Stability Analysis of Discrete Time Fuzzy Systems Based on Piecewise Lyapunov Functions

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Abstract: - This paper presents a stability analysis method for the discrete time fuzzy system based on a piecewise Lyapunov function. It is shown that the stability of the system can be established if a piecewise Lyapunov function can be constructed, and moreover, the function can be obtained by solving a set of Linear Matrix Inequalities (LMI) that is numerically feasible with commercially available software.

Key-Words: - Discrete time, Fuzzy systems, Linear matrix inequality, Piecewise Lyapunov function, Stability.

1 Introduction

Fuzzy logical control (FLC) has recently proved to be a successful control approach for certain complex nonlinear systems, see [1–5] for example. However, conventional fuzzy control system [1] has proved extremely difficult to develop a general analysis and design theory. The reason for this is believed to be due to the fact that no mathematical model is available from the conventional fuzzy system.

Recently, there have appeared a number of stability analysis and controller design results in fuzzy control literature, where the Takagi-Sugeno's fuzzy models are used, see references in [7-11]. The stability of the overall fuzzy system is determined by checking a Lyapunov function or a set of Linear Matrix Inequalities (LMI). It is required that a common positive definite matrix P can be found to satisfy the Lyapunov function or the set of LMIs. However, this is a difficult problem to solve since such a matrix might not exist in many cases, especially for highly nonlinear complex systems. Most recently, a stability result of fuzzy systems using a piecewise quadratic Lyapunov function has been reported [14]. It is also demonstrated in the paper that the piecewise Lyapunov function is a much richer class of Lyapunov function candidates than the common Lyapunov function candidates and thus it is able to deal with a larger class of fuzzy systems. Further references to piecewise Lyapunov functions can be found in [15-18].

During the last few years, we have proposed a number of new methods for the systematic analysis and design of fuzzy logic controllers based on a so-called fuzzy dynamic model which is similar to the Takagi-Sugeno's model [7-11]. These methods include designs based on a nominal model, a common Lyapunov function and a piecewise Lyapunov function. However, for the methods based on the piecewise Lyapunov function, certain restrictive boundary conditions have to be imposed.

Motivated from the results of continuous time piecewise Lyapunov functions in [14], we develop a new stability theorem for discrete time fuzzy systems based on a piecewise Lyapunov function in this paper. This function is guaranteed to be decreasing when the state of the system stays within the subspace or jumps from one subspace to another. It should be noted that with this kind of piecewise Lyapunov function, the restrictive boundary condition existing in our previous analysis can be removed and global stability of the system can be easily established. Moreover, the stability checking procedure is to solve a set of LMIs that is numerically feasible with commercially available software.

The rest of the paper is organized as follows. Section 2 introduces the discrete time fuzzy system. Section 3 defines the piecewise Lyapunov function candidate and the *S*-procedure technique. Then a new stability theorem for discrete time fuzzy systems is presented and an example is given to illustrate the theorem. Finally, conclusions are given in Section 4.

2 Problem Formulation

The fuzzy dynamic model proposed in [7-11] can be used to represent a complex discrete-time system with both fuzzy inference rules and local analytic linear models,

$$R^l: \quad \text{IF} \quad x_1 \text{ is } F_1^l \text{ AND ... } x_n \text{ is } F_n^l$$
$$\text{THEN} \quad x(t+1) = A_l x(t) + a_l,$$
$$l = 1,2,...,m \qquad (1)$$

where R^l denotes the *l*-th fuzzy inference rule, *m* the number of inference rules, F_j^l *(j=1,2,...,n)* are fuzzy sets, $x(t) \in \Re^n$ the system state variables, (A_l, a_l) is the *l-th* local model of the fuzzy system (1), and a_l is the offset.

Let $m_l(x(t))$ be the normalized membership function of the inferred fuzzy set F^l where $F^l = \prod_{j=1}^{n} F_j^l$, and is defined as

$$0 \le m_l(x(t)) \le 1 \text{ and } \sum_{l=1}^{m} m_l(x(t)) = 1. \qquad (2)$$

By using a center-average defuzzifer, product inference and singleton fuzzifier [7-11], the discrete time fuzzy system (1) can be expressed by the following global model,

$$x(t+1) = \sum_{l=1}^{m} m_l(x(t)) \cdot \{A_l x(t) + a_l\},\ x(t) \in \Re^n \qquad (3)$$

Remark 2.1: It is noted that the system models defined in (1) or (3) are in fact affine systems instead of linear systems. They include an additional offset term. These models have much improved function approximation capabilities [7,12].

