Consider a class of linear switched systems, denoted as

$$\begin{cases} \dot{x}(t) = A(\sigma(t))x(t) + B(\sigma(t))u(t), \\ y(t) = C(\sigma(t))x(t) \end{cases} \tag{1}$$

where the state variable $x(t) \in R^n$, (controllable) input variable $u(t) \in R^m$ and output variable $y(t) \in R^p$. $\sigma(t) : R^+ \mapsto N$ is a piecewise constant switching function mapping from the real time line R^+ to an integer set N. Matrices $A(\sigma), B(\sigma)$ and $C(\sigma)$ are piecewise constant matrices depending on values of σ. Obviously, the considered switched system (1) is a specific hybrid system with the *hybrid space* as a cross product of set N and Euclidean space R^n. Denote a *hybrid state* as (n, x) with $n \in N$ and $x \in R^n$, where n and$x$ are usually referred to as the *discrete state* and *continuous state*, respectively.

Based on these notations, we make the following assumptions:

- (i) The integer set N is finite;
- (ii) $\sigma(t)$ is left-continuous, and any time interval within which $\sigma(t)$ is constant is no less than a proper *dwelling time* (Liberzon and Morse (1999));
- (iii) The switching time set $\{t_i\}_{i=1}^k$ and corresponding switching mode set $\{n_i\}_{i=0}^k$ both can be determined by the control design, as well as the continuous-time control signal $u(t)$ within each selected mode;
- (iv) There is no discontinuous state jumps during mode switches.

The practical situation of the considered systems can be found in the design of an autonomous vehicle integrated with an autopilot system, where the autopilot system need select a proper operating mode regarding to different gears at any time, and meanwhile control the engine throttle continuously within each operating mode.

By employing the hybrid controllability concept given in Tittus and Egardt (1998); van Schuppen (1998); Yang and Blanke (2000), we have:

Definition 1: Given any pair of hybrid states, denoted as (n_0, x_0) and (n_k, x_k) respectively, if there exists a timed mode-switching set $\{(n_{i-1}, t_i, n_i)\}_{i=1}^k$ and a corresponding piecewise continuous finite input signal $u(t)$, such that system (1) evolving under these two distinct inputs is reachable from (n_0, x_0) to (n_k, x_k) within a finite time interval, then the considered system (1) is controllable, otherwise, system (1) is uncontrollable.

It is obvious that the controllability analysis can be performed by checking the reachability for any pair of hybrid states. In the following, we show that the controllability analysis of (1) can be efficiently coped with through manipulation of related system matrices.

3. PRELIMINARY KNOWLEDGE

An LTI system can be described by (1) when $size(N) = 1$. The matrix defined as $W_c \hat{=} [B\ AB\ \cdots\ A^{n-1}B]$ is usually used for the controllability analysis of LTI systems (Rugh (1996)). Similarly, when $size(N) \hat{=} q \geq 1$,

Definition 2: The matrix defined as

$$W_C \hat{=} [W_c^1\ \cdots\ W_c^q] \hat{=} [B_1\ A_1B_1\ \cdots\ A_1^{n-1}B_1 \\ \cdots\ B_q\ A_qB_q\ \cdots\ A_q^{n-1}B_q]. \tag{2}$$

is called the *controllability matrix of (1).*

In order to explore the interaction among multiple modes, some adjoint system matrix manipulation need to be further defined.

If the mode-switching order of one switched mode set is fixed as from mode n_{i_1} to mode

n_{i_2}, then n_{i_3}, until reaching n_{i_m}. For any given $j_1, j_2, \cdots, j_k$, matrix $A_{i_k}^{j_k} \cdots A_{i_2}^{j_2} A_{i_1}^{j_1} B_{i_1}$ has m columns. When $j_1, j_2, \cdots, j_k$ take all the possible values - $1, 2, \cdots, n-1$ - respectively, we construct a new matrix by all possible columns of $A_{i_k}^{j_k} \cdots A_{i_2}^{j_2} A_{i_1}^{j_1} B_{i_1}$ as columns of new matrix, which is an $n \times (mn^k)$ order matrix as defined in the following:

Definition 3: The matrix defined as

$$\text{Œ}^k(i_1, \cdots, i_k) \hat{=} [A_{i_k}^{j_k} \cdots A_{i_1}^{j_1} B_{i_1}]_{j_k, \cdots, j_1 = 0, 1, \cdots, n-1},$$

is called the *joint controllability matrix of modes* $n_{i_1}, n_{i_2}, \cdots, n_{i_k}$.

