Discrete Sliding-Mode Adaptive Algorithm For Adaptive Filtering

SENG KAH PHOOI
School of Microelectronics, Griffith University, Kessels Rd, Nathan QLD 4111,AUSTRALIA

Abstract: - This paper presents the discrete adaptive sliding-mode algorithm for adaptive filtering in signal processing. The parameter updated law of the adaptive filter is based on the FIR (*Finite Impulse Response*) model and is the non-switching type. The adaptive gain of the parameter updated law is adaptively adjusted to make the sliding variable to converge to zero in a finite duration. Then the gain is adjusted to keep the error dynamics in the sliding mode so that the desired error dynamics can be achieved in the sliding mode. The error dynamics are insensitive with respect to the bounded disturbances. The concept of the sliding mode was originally used for the design of robust control systems. Now it can be used for adaptive filter design to further improve the performance of adaptive filters. Simulation results are presented to illustrate the features of the proposed scheme.

Key-Words: Adaptive filtering, sliding-mode, signal processing, Lyapunov stability theory

1 Introduction

Sliding-mode control of continuous-time systems offers the advantage of robustness against parameter variations and external disturbances [1]. The robustness property is achieved by using high frequency switching to keep the state on the sliding surface. However this high-frequency switching leads to undesirable chattering of the control input. In recent years, discrete sliding mode controllers have received much attention [2]-[5]. Most of these studies have concentrated on fixed strategies. Some studies of the adaptive control of discrete systems based on sliding mode have been carried out [6]-[7]. The discrete sliding-mode controllers have been developed mainly using state-space models. However, the use of the input-output models in designing discrete sliding-mode controllers has also received some attention [3]-[5].

The advantages of sliding mode controller in control theory are well-known. However, the use the sliding mode techniques [1] in signal processing does not get much attention as the sliding mode techniques in control engineering. It is possible to use sliding-mode techniques to design the adaptive filter in order to further improve the performance of adaptive filters. In this paper, a discrete sliding-mode adaptive algorithm for adaptive filtering in signal processing is proposed. The present work differs from previous works on adaptive filtering. The parameter updated law of the adaptive filter is based on the FIR (*Finite Impulse Response*) model [8]-[10] and is the non-switching type. The adaptive gain of the parameter updated law is adaptively adjusted to make the sliding variable to converge to zero in a finite duration. Then the gain is adjusted to keep the error dynamics in the sliding mode so that the desired error dynamics can be achieved in the sliding mode. The error dynamics are insensitive with respect to the bounded disturbances. The idea of the sliding mode was initially used for the design of robust control systems. In our approach, it can be used for adaptive filter design to further improve the performance of adaptive filters. Simulation results are presented to illustrate the features of the sliding-mode adaptive filters.

The organization of this paper is as follows. Section 2 presents the discrete sliding-mode adaptive filtering. The stability analysis and discussion are presented in Section 3. Simulation results to illustrate the features of the proposed filter are presented in Section 4.

2 Problem Formulation

Let the sliding surface be defined as follows:

$$s(k) = C(q^{-1})\, e(k) = 0 \qquad (2.1)$$

where $e(k) = r(k) - y(k)$ (2.2)

$r(k)$ is the bounded reference input. The $e(k)$ is the error between the reference input and the filter output. q^{-1} is the unit-delay operator. $C(q^{-1})$ is the stable polynomial defined as

$$C(q^{-1}) = 1 + c_1 q^{-1} + \ldots + c_{n+1} q^{-(n+1)} \qquad (2.3)$$

For the purpose of designing the adaptive filter, the following *finite impulse response* (FIR) model is used. The FIR model can be considered as moving average or MA model, in which the filter has only zeros, characterized by the difference equation

$$y(k) = \sum_{i=0}^{N-1} w_i(k)x(k-i) \qquad (2.4)$$

The difference equation in (2.4) can be rewritten as

$$y(k) = W^T(k)X(k) \qquad (2.5)$$

where $W(k) = [w_k(0), \quad w_k(1), \quad ..., \quad w_k(N-1)]^T$

$X(k) = [x(k), x(k-1), ..., x(k-N+1)]^T$

The filter weight vector update equation is

$$W(k) = W(k-1) + u(k)\beta(k) \qquad (2.6)$$

where $u(k)$ is the adaptation gain and $\beta(k)$ is defined as

$$\beta(k) = r(k) - W^T(k-1)X(k) + ce(k-1) \qquad (2.7)$$

The $u(k)$ in (2.6) is updated in the sense that the sliding variable $s(k)$ will converge to zero in a finite duration. In the sliding mode $s(k) = 0$ or $e(k) + c \cdot e(k-1) = 0$, the error $e(k)$ will asymptotically converge to zero with the convergence rate specified by the constant c.

$$u(k) = \frac{X(k)}{\| X(k) \|^2}\left(1 - \tau\frac{|e(k-1)+ce(k-2)|}{\beta(k)}\right) \qquad (2.8)$$

where $0 \le \kappa < 1$.