Define *L* as the set of subspace indexes, $L_0 \subseteq L$ as the set of indexes for subspaces that contain the origin and $L_1 \subseteq L$ the set of indexes for the subspaces that do not contain the origin. Since the rules of the fuzzy system (1) induce a polyhedral partition $\{S_i\}_{i \in L} \subseteq \Re^n$ of the state space, the fuzzy system (3) can be viewed as a set of individual subspaces, which consist of crisp (operating) and fuzzy (interpolation) subspaces.

The crisp subspace is defined as the subspace where $m_l(x) = 1$ for some l, all other membership functions evaluate to zero. The system dynamics of crisp subspace is given by *l-th* local model of the fuzzy system (1). On the other hand, the fuzzy subspace is defined as the subspace where $0 < m_l(x) < 1$ and the system dynamics is given by a convex combination of several affine systems.

In the extreme case where all the subspaces of a fuzzy system are crisp, that is, $m_l(x(t)) = 1$ for some l and all other membership functions are equal to zero, then fuzzy system (3) becomes a piecewise linear system, $x(t+1) = A_l x(t) + a_l$. However, in terms of fuzzy system (3), the membership function, $m_l(x(t))$ for some l, could be between 0 and 1. Thus we are required to find a mean to evaluate this fuzzy subspace.

In our previous attempts [7-11], we treated the fuzzy subspace in terms of uncertainties and modelled them in terms of upper bound approximation to perform stability analysis. As noted in [14], these same subspaces can be considered as a region with crisp subspaces blend and overlap each other. Hence, in each subspace, we can write the fuzzy system (3) as a convex combination of affine systems

$$x(t+1) = \sum_{k \in K(i)} m_k(x(t)) \{A_k x(t) + a_k\},\ x \in S_i \qquad (4)$$

with $0 \le m_k(x) \le 1$, $\sum_{k \in K(i)} m_k(x) = 1$. For each subspace S_i, the set $K(i)$ contains the indexes for the system matrices used in the interpolation within that subspace. For crisp subspace, $K(i)$ contains a single element.

Assumption 2.1: We assume that given any initial condition $x(0) = x_0$, the global model (4) has a unique solution for all $t \ge 0$.

Assumption 2.2: We also assume that when the state of the system transits from the subspace S_i to S_j at the time *t*, the dynamics of the system is governed by the dynamics of the local model of S_i at that time.

For future use, we also define a set Ω that represents all possible transitions from one subspace to another, that is,

$$\Omega := \{i, j \mid x(t) \in S_i,\ x(t+1) \in S_j,\ i \ne j\} \qquad (5)$$

Remark 2.2: Due to the discrete nature of the system, it is noted that Ω as in (5) could include transitions occurred between non-adjacent subspaces in one step.

For convenient notation, we also introduce

$$\bar{A}_k = \begin{bmatrix} A_k & a_k \\ 0 & 0 \end{bmatrix},\ \bar{x} = \begin{bmatrix} x \\ 1 \end{bmatrix}, \qquad (6)$$

where it is assumed that $a_k = 0$ for all $k \in K(i)$ with $i \in L_0$. Then using this notation, the system model (4) can be expressed as

$$\bar{x}(t+1) = \sum_{k \in K(i)} m_k(x(t)) \cdot \bar{A}_k \bar{x}(t), \; x(t) \in S_i, \qquad (7)$$

with $0 \le m_k(x) \le 1$, $\sum_{k \in K(i)} m_k(x) = 1$. Note in comparison with (3), the state space in fuzzy system (7) is partitioned and analyzed in terms of subspaces. The following example illustrates the idea.

