QUANTITATIVE AND QUALITATIVE ANALYSIS OF WAVELET GALERKIN APPROACH TO SOLVE RADIONUCLIDE TRANSPORT IN INHOMOGENEOUS CRYSTALLINE ROCKS

ATSUSHI NEYAMA, HESHAM NASIF
Environmental Engineering Division
Computer Software Development Company
Tokyo-Japan
HIROYUKI UMEKI
Research and Development-Science and Technology Division
Nuclear Waste Management Organization of Japan
Atsuyuki SUZUKI
DEPARTMENT OF QUANTUM ENGINEERING AND SYSTEMS SCIENCE
THE UNIVERSITY OF TOKYO

Abstract: A quantitative and qualitative analysis for the wavelet Galerkin approach to solve radionuclide transport in inhomogeneous crystalline rocks including single nuclides and radioactive chains is presented in this work. The several radionuclides release calculations made with this approach have shown the capability of this to handle situations rapidly and easily. The particularity of this, the high accuracy obtained with relatively low wavelet-dilation orders pair and the small requirements of computing time. Using higher wavelet order to ensure the smoothness and higher dilation order to get fine resolution especially at the meshes close to the boundaries can considerably reduce the error.

Keywords: Wavelet, geological disposal, geosphere, parallel plate, pathways

1. Introduction

Radionuclides are transported in solution through the host rock by groundwater flowing through the fractures. Flow does not occur uniformly in the fractures, but rather is concentrated in the channels that arise, for example, from variation in the aperture and the presence of infill material [1]. There are many potential transport paths through the host rock, each comprised of one or series of channels. The variability in length, transmissivity and other transport-relevant properties of the transport paths, as well as interconnections between paths, gives rise to mechanical dispersion. Transport along these paths is retarded by diffusion into stagnant water in the pores of the rock matrix surrounding the channels and by sorption on infill and rock matrix pores. The matrix itself may be heterogeneous, comprising zones of different properties created by a succession of geological and geochemical processes.

The multi-pathway model represents a step toward realism taking into consideration the variability in the transport paths. The model is formulated and solved using the wavelet Galerkin approach using the compactly supported advantage of Daubechies' wavelet [2]. Using the boundaries of wavelet series expansion as the real boundary cures the difficulties in the treatment of boundary and interface conditions [3]. The inner products, which are called connection coefficients, arising from using the wavelet as a base function are analytically evaluated, and tabulated [4]. Analytical solution of the inner

products allows using higher wavelet order, which gives better accuracy.

This paper will show the applicability of wavelet Galerkin method to calculate the radionuclide transport through inhomogeneous crystalline rock. An insight of how distribution coefficient and transmissivity mainly influence the migration of the species through the host rock. In Sec. 2 the mathematical formulation of the multi-pathway model is presented. Sec. 3 shows the results including the quantitative and qualitative analysis. In sec.4 the conclusion of this work is shown.

2. Problem Formulation

The features and processes for radionuclides transport in the Far Field are modeled as parallel-walled openings in which transport occurs by advection and dispersion, and at a given time, radionuclide concentration varies only in the flow direction. According to that, within the fracture, advection and dispersion are modeled in one spatial dimension. Matrix diffusion is also modeled in one spatial dimension normal to the plane of the fractures. The radioactive nuclides then transport through the Main Water Conducting Fault (MWCF) to the biosphere. The Model of the radionuclide transport through MWCF is the same as in the geosphere except that single channel with fixed transmissivity is considered.

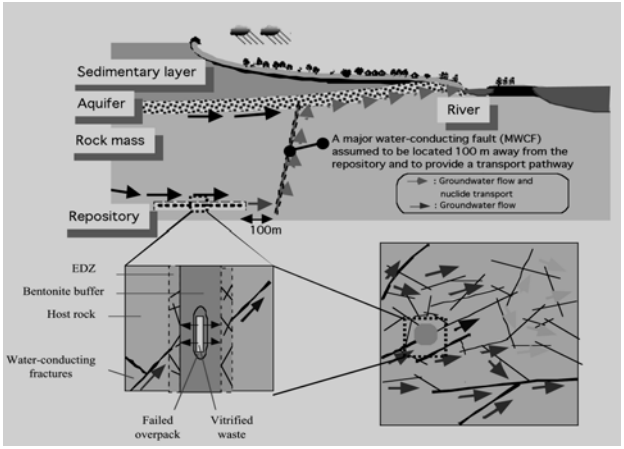

Fig.1. Illustration of the processes considered in the parallel plate model

The governing equation can be written as follows:

$$R_n^f \frac{\partial C_n^f}{\partial t} = \frac{\partial}{\partial x} D_L \frac{\partial C_n^f}{\partial x} - \nu \frac{\partial C_n^f}{\partial x} - R_n^f \lambda_n C_n^f + R_{n-1}^f \lambda_{n-1} C_{n-1}^f + \frac{F}{b} D_e^m \frac{\partial C_n^m}{\partial \omega}\bigg|_{\omega=0} \quad (1)$$

$$R_n^m \frac{\partial C_n^m}{\partial t} = \frac{\partial}{\partial \omega} D^m \frac{\partial C_n^m}{\partial \omega} - R_n^m \lambda_n C_n^m + R_{n-1}^m \lambda_{n-1} C_{n-1}^m \quad (2)$$

Where, b half of the fracture aperture m, ν dispersion coefficient in the fractures $m^2 s^{-1} (= \alpha_L \nu + D_0)$, α_L longitudinal dispersion length m, D_0 diffusion coefficient in free water $m^2 s^{-1}$, λ_n decay constant s^{-1}, F proportion of fracture surface from which nuclides can diffuse into the matrix , D_e^m effective diffusion coefficient $m^2 s^{-1}$ $(= \theta^m D^m)$, θ^m matrix porosity [-], D^m diffusion coefficient in the matrix pores $m^2 s^{-1}$, C_n, C_n^m nuclide concentrations in the fracture and in the matrix mol/m^3, x transport distance m, ω perpendicular distance into the matrix from the fracture surface m, and t the time. The suffices n, f and m are represented a nuclide, fracture and matrix respectively. In addition, R_n and R_n^m represent the retardation coefficients for nuclide n in fracture and in the matrix respectively, and are expressed by the following formulas with the assumption of linear, reversible and instantaneous sorption:

$$R_n = 1 + \frac{Ka_n}{b}, \quad R_n^m = 1 + \frac{\rho^m Kd_n^m}{\theta^m} \quad (3)$$

where, Ka_n is the distribution coefficient for the fracture surface m, Kd_n^m is the distribution coefficient for the rock minerals in the matrix

$m^3 kg^{-1}$ and ρ^m is the dry density of the matrix kgm^{-3}.

The initial concentration in the fracture and matrix assumed to be zero. The weighted nuclide release from the EBS [4] is taken as the boundary condition for the nuclide transport in the model pathway with tramsmissivity T_i,

$$A_f\left[\nu C_n^f - D_L \frac{\partial C_n^f}{\partial x}\right]_{x=0} = f_n(T_i,t) \qquad (4)$$

where A_f is the fracture cross section (= aperture x fracture unit width)[m^2]. Based on the above the weighted nuclide release rate from the EBS to the model pathways $f_n(T_i,t)$, is defined as:

$$f_n(T_i,t) = P(T_i).f_n^{buffer\rightarrow hostrock}(t)$$

where, T_i is the median value of transmissivity in segment i [m2s-1], $P(T_i)$ probability of segment i, which is designed according to the probability that transmissivity of randomly sampled channel in the host rock lies in the range of transmissivity in segment i, $f_n^{buffer\rightarrow hostrock}(t)$ is the nuclide release rate from the engineered barrier system. The boundary condition at the fracture surface and at the maximum depth for matrix diffusion, d [m], are expressed as follows:

$$C_n^f(x,t) = C_n^m(x,0,t) \qquad (\omega = 0) \qquad (5)$$

$$\frac{\partial C_n^m}{\partial \omega} = 0 \qquad (\omega = d) \qquad (6)$$

The release rate from the host rock per waste package, $g_n(t) = \sum_{i=1}^{J} h_n(T_i,t)$

Where, $h_n(T_i,t)$ is the nuclide release rate from the model pathway with transmissivity T_i, T_i is the median value of the transmissivity in segment i $m^2 \sec^{-1}$, J is the total number of segment, and t is the time in *sec*.