Specially, for any single mode $i \in N$, $\text{Œ}^1(i)$ is actually the controllability matrix W_c^i. If the space spanned by columns of matrix M is denoted as $span(M)$, then

Proposition 1: $\text{Œ}^m(i_1, i_2, \cdots, i_m)$ has the properties

- $span(\text{Œ}^{k-1}(i_1, i_2, \cdots, i_{k-1})) \subseteq span(\text{Œ}^k(i_1, i_2, \cdots, i_{k-1}, i_k))$.
- When the mode indices $i_l = i_s$ with $l \neq s$. Assume $l - s \geq 1$, then if the multiplication of A_{i_s} and A_{i_j} for $j = s+1, \cdots, l$ satisfy the immutable property, i.e., $A_{i_s} A_{i_j} = A_{i_j} A_{i_s}$, there is

$$span(\text{Œ}^k(i_1, \cdots, i_s, \cdots, i_{l-1}, i_l, i_{l+1}, \cdots, i_k)) = span(\text{Œ}^{k-1}(i_1, \cdots, i_s, \cdots, i_{l-1}, i_{l+1}, \cdots, i_k)).$$

Assume only the initial mode of a mode switching sequence which contains $k+1$ elements is fixed and denoted as $i \in N$, we can construct a new matrix, denoted as $\text{Æ}^k(i)$, based on the definition of $\text{Œ}^k(i_1, \cdots, i_k)$, and its columns consist of all possible columns of matrix $A_{i_k}^{j_k} A_{i_{k-1}}^{j_{k-1}} \cdots A_{i_1}^{j_1} A_i^j B_i$, where mode indices $i_k, i_{k-1}, \cdots, i_1$ take all possible values from $\{1, 2, \cdots, q\}$ respectively, under condition that the successive mode is different from the former, and the power indices $j_k, j_{k-1}, \cdots, j_1, j_i$ take all possible values from $\{0, 1, \cdots, n-1\}$ respectively. When parameter k is assigned as $0, 1, \cdots, k, \cdots$ respectively, an iterative definition of $\text{Æ}^k(i)$ is given in the following:

Definition 4: The matrix defined iteratively as

$$\begin{aligned} &\text{Æ}^0(i) \hat{=} \text{Œ}^1(i) = W_c^i, \\ &\text{Æ}^1(i) \hat{=} [\text{Œ}^2(i,j)]_{\substack{j = 1, \cdots, q \\ \text{with } j \neq i}} \\ &\vdots \\ &\text{Æ}^k(i) \hat{=} [\text{Œ}^{k+1}(i, i_1, \cdots, i_k)]_{\substack{i_1, \cdots, i_k = 1, \cdots, q \\ \text{with } i_1 \neq i, \cdots, i_k \neq i_{k-1}}} \end{aligned}$$

is called the *kth order joint controllability matrix of mode* n_i.

Proposition 2: $\text{Æ}^k(i)$ has the properties

- It is an $n \times (mn^{k+1}(q-1)^k)$ order matrix; and
- $span(\text{Æ}^k(i)) = span\left([A_j^l \text{Æ}^{k-1}(i)]_{\substack{j = 1, 2, \cdots, q \\ l = 0, 1, \cdots, n-1}} \right)$.

If there is no any specific mode or mode-switching order which has been fixed within a timed switching set, we should consider any possible mode-switching sequences. As the construction in the above analysis, a system joint controllability matrix is defined as:

Definition 5: The matrix defined iteratively as

$$\begin{aligned} \overline{W}^0 &= [\text{Æ}^0(1)\ \text{Æ}^0(2)\ \cdots\ \text{Æ}^0(q)] \\ &\vdots \\ \overline{W}^k &= [\text{Æ}^k(1)\ \text{Æ}^k(2)\ \cdots\ \text{Æ}^k(q)] \end{aligned}$$

is called the *kth order joint controllability matrix of (1)*.

Proposition 3: $\overline{W}^k$ has the properties:

- It is an $n \times (qmn^{k+1}(q-1)^k)$ order matrix;
- $span(\overline{W}^k) \subseteq span(\overline{W}^{k+1})$; and
- When the power index l satisfies $rank(\overline{W}^l) = rank(\overline{W}^{l+1})$, then $\forall k \geq l$, there is $rank(\overline{W}^k) = rank(\overline{W}^l)$.