Remark 2.1: By observing the expressions (2.1)-(2.8), this scheme is independent of stochastic properties of the input disturbance and input signal. Only the input signal and the reference input are required to update the filter parameters.

3 Stability Analysis

To proof the stability of the proposed sliding-mode adaptive algorithm, the following Lyapunov function is first defined.

$$V(k) = s^2(k) \qquad (3.1)$$

where

$$s(k) = C(q^{-1})e(k) = e(k) + c \cdot e(k-1) \qquad (3.2)$$

$$\Delta V(k) = V(k) - V(k-1)$$
$$= [e(k) + c(e(k-1)]^2 - [e(k-1) + ce(k-2)]^2$$
$$= [r(k) - W^T(k)X(k) + ce(k-1)]^2 - [e(k-1) + ce(k-2)]^2$$
$$= [r(k) - W^T(k-1)X(k) - u^T(k)\beta(k)X(k) + ce(k-1)]^2 - [e(k-1) + ce(k-2)]^2$$
$$= [r(k) - W^T(k-1)X(k) - u^T(k)\beta(k)X(k)) + ce(k-1)]^2 \quad - [e(k-1) + ce(k-2)]^2$$
$$= [r(k) - W^T(k-1)X(k) + ce(k-1) - u^T(k)\beta(k)X(k))]^2 - [e(k-1) + ce(k-2)]^2$$
$$= [\beta(k) - u^T(k)\beta(k)X(k)]^2 - [e(k-1) + ce(k-2)]^2 \qquad (3.3)$$

Substituting the equation (2.8) in the equation (3.3),

$$\Delta V(k) = [\tau \, |e(k-1) + ce(k-2)|]^2 - [e(k-1) + ce(k-1)]^2 = -(1-\tau^2)[e(k-1) + ce(k-1)]^2 < 0 \qquad (3.4)$$

Remark 3.1: According to Lyapunov stability theory [11], the system is stable.

Remark 3.2: If $u(k)$ in (2.8) is updated in the sense that $\Delta V(k) = -(1-\tau^2)[e(k-1) + ce(k-1)]^2 < 0$, the sliding variable $s(k)$ will converge to zero in a finite duration. In the sliding mode $s(k) = 0$ or $e(k) + c \cdot e(k-1) = 0$, the error $e(k)$ will asymptotically converge to zero with the convergence rate specified by the constant c.

Remark 3.3: Due to the reason that the error dynamics are only determined by the parameter c in the sliding mode (3.2), the error dynamics are insensitive with respect to the bounded disturbances. It can be seen from the above that $u(k)$ in (2.8) is adaptively adjusted to make the sliding variable $s(k)$ to converge to zero, and then $u(k)$ is adaptively adjusted to keep the error dynamics in the sliding mode (3.2) so that the desired error dynamics can be achieved in the sliding mode.

Remark 3.4: The convergence rate of the error $e(k)$ is also dependent on the constant τ. The smaller the constant τ is, the faster the error convergence.

Remark 3.5: In order to prevent singularities due to the denominator terms of $\beta(k)$ and $\|X\|^2$, the adaptive gain, $u(k)$ can be modified as follow

$$u(k) = \frac{X(k)}{\| X(k) \|^2 + \delta_1}\left(1 - \tau\frac{|e(k-1)+ce(k-2)|}{\beta(k)+\delta_2}\right) \qquad (2.5)$$

where $0 \le \kappa < 1$, δ_1, δ_2 are small positive numbers, for example, 0.001. Same values of these constants can be selected. Smaller these values provide smaller error, $e(k)$.

4 Simulation

In this section, we illustrate the performance of the proposed sliding-mode filter to the adaptive noise filtering problem. The speech signal, which is

denoted *S2* and identical to that used by Prof. S. Haykin is used. The signals are available from the *WWW* homepage [12] and are described as follow:
S2: speech sample "*When recording audio data …*", length 10000, sampled at 8kHz. (*Fig. 1*)

The FIR filter has the following structure:

$$y(k) = \sum_{i=0}^{2} w_i(k)x(k-i) \qquad (4.1)$$

In this simulation, the additive bounded random noise, $n(k)$ is used and this additive noise satisfies the following bounded condition:

$$0 \le n(k) \le 0.01.$$

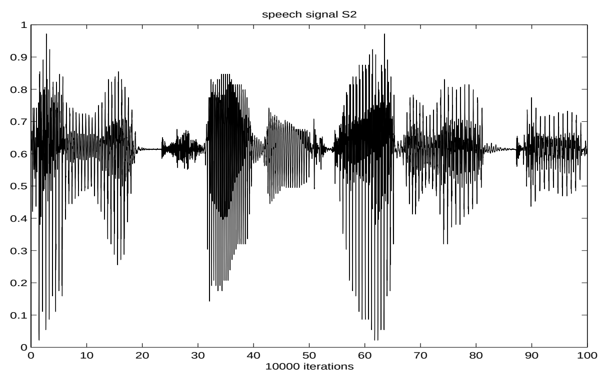

Fig. 1: The speech signal, S2

The following specifications have been used:
$C(q^{-1}) = 1 - 0.9q^{-1}$
Parameters $\delta_1 = \delta_2 = 0.01$
Parameter $\tau = 0.01$