In order to reach biosphere, it is assumed that the nuclides are transported horizontally from the repository through 100 m of the host rock, and are then transported vertically through a regional scale MWCF as shown in Fig. 1. The fractures within the MWCF are assumed to have identical properties (unlike the model of the host rock, variability of transmissivity is ignored), and for modeling purpose, a single representative fracture is considered. The transport distance of the MWCF is set to 800 *m* based on a repository depth of 1000 *m* with 200 *m* of overburden sediments. The transport processes are identical to those of the host rock. The source term is the release rate from the host rock.

The problem is two-dimensional in space in the sense that equation (2) has to be solved for each x-value. The solution of this system can be represented in a continuous function form with the series of Daubechies' scaling function and coefficients. The unknown is not the solution at the nodes but the coefficients of the scaling function and we have to recalculate the concentrations at all the points.

In order for the algorithm to resolve all the structures the basis of active wavelet and, consequently the computational grid should adapted dynamically in the time to reflect local changes in the solution. The system is solving using fifth order Gear implicit integration [5].

3. Problem Solution

To give picture of the capability of wavelet-Galerkin approach to calculate the radionuclide transport, release calculations for several nuclides are discussed considering several situations of transport existing in the repository system. The required data for the calculations are shown in Table 1, for single radionuclides and decay chains.

Figures 2, and 3 show the nuclide release from the host rock for single isotope and radionuclides chain respectively. Se-79 has the highest release rate about 4,000 years after disposal, when it is over taken by Cs-135. Those nuclides have high release rate because, their release rate from the EBS is also high. Beside the initial high release rate Se-79 and Cs-135 have low distribution coefficient, which

affects their migration through the host rock. Also, Cs-135 has relatively long half-life. Figure 3 shows the release rate of the decay chains. Nb-93m and Zr-93 have the same release rate and that is due to the short half-life of the Nb-93 m comparing to its parent Zr-93 which let it comes in equilibrium with Zr-93. Moreover, they have the same distribution coefficient. In particular, the release rate of nuclides with relatively short half lives (e.g. Pu-239 and Am-243) are drastically reduced below 10^{-3} Bq/yr with the exception of those (e.g. Ra-226 and Pb-210) which are in radioactive equilibrium with their parents.

The release rate from the MWCF to the biosphere for different isotopes including single isotope calculations and decay chains calculations is shown in Fig. 4. Se-79 again has the highest release rate initially, and is overtaken by Cs-135 after about one hundred thousand years. Comparing figures 2 and 3 with 4, it can be seen that the times of peak release for Se-79 are somewhat delayed following transport through MWCF. However, the magnitude of the peak releases is almost unaffected due to their relatively low distribution coefficients and long half-life. The releases of nuclides with relatively high distribution coefficient are considerably retarded in the MWCF.

Table. 1 Parameters for the geosphere models

Parameter	**Description**
Rock Type	Granite
Groundwater	Fresh type groundwater
Hydraulic gradient	0.01
Transport distance	100 m
Transmissivity Dist.(T)	$10^{-13} \sim 10^{-7}$
Aperture, $2b$	$2b = 2\sqrt{T}$
Dispersion length	10 m
Portion of fracture F	50%
Diffusion depth	0.1 m
Porosity	2%
Dry density	2640 kg/m^3
Effective diffusion Coefficient.	$3 \times 10^{12} m^2 s^{-1}$