From the third property of $\overline{W}^k$, we can define a system parameter as:

Definition 6: The parameter k_r defined as

$$k_r \hat{=} \min\{l | rank(\overline{W}^l) = rank(\overline{W}^{l+1})\} \quad (3)$$

is called the *joint controllability coefficient of system (1)*.

Proposition 4: k_r has the property $0 \leq k_r \leq n - n_0$, where n_0 is the rank of controllability matrix W_C defined in (2).

4. MAIN RESULTS

Lemma 1: Consider a matrix defined as

$$F \hat{=} \begin{bmatrix} \int_{t_1}^{t_2} b_0(t_{n+1} - \tau) d\tau & \cdots & \int_{t_n}^{t_{n+1}} b_0(t_{n+1} - \tau) d\tau \\ \vdots & \vdots & \vdots \\ \int_{t_1}^{t_2} b_{n-1}(t_{n+1} - \tau) d\tau & \cdots & \int_{t_n}^{t_{n+1}} b_{n-1}(t_{n+1} - \tau) d\tau \end{bmatrix} \quad (4)$$

where functions $b_i(t)$ are obtained from

$$e^{At} \hat{=} b_0(t) I + b_1(t) A + \cdots + b_{n-1} A^{n-1}, \quad (5)$$

there exists a real sequence $\{t_1, t_2, \cdots, t_{n+1}\}$, which satisfies $0 < t_1 < t_2 < \cdots < t_{n+1}$ and makes F defined in (4) non-singular.

Theorem 1: A binary-mode (1) is controllable if matrix W_C defined in (2) is of full row rank.

Proof: For any given initial and final hybrid states, denoted as (n_0, x_0) and (n_f, x_f) respectively, where $n_0, n_f \in N$ and $x_0, x_f \in R^n$, and initial and final time instants t_0 and t_f with $t_f > t_0$, without lose of generality, we assume that there are $n_0 = n_1$ and $n_f = n_2$ firstly.

Select a timed switching set as $\{(n_1, t_1, n_2)\}$ with property $t_0 < t_1 < t_f$, then the state trajectory of the binary system (1) at time t_f has the form:

$$x(t_f) = e^{A_2(t_f - t_1)} e^{A_1(t_1 - t_0)} x(t_0) + \int_{t_0}^{t_1} e^{A_2(t_f - t_1)} e^{A_1(t_1 - \tau)} B_1 u(\tau) d\tau + \int_{t_1}^{t_f} e^{A_2(t_f - \tau)} B_2 u(\tau) d\tau. \quad (6)$$

Denote $\overline{x}_f \hat{=} e^{-A_2(t_f - t_1)} x_f - e^{A_1(t_1 - t_0)} x_0$. With respect to expression (5), equation (6) can be further expressed as

$$\overline{x}_f = \sum_{i=0}^{n-1} A_1^i B_1 \alpha_i + \sum_{j=0}^{n-1} A_2^j B_2 \beta_j,$$

where for any $i, j = 0, 1, \cdots, n-1$, there are

$$\alpha_i \hat{=} \int_{t_0}^{t_1} b_i(t_1 - \tau) u(\tau) d\tau, \quad (7)$$

$$\beta_j \hat{=} \int_{t_1}^{t_f} c_j(t_1 - \tau) u(\tau) d\tau. \quad (8)$$

Here $b_i(t)$ and $c_j(t)$ are the expansion coefficients of $e^{A_1 t}$ and $e^{A_2 t}$ respectively. Equation (6) can be further expressed as a matrix multiplication form:

$$\overline{x}_f = [B_1 \ \cdots \ A_1^{n-1} B_1 \ \ B_2 \ \cdots \ A_2^{n-1} B_2] \begin{bmatrix} \alpha_0 \\ \vdots \\ \alpha_{n-1} \\ \beta_1 \\ \vdots \\ \beta_{n-1} \end{bmatrix}$$

Due to the assumption that $rank(W_C) = n$, we know that there always exists a solution $([\alpha_0 \ \cdots \ \beta_{n-1}]^T)$ of above equation for any given $\overline{x}_f \in R^n$. Assume one solution determined by the given continuous states x_f and x_0 is $[\alpha_0^\star \cdots \alpha_{n-1}^\star \ \beta_0^\star \cdots \beta_{n-1}^\star]^T$, then with respect to definitions (7), there exist $2n$ integral equations need to be considered:

$$\begin{cases} \alpha_0^\star = \int_{t_0}^{t_1} b_0(t_1 - \tau) u(\tau) d\tau \\ \vdots \\ \alpha_{n-1}^\star = \int_{t_0}^{t_1} b_{n-1}(t_1 - \tau) u(\tau) d\tau \\ \beta_0^\star = \int_{t_1}^{t_f} c_0(t_1 - \tau) u(\tau) d\tau \\ \vdots \\ \beta_{n-1}^\star = \int_{t_1}^{t_f} c_{n-1}(t_1 - \tau) u(\tau) d\tau \end{cases} \quad (9)$$

Now we need to prove that there exists a piecewise continuous function $u(t)$ defined within period $[t_0, t_f]$, which makes these $2n$ integral equations valid.