Fig. 2 shows the plots of 10000 samples of the filtered output signals for *S2* versus time. *Fig. 3* illustrates the plots of 10000 samples of the filter output error, $e(k)$ or the difference between the reference input and the filter output of the proposed sliding-mode filter for *S2*.

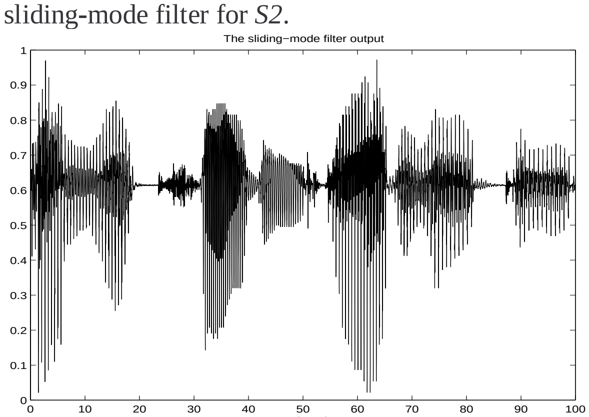

Fig. 2: The sliding-mode filter output, $y(k)$ for S2

The simulation is then repeated for the speech signal *S2* using difference polynomial, $C(q^{-1})$ to reveal the effect of constant *c*. In this case, the specifications are selected as follow:
$C(q^{-1}) = 1 - 0.01q^{-1}$
Parameters $\delta_1 = \delta_2 = 0.01$
Parameter $\tau = 0.01$

Fig. 4 shows the output error of the sliding-mode filter. The result has indicated that in the sliding mode $s(k) = 0$ or $e(k) + c \cdot e(k-1) = 0$, the error $e(k)$ will asymptotically converge to zero with the convergence rate specified by the constant *c*. A small constant *c* value gives faster error convergence.

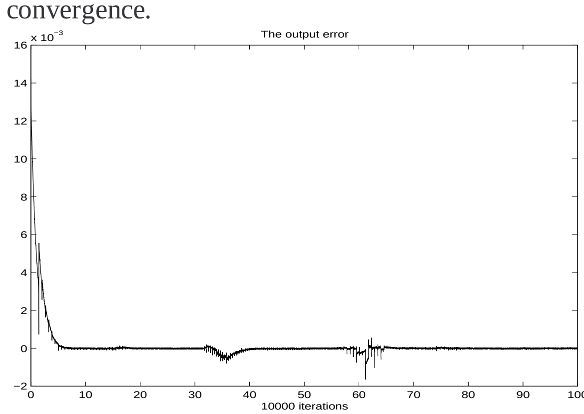

Fig. 3: The output error of the filter, $e(k)$ for S2 *(Note: y-axis: x10^-3)*

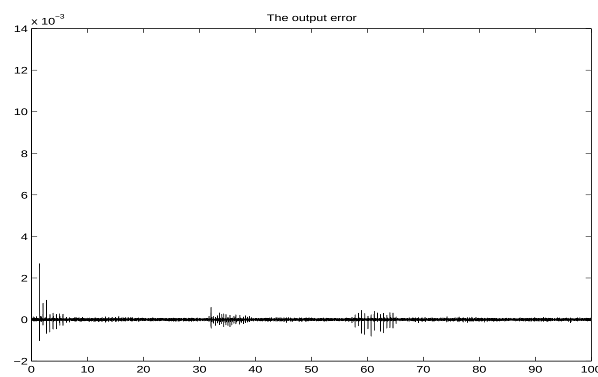

Fig. 4: The error, $e(k)$ for S2 - $C(q^{-1}) = 1 - 0.01q^{-1}$
$\delta_1 = \delta_2 = 0.01, \tau = 0.01$ *(Note: y-axis: x10^-3)*

Finally, the simulation is replicated with the following specifications:
$C(q^{-1}) = 1 - 0.01q^{-1}$
Parameters $\delta_1 = \delta_2 = 0.001$
Parameter $\tau = 0.001$

Fig. 5 illustrates the squared error, $e^2(k)$. The convergence rate of the error $e(k)$ is also dependent on the constant τ. The smaller the constant τ is, the faster the error convergence. Smaller the values of δ_1, δ_2 contribute smaller error, *e(k)*.Further Smaller error can be achieved if smaller values of these parameters are selected. The adaptive parameters of the sliding-mode adaptive filter are shown in *Fig.* 6.