Example 1: Consider a discrete time fuzzy system that switches between 3 rules,

R^1 : IF x_1 is about negative
THEN $x(t+1) = A_1 x(t) + a_1$

R^2 : IF x_1 is about zero
THEN $x(t+1) = A_2 x(t) + a_2$

R^3 : IF x_1 is about positive
THEN $x(t+1) = A_3 x(t) + a_3$

where the membership function for "about negative", "about zero" and "about positive" is defined as in Fig.1. The effect of these membership functions on state space is shown in Fig. 2.

As shown, rather than consider the fuzzy system as in the form (3),

$$\bar{x}(t+1) = m_1(x(t))\bar{A}_1\bar{x}(t) + m_2(x(t))\bar{A}_2\bar{x}(t) + m_3(x(t))\bar{A}_3\bar{x}(t),$$

we considered it in this paper as in the form (7),

$$\bar{x}(t+1) = \bar{A}_1\bar{x}(t), \qquad x \in S_1$$
$$\bar{x}(t+1) = m_1\bar{A}_1\bar{x}(t) + m_2\bar{A}_2\bar{x}(t), \qquad x \in S_2$$
$$\bar{x}(t+1) = \bar{A}_2\bar{x}(t), \qquad x \in S_3$$
$$\bar{x}(t+1) = m_2\bar{A}_2\bar{x}(t) + m_3\bar{A}_3\bar{x}(t), \qquad x \in S_4$$
$$\bar{x}(t+1) = \bar{A}_3\bar{x}(t), \qquad x \in S_5$$

so the stability of the system can be checked by a piecewise Lyapunov function as in section 3.

Remark 2.3: During simulation, subspace S_3 is further separated by origin into 2 subregions. This operation is carried so the *S*-procedure as in section 3 can be performed.

3 Problem Solution

It is demonstrated in [14] that continuous time fuzzy systems can be analysed by piecewise quadratic Lyapunov function. We extend the same idea to discrete time case. The general idea of our approach is to consider the state space of the fuzzy system (1)

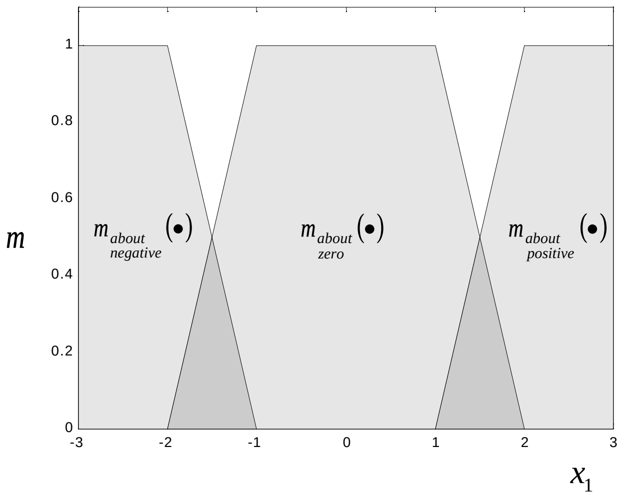

Fig.1. The membership functions for x1 for the fuzzy system in Example 1 and 2.

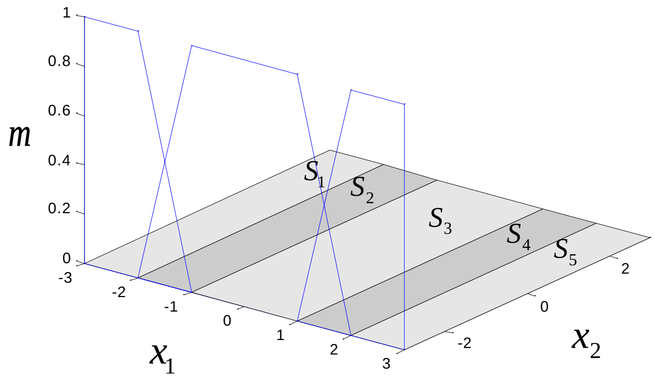

Fig.2. The membership functions for x1 divide the state space into 5 subspaces.

in terms of subspace form as in (7). Then we can use the piecewise Lyapunov function to check for the stability of discrete time fuzzy systems in each subspace.