Divide the time interval $[t_0, t_1]$ into n subintervals with property $t_0 \hat{=} t_{0,1} < t_{0,2} < \cdots < t_{0,n+1} \hat{=} t_1$, and define the input $u(t)$ within $[t_0, t_1)$, denoted as $u(t/[t_0, t_1))$, as a piecewise constant vector function, i.e.,

$$u(t/[t_0, t_1)) \hat{=} U_{0i}, \ t_{0,i} \le t < t_{0,i+1}, \quad U_{0i} \in R^m, i = 1, \cdots, n. \quad (10)$$

Similarly, we also divide the time interval $[t_1, t_f]$ into n subintervals with property $t_1 \hat{=} t_{1,1} < t_{1,2} < \cdots < t_{1,n+1} \hat{=} t_f$, and define $u(t/[t_1, t_f])$ as a piecewise constant function:

$$u(t/[t_1, t_f]) \hat{=} U_{1i}, \ t_{1,i} \le t \le t_{1,i+1}, \quad U_{1i} \in R^m, i = 1, \cdots, n. \quad (11)$$

Here U_{0i} and U_{1i} for $i = 1, \cdots, n$ need to be determined in the following.

Substitute the assigned control (10) and (11) into equation (9), then we have

$$\begin{bmatrix} \alpha_0^\star \\ \vdots \\ \alpha_{n-1}^\star \end{bmatrix} = F(t_{0,1}, t_{0,2}, \cdots, t_{0,n+1}) \begin{bmatrix} U_{01} \\ \vdots \\ U_{0n} \end{bmatrix} \quad (12)$$

$$\begin{bmatrix} \beta_0^\star \\ \vdots \\ \beta_{n-1}^\star \end{bmatrix} = F(t_{1,1}, t_{1,2}, \cdots, t_{1,n+1}) \begin{bmatrix} U_{11} \\ \vdots \\ U_{1n} \end{bmatrix} \quad (13)$$

where $F(t_{i,1}, t_{i,2}, \cdots, t_{i,n+1})$ for $i = 1, 2$ are matrices defined as (4). With respect to Lemma 1, we know that there exists a proper set of switching time instants $t_{0,1}, \cdots, t_{0,n+1}$ and $t_{1,1}, \cdots, t_{1,n+1}$, such that equations (12) and (13) will always have unique solution, respectively. Denote the solutions of (12) and (13) as $[U_{01}^\star \ \cdots \ U_{0n}^\star]^T$

and $[U^\star_{11} \ \cdots \ U^\star_{1n}]^T$, respectively. Then the timed switching set $\{(n_1, t_1, n_2)\}$ and the piecewise constant control input

$$U^\star(t/[t_0,t_f]) \hat{=} \begin{cases} U^\star_{0i}, \ t_{0,i} \le t < t_{0,i+1}, \ i = 1, \cdots, n \\ U^\star_{1i}, \ t_{1,i} \le t < t_{1,i+1}, \ i = 1, \cdots, n \end{cases}$$

make the final state (n_2, x_f) reachable from (n_1, x_0) within period $[t_0, t_f]$, where $t_{0,n+1} = t_{1,1} = t_1$ and $t_{1,n+1} = t_f$.

Once the given initial and final discrete states have the property $n_0 = n_f$, without lose of generality, we assume there is $n_0 = n_f = n_1$.

