Innovations in On-Line Mathematics Education

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Abstract: - This paper describes the development and delivery of the first fully on-line university-level mathematics course presented through Open Learning Australia. It documents the difficulties and challenges of such a course, and the solutions that have been found to deal with the inherent problems of on-line delivery. Finally it canvasses strategies for the improvement and further development of on-line mathematics education.

Key-Words: - On Line Open Learning Discrete Mathematics Education

1 Introduction

Open Learning Australia (hereafter known as OLA) is a conglomerate of Australian universities which came into being to provide greater opportunities for university education within Australia and neighbouring countries.

The Royal Melbourne Institute of Technology (also known as RMIT University, or more simply as RMIT) provides several programs through OLA, one of which is the program leading to a Bachelor of Applied Science degree in Information Technology. Similar programs delivered in face-to-face mode have traditionally included a foundational course in Discrete Mathematics, and in 1998 it was decided to develop an on-line Discrete Mathematics course as a core subject within the Applied Science (Information Technology) program. The course was scheduled for inaugural delivery in 1999.

2 The Initial Challenge

An on-line course in Discrete Mathematics needs to have several features: a front page as a first point of contact for students; the provision of lecture notes; a means of communication between staff and students, and among students; a self-assessment feature for students (often called “formative” assessment); and a way of evaluating student performance in tests and examinations (often called “summative” assessment).

The front page needed to be located at a web site where students could access course information and find links to all other components of course delivery.

The lecture notes had to be presented in a more conversational style than would normally be found in a text book, yet be somewhat more formal than a traditional face-to-face lectures need to be.

Communication from staff to students needed to take place both collectively and individually. A place for announcements was needed, together with a capacity for general explanations of difficult concepts. But also there was a need for staff to communicate with students individually. This would happen if an individual student had difficulty understanding some part of the course, or if special arrangements needed to be made concerning the assessment, or if a student were having log-in difficulties, and so on.

2.1 Assessment

A bank of exercises was needed, which students could use to evaluate their own progress. And a means of official assessment was needed, so that staff could determine the level of understanding and competence that students had achieved and could assign appropriate grades to the students.

2.2 Communication

The feature of the traditional class room which is most difficult to reproduce in an on-line environment is the sense of personal interaction. Without face-to-face contact it is harder for staff to explain difficult concepts and techniques to students, especially where some dynamic process is involved that can best be understood when it can be viewed while it is happening.

3 The Design

During the period from mid-1998 to mid-1999, Gary Fitz-Gerald (also from RMIT's Department of Math-

ematics) and the author developed a strategy for meeting the challenges described in the previous section.

The design was based on four components - three web sites and an electronic mail system. One site was the location for the lecture notes, and the others were for “Serf” and “WebLearn”.

3.1 The Delivery Platform

Serf (Server-side educational records facilitator) is a product developed at the University of Delaware. It provides a high degree of structure for a web site used as a platform for computer-assisted course delivery. Some of the most useful features are these: an Announcements page; a Discussion Forum area; and a Calendar which can be linked to the sequence of tasks needing to be completed by students. Another advantage of Serf is that it is easy to “port” information from Serf across to other distributed learning systems.

The calendar associates a specific date with a particular activity, known as an event. An event may be a lecture, a quiz, a test, or some other activity associated with the course.

Serf was already being widely used in other courses of the degree program, under the overall direction of Ed Morris of RMIT's School of Computer Science and Information Technology. The features of Serf seemed entirely suitable for the Discrete Mathematics delivery, and so we decided to adopt it for our course.

3.2 The Lecture Notes

To reflect the content of the traditional face-to-face lecture course of three lectures per week for thirteen weeks, thirty-nine lectures were created using LaTeX - Leslie Lamport's version of Donald Knuth's TeX typesetting package. These were converted to pdf (“portable document format”) files and uploaded to a dedicated web site. This meant that students needed a reasonably up-to-date version of the Adobe Acrobat Reader package, if they were to be able to view the lecture notes. Fortunately, the Reader is freely downloadable. Hot links were created from Serf to the pdf file for each lecture.

3.3 The Assessment Site

The WebLearn package was designed and developed by George Fernandez of RMIT's School of Computer Science and Information Technology. It provides two main kinds of assessment - “quizzes” for self-assessment and “tests” for formal assessment. An instructor is able to program randomness in the way the system selects questions for each student. A student's result on a quiz is displayed on screen immediately after the quiz is submitted. A test, however, is not marked until the instructor activates the test-marking facility, after which a student's score is automatically sent to the registered e-mail address of the student. Hot links were created from Serf and from the lecture notes to WebLearn.