Due to the fact that the state in discrete time system may jump between non-adjacent subspaces, the structural information, like the matrices *F*'s in [14], cannot be used to characterize the state transition from one subspace to another as dealt with in the case of continuous time systems. More specifically, it may not be helpful to construct a piecewise Lyapunov function that is continuous across boundaries for the discrete time systems to analyse stability of the system as in [14] for the continuous time systems. Nevertheless, it may also be unnecessary to require the piecewise Lyapunov function to be continuous across boundaries for the discrete time piecewise linear systems since the state of such systems may never pass through the boundaries.

As Lyapunov function candidate we consider function of the form

$$V(t)=\begin{cases} x^t P_i x, & x\in S_i,\ i\in L_0 \\ \begin{bmatrix} x \\ 1\end{bmatrix}^t \overline{P}_i \begin{bmatrix} x \\ 1\end{bmatrix}, & x\in S_i,\ i\in L_1 \end{cases}, \quad (8)$$

This function combines the power of quadratic Lyapunov functions near an equilibrium point with the flexibility of piecewise linear functions in the large.

Since the matrix P_i or $\overline{P}_i$ is only used to describe the Lyapunov function in subspace S_i, it is natural to use the *S*-procedure to allow the Lyapunov function search in a less conservative way. To this end, construct matrices, $\overline{E}_i=[E_i \quad e_i],\ i\in L$ with $e_i=0$ for $i\in L_0$ such that

$$\overline{E}_i\begin{bmatrix} x \\ 1\end{bmatrix}\geq 0,\quad x\in S_i,\quad i\in L. \quad (9)$$

It should be noted that the above vector inequality means that each entry of the vector is nonnegative. Now, for every symmetric matrix U_i with nonnegative entries, condition (9) implies that

$$\begin{bmatrix} x \\ 1\end{bmatrix}^t \overline{E}_i^t U_i \overline{E}_i \begin{bmatrix} x \\ 1\end{bmatrix}>0,\ \forall x\in S_i,\ i\in L. \quad (10)$$

Remark 3.1: A systematic procedure for constructing these matrices $\overline{E}_i,\ i\in L$ for a given fuzzy system can be found in [14]. The procedure is directly based on the information in the fuzzy rule base. The interested readers please refer to [14] for details.

Then we come to our theorem.

Theorem 3.1: Consider the discrete time fuzzy system (7). If there exist symmetric matrices, $P_i,P_j,\ i,j\in L_0$, $\overline{P}_i,\overline{P}_j,\ i,j\in L_1$, symmetric matrices U_i,W_{ik} and Q_{ijk} such that U_i,W_{ik} and Q_{ijk} have nonnegative entries, and the following LMIs are satisfied,

$$0<P_i-E_i^T U_i E_i, \quad (11)$$

$$A_k^T P_i A_k - P_i + E_i^T W_{ik} E_i<0, \quad (12)$$

for $i\in L_0$, $k\in K(i)$ and

$$0<\overline{P}_i-\overline{E}_i^T U_i \overline{E}_i, \quad (13)$$

$$\overline{A}_k^T \overline{P}_i \overline{A}_k - \overline{P}_i + \overline{E}_i^T W_{ik} \overline{E}_i<0, \quad (14)$$

for $i\in L_1$, $k\in K(i)$ and

$$\overline{A}_k^T \overline{P}_j \overline{A}_k - \overline{P}_i + \overline{E}_i^T Q_{ijk} \overline{E}_i<0, \quad (15)$$

for $i,j\in\Omega$, $i\in L_1$, $j\in L$, $k\in K(i)$ and

$$A_k^T \tilde{P}_j A_k - P_i + E_i^T Q_{ijk} E_i<0, \quad (16)$$

for $i,j\in\Omega$, $i\in L_0$, $j\in L$, $k\in K(i)$,

where we define

$\overline{P}_j=[I_{n\times n} \quad 0_{n\times 1}]^T P_j [I_{n\times n} \quad 0_{n\times 1}]$ for $j\in L_0$ in (15),

$\tilde{P}_j=[I_{n\times n} \quad 0_{n\times 1}]\overline{P}_j[I_{n\times n} \quad 0_{n\times 1}]^T$ for $j\in L_1$ in (16),

then the discrete time fuzzy system is globally exponentially stable, that is, *x(t)* tends to the origin exponentially for every trajectory in the state space.