Select a timed switching set as

$$\{(n_1, t_1, n_2), (n_2, t_2, n_1)\}$$

with property $t_0 < t_1 < t_2 < t_f$, and define the piecewise continuous input $u(t/[t_2, t_f]) \equiv 0$. Then when denoting

$$\overline{x}_f \hat{=} e^{-A_1(t_f - t_2)} e^{-A_2(t_2 - t_1)} x_f - e^{A_1(t_1 - t_0)} x_0,$$

we can get the same expression of the system state trajectory as (6). Therefore, we can further assign properly a piecewise constant input function as

$$U^\star(t/[t_0,t_f]) \hat{=} \begin{cases} U^\star_{0i}, \ t_{0,i} \le t < t_{0,i+1}, \ i = 1, \cdots, n \\ U^\star_{1i}, \ t_{1,i} \le t < t_{1,i+1}, \ i = 1, \cdots, n \\ 0 \qquad t_2 \le t \le t_f, \end{cases}$$

which makes (n_1, x_f) reachable from (n_1, x_0) within period $[t_0, t_f]$, where $t_{0,n+1} = t_{1,1} = t_1$ and $t_{1,n+1} = t_2$. □

Actually, Theorem 1 can be extended for any $q \ge 2$ case as stated in Yang et al. (1998), i.e.,

Theorem 2: System (1) with q-mode is controllable, if W_C defined in (2) is of full row rank.

In addition to that, through detailed analysis, it can be proved that the necessary condition stated in Yang et al. (1998) is also a sufficient condition for controllability, i.e.,

Theorem 3: System (1) is controllable, if and only if the k_rth order system joint controllability matrix $\overline{W}^{k_r}$ is of full row rank, i.e., $rank(\overline{W}^{k_r}) = n$, where k_r is the joint controllability coefficient of (1) according to (3).

Proof: (omitted)

Remark 1: It can be observed that when the considered system (1) has only one mode, i.e., it's an ordinary LTI system, with respect to the definitions of W_C, k_r and $\overline{W}^{k_r}$, the considered system is completely controllable if and only if $W_C = \overline{W}^{k_r}$ is full rank in row with $k_r = 0$. It's obvious that this conclusion is actually the controllability matrix test for the LTI systems.

5. RECONFIGURABILITY ANALYSIS

Fault-tolerance in control is the ability of a controlled system to maintain or gracefully degrade control objectives despite the occurrence of a fault. *Reconfiguration* means to change the input-output between the controller and plant through change of the controller structure and parameters, so as to maintain the original control objective (Blanke et al. (2001)). The reconfigurability can be evaluated according to different control objectives, such as satisfying some performance requirements or preserving some system properties (Yang et al. (2000)).

Motivated by the work in Frei et al. (1999); Blanke et al. (2001) which regard the recoverability as a kind of system properties, we discuss the configurability for the considered system (1) with respect to the controllability concept.

A *fault* means an unpermitted deviation of at least one characteristic property or parameter of the considered system from a usual condition (Blanke et al. (2001)). Without any fault, the system (1) is referred to as the nominal system and denoted as $M^n \hat{=} \{(A_i, B_i)\}_{i=1}^q$. When some fault happened in the considered system, the system (1) is usually referred to as the *faulty system* and denoted as $M^f \hat{=} \{(A_i^f, B_i^f)\}_{i=1}^q$.

Definition 7: The nominal system $\mathcal{M}^f$ is called reconfigurable with respect to the considered faults if $\mathcal{M}^f$ preserves the controllable subspace of $\mathcal{M}^n$.

By employing Theorem 2 obtained in last section, we have

Theorem 4: The faulty system M^f can be reconfigured with respect to the considered faults if the matrix

$$W_C^f \hat{=} [W_1^f \ \cdots \ W_q^f] \hat{=} [B_1^f \ \cdots \ (A_1^f)^{n-1} B_1^f \\ \cdots \ B_q^f \ \cdots \ (A_q^f)^{n-1} B_q^f]. \qquad (14)$$

is of full row rank.

When some specific fault happened such that some mode disappears from the nominal system, such as the situation that the gear stick can not switch into some gear position in an automobile power system due to some possible mechanical problem, this kind of faults corresponds to the disappearance of some specific discrete state. Without loss of generality, we assume that the subsystem (A_q, B_q) can not appear in the faulty system M^f which is denoted as $M^f \hat{=} \{(A_i, B_i)\}_{i=1}^{q-1}$. Then there is

Theorem 5: The faulty system M^f can be reconfigured with respect to the considered faults if the matrix

$$W_C^f \hat{=} [W_1 \ \cdots \ W_{q-1}] \hat{=} [B_1 \ \cdots \ A_1^{n-1} B_1 \\ \cdots \ B_q \ \cdots \ A_{q-1}^{n-1} B_{q-1}]. \qquad (15)$$

is of full row rank.