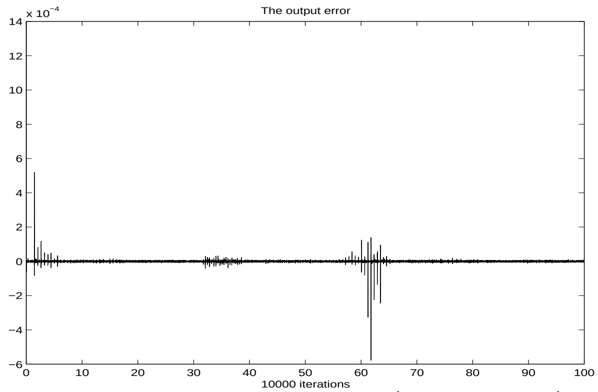

Fig.5: The error, *e(k)* for S2 - $C(q^{-1}) = 1 - 0.01q^{-1}$, $\delta_1 = \delta_2 = 0.001, \tau = 0.001$ *(Note: y-axis: x10^{-4})*

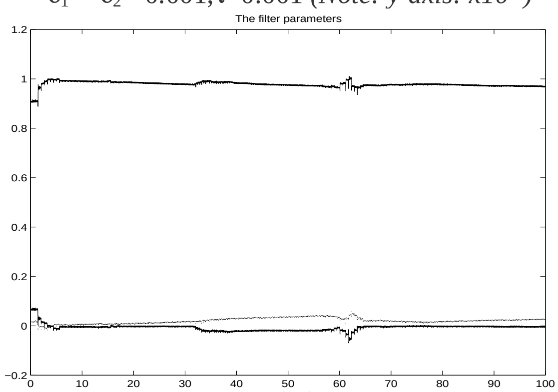

Fig. 6: The filter parameters, $w_0(k)$, $w_1(k)$, $w_2(k)$.

5 Conclusion

This paper has presented a new approach in designing an adaptive algorithm using the sliding-mode technique. The parameter updated law of the adaptive filter is based on the FIR model and it can be extended to IIR (*infinite impulse response*) model easily. The adaptive gain of the parameter updated law is adaptively adjusted to make the sliding variable to converge to zero in a finite duration. Then the gain is adaptively updated to keep the error dynamics in the sliding mode so that the desired error dynamics can be achieved in the sliding mode. The error dynamics are insensitive with respect to the bounded disturbances. The concept of the sliding mode was originally used for the design of robust control systems. In this paper, sliding mode technique has been be used for adaptive filter design to further improve the performance of adaptive filters. However, further work needs to be conducted in this area. Many issues need to be addressed regarding simulations, practical implementations, and the further analysis on the improvement of the tracking precision, stability, and robustness for the sliding mode adaptive filters.

References:

[1] Utkin, V. I, *Sliding modes in control optimisation. New York*: Springer, 1992.

[2] Drakunov. S.V and Utkin, V. I, On discrete-time sliding modes. In *Proc. IFAC Conf. On Nonlinear Control Systems Design, Capri, Italy*, 99. 1989, pp. 273-277.

[3] Furuta, K., *Sliding mode control of a discrete time system*. Syst. Control Letter, Vol.14, 1990, pp.145-152.

[4] Kaynak, O. and Denker. A, Discrete-time sliding mode control in the presence of system uncertainty. *Int. J. Control* No. 57, ,1993, pp. 1177-1189.

[5] Chan, C. Y. Robust discrete quasi-sliding mode tracking controller. *Automatica* 31, , 1995, pp.1509-1511.

[6] Bartolini, G., Ferrara. A. and Utkin, V. I, Design of discrete-time adaptive sliding mode control. In *Proc. 31st IEEE Conf. On Decision and Control, Tucson, AZ*, ,1992, pp. 2387-2391.

[7] Furuta, K., VSS-type self-tuning control - β equivalent control approach. In *Proc. American Control Conf., San Francisco. CA*, 1993, pp. 980-984.

[8] C.F.N. Cowan, P.M. Grant, *Adaptive Fitlers: Prentice-Hall Signal Processing Series*, 1985

[9] Bernard Mulgrew, Colin F. N. Cowan, *Adaptive filters and equalisers: Kluwer Academic Publishers*, 1984.

[10] S. Haykin, *Adaptive filter theory. Englewood Cliffs, NJ: Prentice-Hall*, 1985.

[11] Slotine, J-J. E. and Li, W. *Applied nonlinear control, Prentice-Hall, Englewood Cliffs, NJ*, 1991.

[12] http://www.ert.rwth-aachen.de/Presonen/balterse.html.