3.4 Electronic Mail

The course design involved the use of e-mail for two purposes. One was to send information bulletins to all students, and the other was for contact with individual students. An initial bulk e-mail was needed to inform all students of the locations of the course web sites. The possibility of individual contact meant that a student could make specific requests for help, involving either further explanations of some topic the student found difficult or making arrangements to allow late submission of a test in the case of illnesses or other problems.

4 The Launch

The mathematics course MAT17 “Discrete Mathematics 1” was launched in June 1999 with an initial intake of 24 students. Of these, 5 deferred and 18 passed. Of those who passed, 6 gained High Distinctions, 3 earned Distinctions and 6 were awarded Credits.

Communication was conducted almost entirely by e-mail, with very little use being made of Serf's Discussion Forum facility.

Students were very positive about the course, with comments including the following.

“I have just completed the subject MAT17 (Discrete Mathematics) and thoroughly enjoyed it. I appreciated the lecture notes, they were inciteful with ample examples and explanations and left no unanswered questions in my mind, with regards to the topics they covered. I liked how the topics had practical implications. The quizzes I completed correctly prepared me for the tests and the exam. Your response to my Email's were refreshingly quick and helpful. I look forward to continuing with subjects of this nature in the future. Thank you.”

A report on this first delivery of MAT17 was presented by Gary Fitz-Gerald at the Fourth Asian Technology Conference in Mathematics in Guangzhou, China in December 1999, and is published in the proceedings of that conference [1].

5 Later Deliveries

From March 2000 onwards, MAT17 has been delivered twice a year with an average of 110 students each time. Students have been located all over Australia, with a small number studying the course from overseas. The Discussion Forum became a major focus of the course, with students often answering each others questions.

The students have continued to be successful, and their responses have been very positive. Here is one comment from a student.

"I personally really enjoyed the opportunity of this kind of study and as this was my first class I found it ran smoothly, and professionally."

6 Further Challenges

During the first version of MAT17, there appeared certain problems requiring both immediate attention and the development of long-term solutions. Some of these problems were addressed in later versions of the course, but others continue to be of concern.

Many of the problems come down to one single issue: how can mathematics be presented on line?

Normal text, such as what can be used in an e-mail message or in the Notepad facility under Microsoft Windows, is very limited in the range of mathematical symbols that are available to it. While a linear equation can be written (using the numbers and letters on a normal keyboard, together with the "equals" sign), even a quadratic equation can't be displayed in a satisfactory way.

One consequence of this is that replies to student queries concerning mathematical topics take much longer to create than they might in a topic that only involved words in a normal alphabet. During the first incarnation of MAT17 an inordinate amount of time was devoted to individual replies to students. It became apparent that this could not continue, especially as the number of students was set to rise. In later versions, students were encouraged to post questions in the Discussion Forum. While this meant that a question only needed to be answered once and that the question and answer could be viewed by all students, the Discussion Forum in Serf still did not allow a large amount of mathematics to be included.

Another major problem concerns the difficulty of on-line assessment, both formative and summative. While WebLearn does allow for different students to be asked different questions, it doesn't allow for different sequences of questions. This makes it very difficult to ask several questions based on an earlier question, unless all students had seen the earlier question. Furthermore, WebLearn has the same limitations as Serf and the standard e-mail systems, in that only a very small number of mathematical symbols are available.

Another difficulty with the assessment is that the system doesn't send students a statement of the correct answer, so that except in the case of "true or false" questions a student may have difficulty knowing what the correct answer is to a question which he or she got wrong. Furthermore, telling students the correct answer doesn't tell them why their answers were wrong.

Finally, it seems very difficult to guarantee that students in on-line examinations are not receiving help. The process of authorising or licencing invigilators is made more complicated by the fact that some students are in remote areas, and some are even in overseas countries.

7 Meeting the Challenges

Discrete mathematics has some inherent advantages over other areas of mathematics. Its symbols are often much less complicated than those that are found in other topics. MAT17 involved three major topic areas - Logic, Graphs and Trees, and Algebra and Coding. Especially in the area of Logic there is quite a lot that can be done using ordinary words and keyboard symbols.

Logical propositions are often represented by capital letters, such as P. And predicates are represented by expressions such as P(x). Although these might better be represented in mathematical italics, students can understand what is meant by "the proposition P" or "the predicate P(x)".

The next step is to find close approximations to the five main logical connectives - negation, conjunction, disjunction, implication and equivalence. For these we chose respectively a tilde, a wedge, a vee, a right-pointing arrow, and a two-headed arrow pointing both to the left and to the right. In ordinary text these can be represented roughly as ~, ^, v, -> and <-> respectively.

Compound propositions are often compared to see if they are logically equivalent to each other or if one logically implies another. In the lecture notes we used two symbols for logical equivalence, one resembling an "equals" sign with an extra horizontal bar and the other being essentially an "equals" sign with outward-pointing arrows on both ends. In ordinary text we used <=> to represent the second of these, but were unable to represent the first one. For logical implication we used => or <= (depending on the required direction).