In a quite general way, using Theorem 3 there is

Theorem 6: The faulty system M^f can be reconfigured with respect to the considered faults if

$$span(\overline{W_f}^{k_r^f}) = span(\overline{W_n}^{k_r^n}) \qquad (16)$$

where $\overline{W_f}^{k_r^f}$ ($\overline{W_n}^{k_r^n}$) is the k_r^fth (k_r^nth) order system joint controllability matrix of M^f (M^n), and k_r^f (k_r^n) is the joint controllability coefficient of M^f (M^n) according to (3).

Proof: (Omitted)

Example: Consider a binary-mode system, which parameters under nominal case are:

$$A_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, B_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}.$$
$$A_2 = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

It can be checked through Theorem 1 that the nominal system is hybrid controllable. When some fault happens such that in mode-1 the matrix B_1 changes to be $B_1 \hat{=} [1\ 0\ 0]^T$, i.e., the second actuator of the considered system is out of order. Through theorem 6, it can be observed that this faulty system is still reconfigurable with respect to this fault.

6. CONCLUSIONS

The controllability of a class of linear hybrid systems was analyzed using an algebraic approach. Some sufficient and necessary conditions were obtained based on the manipulation of related system matrices. Using the obtained results for controllability, the reconfigurability as one fundamental property of fault-tolerant control can also be efficiently examined for the considered systems.

References

R. Alur, C. Courcoubetis, T.A. Henzinger, and P. Ho. Hybrid automata: an algorithm approach to the specification and verification of hybrid systems. In A.R. Ravn, H. Rischel, and R.L. Grossman, editors, *Lecture Notes in Computer Science*, volume 736, pages 209–229. Springer-Verlag, 1993.

P.J. Antsaklis and A. Nerode. Hybrid control systems: An introductory discussion to the special issue. *IEEE Automatic Control*, 43(4):457–460, April 1998. special issue.

A. Bemporad, G. Ferrari-Trecate, and M. Morari. Observability and controllability of piecewise affine and hybrid systems. *IEEE Automatic Control*, 45(10):1864–1876, 2000.

M. Blanke, M. Staroswiecki, and N.E. Wu. Concepts and methods in fault-tolerant control. In *Proc. of ACC01*, 2001.

V.D. Blondel and J.N. Tsitsiklis. Complexity of stability and controllability of elementary hybrid systems. *Automatica*, 35(3):479–490, 1999.

Michael S. Branicky, Vivek S. Borkar, and Sanjoy K. Mitter. A unified framework for hybrid control: Model and optimal control theory. *IEEE Automatic Control*, 43(1):31–45, January 1998.

P.E. Caines and Y.J. Wei. Hierarchical hybrid control systems: A lattice theoretic formulation. *IEEE Automatic Control*, 43(4):501–508, April 1998.

J. Ezzine and A.H. Haddad. Controllability and observability of hybrid systems. *International Journal of Control*, 49(6):2045–2055, 1989.

C.W. Frei, F.J. Kraus, and M. Blanke. Recoverability viewed as a system property. In *Proc. of ECC99*, 1999.

D. Liberzon and A. S. Morse. Basic problems in stability and design of switched systems. *IEEE Control Systems*, pages 59–70, October 1999.

W.J. Rugh, editor. *Linear System Theory*. Prentice Hall, 1996.

J.A. Stiver and P.J. Antsaklis. On the controllability of hybrid control systems. In *Proc. of 32st CDC*, pages 294–299, San Antonio, Texas, 1993.

M. Tittus and B. Egardt. Control design for integrator hybrid systems. *IEEE Automatic Control*, 43(4):491–500, April 1998.

Jan H. van Schuppen. A sufficient condition for controllability of a class of hybrid systems. In T.A. Henzinger and S. Sastry, editors, *Hybrid Systems: Computation and Control*, number 1386 in LNCS, pages 374–383. Springer, 1998.

Z. Yang and M. Blanke. A unified approach for controllability analysis of hybrid control systems. In *Proc. of IFAC CSD2000*, pages 158–163, Bratislava, Slovak Republic, June 2000.

Z. Yang, R. Izadi-Zamanabadi, and M. Blanke. On-line multiple-model based adaptive control reconfiguration for a class of nonlinear control systems. In *Proc. of IFAC Safeprocess 2000*, pages 745–750, Budapest, Hungary, June 2000.

Z. Yang, M. Verhaegen, Y.J. Wang, and Z.J. Chen. Hybrid controllability of linear switched systems. In *Proc. of 37th CDC*, pages 3920–3925, Tampa,FL, USA, December 1998.