Proof: See Appendix.

The above conditions are linear matrix inequalities in the variables P_i,P_j, $\overline{P}_i,\overline{P}_j$, U_i,W_{ik}, and Q_{ijk}. A solution to those inequalities ensures *V(t)* defined in (8) to be a piecewise Lyapunov function for the system. The LMI in (11) or (13) for each subspace guarantees that the function is positive and the LMI in (12) or (14) guarantees that the function decreases along all system trajectories in each subspace. The LMIs (15)-(16) guarantee that the function is decreasing when the state transits from one subspace to another. The terms involving $E_i,\overline{E}_i,U_i,W_{ik}$ and Q_{ijk} are related to the *S*-procedure to reduce the conservatism of those inequalities.

Remark 3.2: For each fuzzy subspace, we are seeking for a "common" piecewise quadratic Lyapunov function which can satisfy all the partial influencing state matrices, and which decrease in time within or between the subspace.

Remark 3.3: Matrices E_i, $\overline{E}_i$ are the structural information for each subspace. We exploit this information even for the case when the state trajectory $x(t)$ transits between different subspaces as in (15)-(16), based on *Assumption 2.2*.

Remark 3.4: Due to the discrete nature of the system, it is noted that transitions could occur between non-adjacent subspaces in one step. Thus, every subspace pair, (S_i,S_j) as defined by (5), has to be computed in (15)-(16).

Remark 3.5: It is noted that when the state of the system does transit across the boundaries, that is, $x(t)\in S_i\cap S_j$ for some *t*, the result in Theorem 3.1 still holds since the case can be covered by considering the transition from the subspace S_i to S_j at the time *t* or $t+1$.

Remark 3.6: Theorem 3.1 is only a sufficient condition for system stability. Thus, the discrete time fuzzy system may still be stable even if the piecewise Lyapunov function (8) can not be identified from the above inequalities. Shall

Theorem 3.1 fail to generate solutions, one may refine the partition in order to increase the flexibility of the Lyapunov function candidate and try anew [14].

Remark 3.7: The stability test of the discrete time fuzzy system in (11)-(16) can be easily facilitated by a commercially available software package Matlab LMI toolbox [19-20].

Example 2: Consider the discrete time fuzzy system given in Example 1. The system matrices are given by

$$A_1 = \begin{bmatrix} 0.9 & -0.1 \\ 0.1 & 1 \end{bmatrix}, \quad a_1 = \begin{bmatrix} 0 \\ -0.02 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 1 & -0.02 \\ 0.02 & 0.9 \end{bmatrix}, \quad a_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$
$$A_3 = \begin{bmatrix} 0.9 & -0.1 \\ 0.1 & 1 \end{bmatrix}, \quad a_3 = \begin{bmatrix} 0 \\ 0.02 \end{bmatrix}.$$

The trajectory of simulations result with initial conditions $x(0) = [3 \quad 3]^T, [3 \quad -3]^T$, $[-3 \quad 3]^T$, and $[-3 \quad -3]^T$ indicates that the fuzzy system is stable though there does not exist a common positive definite matrix P for the system, see Fig. 3. The matrices characterizing the subspaces are given by

$$\bar{E}_1 = \begin{bmatrix} -1 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}, \quad \bar{E}_2 = \begin{bmatrix} -1 & 0 & -1 \\ 1 & 0 & 2 \end{bmatrix},$$
$$E_{3a} = \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}, \quad E_{3b} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},$$
$$\bar{E}_4 = \begin{bmatrix} -1 & 0 & 2 \\ 1 & 0 & -1 \end{bmatrix}, \quad \bar{E}_5 = \begin{bmatrix} -1 & 0 & 3 \\ 1 & 0 & -2 \end{bmatrix}.$$