In Predicate Logic and in Methods of Proof we met the difficulties of displaying equations and inequalities. There are no problems with the relations of “equal”, “less than” or “greater than”, but for “greater than or equal to” and “less than or equal to” we had to use composite symbols such as >= and =< respectively.

A rough solution to the problem of superscripts and subscripts can be found by using the “carat” and “underscore” symbols ^ and _ respectively. So in WebLearn quizzes and tests, in e-mail messages and in the Discussion Forum students were told that x^2 is to be interpreted as “x squared” and x_2 as “x subscript 2”.

When it came to truth tables and other kinds of tables, the limitations of ordinary text again created difficulties. The variability of fonts is a major problem. The Courier font with standard spacing makes it easy to create a table with the columns properly aligned, but students reading their e-mail in other fonts don't get a satisfactory picture.

The HyperText Markup Language (html) provided a partial solution, because the Serf Discussion Forum recognises the html commands. But even a short table requires a long list of commands, and positioning such a table in the right place in a Discussion Forum message proved difficult. However, these tables in WebLearn were reasonably satisfactory.

The topic of Graph Theory, requiring lots of diagrams, has obvious difficulties. Quite simply, diagrams can't be placed in e-mail messages or in the Discussion Forum. We did experiment with embedding a postscript file in a WebLearn quiz, but many students had difficulty viewing the graph because of compatibility problems with their own computer systems.

Attaching files to e-mail messages doesn't solve the problem, because many students would not have the software necessary to be able to view the attachments.

A primitive strategy which worked surprisingly well was to use words to describe a graph. For graphs which weren't very complicated, this could be done from scratch. For example, here is one of the WebLearn questions about graphs. “Draw a triangle. Put a big dot on each corner, so that the corners become the vertices of a graph. Then the sides of the triangle are the edges of the graph. What are the degrees of the vertices?”

More complicated graphs were described in relation to existing graphs in the lecture notes. For example, here is another WebLearn question. “Draw the graph shown in Figure 2 of Lecture 25. Remove edges cb and cd and vertices b and d, and drag vertex f leftwards so that edge ef slopes down and to the left. This gives a binary tree. What is the order in which the vertices are selected by in-order traversal?”

Students never reported any difficulties in interpreting these descriptions. But an obvious limitation is that complicated graphs which were very different from those in the lecture notes couldn't be described in this way.

The topic of Algebra and Coding is a bit more difficult than Logic to present in ordinary text. Coding involves matrices and tables. The difficulty of representing tables has already been discussed. A matrix can be regarded as a structure similar to a a table, but it's not obvious how to display the enclosing brackets.

Some recent advances have given hope that major improvements can be made in presenting on-line discrete mathematics. Gary Fitz-Gerald and George Fernandez have been developing a facility for incorporating the computer algebra system “Maple” into WebLearn. This will allow a wider class of exercises to be presented to students in the quizzes and tests, without requiring the students to learn how to use Maple. Ongoing developments in “mathML” (a mathematical mark-up language) are expected to make it easier to present mathematics in environments which currently recognise html commands, such as Serf's Discussion Forum facility and WebLearn.

Mirko Lukic of RMIT's Department of Mathematics has been developing pdf files which are directly accessible by the use of hot links created with html commands in the Discussion Forum facility. As the number of these files increases, more and more questions can be answered with the help of clear and accurate diagrams and mathematical notation.

The large number of answers developed over the last two and a half years in response to students' questions is forming the basis of an FAQ site (for Frequently Asked Questions) which is currently in development. The task of collating and indexing these questions and answers, and rendering them where appropriate in pdf format, is now under way.

The problem of formal assessment continues to be a challenge. On occasions when two students have been living at the same address, we have required them to do different versions of the final examination at different times or different places. One suggestion that has been canvassed is the possibility of requiring each student to have a video camera connected to his or her computer so that the examiners could see that the student was the person doing the examination. But we're not necessarily familiar with the appearances of all students, especially

as some of them are in overseas countries so that we have probably never met them. A more systematic procedure for accrediting interstate and overseas invigilators perhaps offers the best hope for guaranteeing the validity of the students' grades.

8 Conclusion

The delivery of on-line discrete mathematics has been a successful initiative, welcomed by students and resulting in good student outcomes. With further improvements already being trialled, it is expected that the course will continue to develop to meet the needs of a growing number of students.

References:

[1] G. Fitz-Gerald, Mathematics Delivery Using On-line Distributed Learning Systems at RMIT, *Proceedings of the Fourth Asian Technology Conference in Mathematics*, pp. 321-328, ATCM Inc., 1999