Using the Theorem 3.1, we can find the following solutions to those 54 LMIs,

$$\bar{P}_1 = \begin{bmatrix} 41.5426 & 8.5689 & -102.3962 \\ 8.5689 & 89.4397 & -1.3397 \\ -102.3962 & -1.3397 & -188.2465 \end{bmatrix},$$
$$\bar{P}_2 = \begin{bmatrix} 45.6622 & 8.3698 & -57.7338 \\ 8.3698 & 89.4603 & -1.1021 \\ -57.7338 & -1.1021 & -20.6759 \end{bmatrix},$$
$$P_{3a} = P_{3b} = \begin{bmatrix} 74.5758 & -2.6340 \\ -2.6340 & 81.1973 \end{bmatrix},$$
$$\bar{P}_4 = \begin{bmatrix} 45.6622 & 8.3698 & 57.7338 \\ 8.3698 & 89.4603 & 1.1021 \\ 57.7338 & 1.1021 & -20.6759 \end{bmatrix},$$

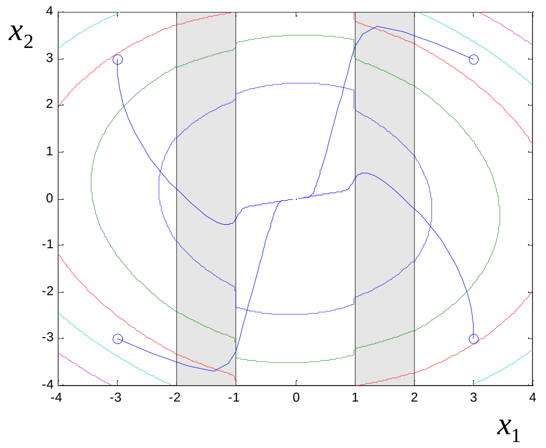

Fig.3. Simulation and level curves of the computed piecewise quadratic Lyapunov function in Example 2.

$$\bar{P}_5 = \begin{bmatrix} 41.5426 & 8.5689 & 102.3962 \\ 8.5689 & 89.4397 & 1.3397 \\ 102.3962 & 1.3397 & -188.2465 \end{bmatrix},$$

and thus one can verify that the fuzzy system is exponentially stable. Note LMI solutions $\bar{P}_1$, $\bar{P}_2$, $\bar{P}_4$ and $\bar{P}_5$ are indefinite. This is because the conservatism which requires them to be positive definite was reduced by the *S*-procedure. The piecewise Lyapunov function obtained is shown in dashed line in Fig. 3.

4 Conclusion

In this paper, a new method is developed to test stability of discrete time fuzzy system based on a piecewise Lyapunov function. It is shown that the stability can be determined by solving a set of LMIs. The approach can be extended to performance analysis of such systems as in [14] for their continuous counterparts.

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Appendix

Proof of Theorem 3.1: Consider the Lyapunov function candidate (8), or in a more compact form

$$V(t) = \bar{x}^t \bar{P}_i \bar{x}, \; x \in S_i, \; i \in L. \qquad (17)$$

It is obvious from (8) that in an open neighborhood of the origin there exists a constant $b > 0$ such that

$V(t) \le b \| x \|^2$, since the affine term does not appear in this case. Moreover, (11) and (13) imply that there exists a constant $a > 0$ such that $a \| \bar{x} \|^2 \le \bar{x}^T \bar{P}_i \bar{x}$ for $x \in S_i$.

Thus we have,

$$a \| x \|^2 \le V(t) \le b \| x \|^2 \quad (18)$$

In addition, it follows from (12), (14)-(16) that there exists a constant $r > 0$ such that $\bar{A}_k^T \bar{P}_j \bar{A}_k - \bar{P}_i + r\,\mathrm{I} < 0$, where $j = i$ when *x(t)* stays in the subspace S_i, $j \ne i$ when *x(t)* transits from the subspace S_i to S_j.

Then along trajectories of the system, we have

$$\Delta V(t) = V(t+1) - V(t)$$
$$= \sum_{k \in K(i)} m_k(x) \cdot \bar{x}^T [\bar{A}_k^t \bar{P}_j \bar{A}_k - \bar{P}_i] \bar{x}$$

Since $\sum_{k \in K(i)} m_k(x) = 1$ for all x, we have

$$\Delta V(t) \le \bar{x}^T (-r\,\mathrm{I}) \bar{x}$$
$$\le -r \| x \|^2 \quad (19)$$

Therefore, the desired result follows directly from (18) and (19) based on the standard Lyapunov theory. ∇∇

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