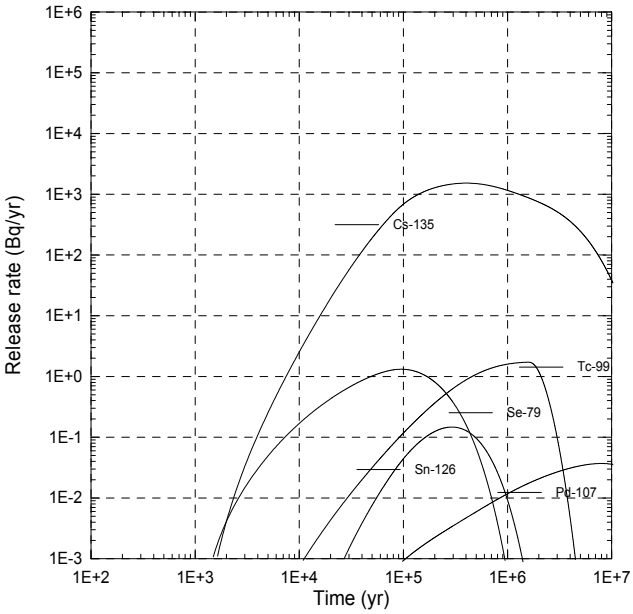

Fig. 2. Release rate of fission products from the host rock to the MWCF per waste package

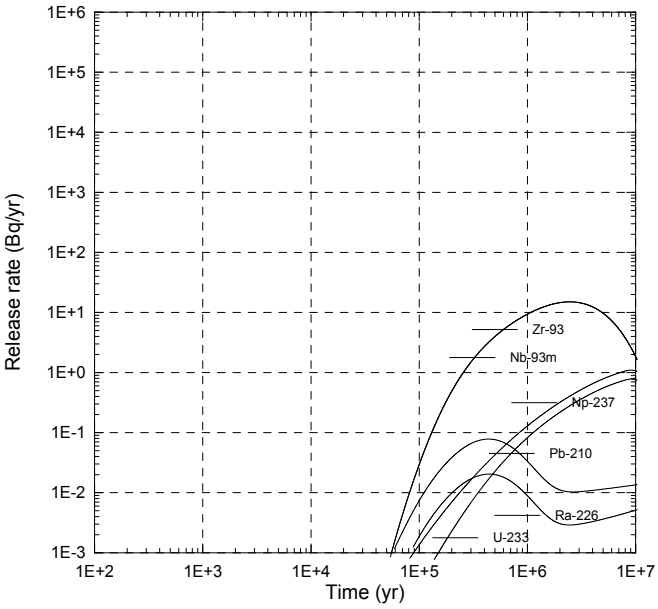

Fig. 3. Release rate of decay chains from the host rock to the MWCF per waste package

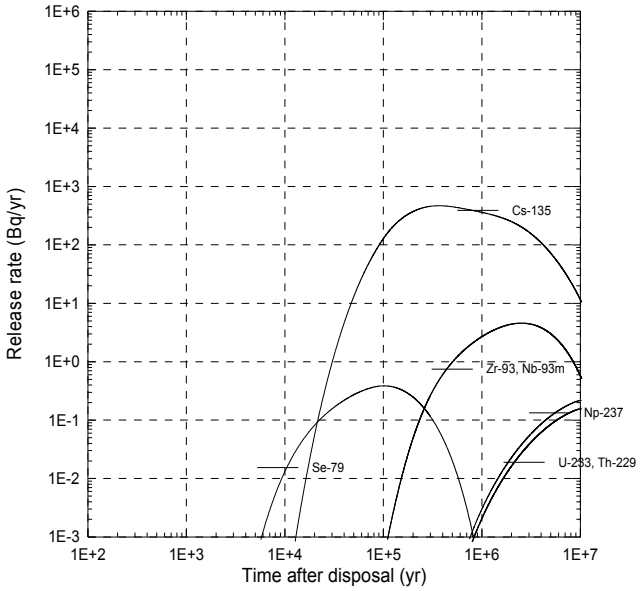

Fig.4 Release rate from the MWCF per waste package

The uncertainty in the modeling data is also analyzed. The first parameter analyzed is the transmissivity. The transmissivity distribution affects the groundwater velocity in the host rock fracture and thus effectiveness of the barrier. Figure 5, shows the effect of the transmissivity distribution on the release rate is effective with high sorptive Thorium case nuclide than less sorptive cesium nuclide case.

The uncertainty in the dispersion length shows that, increasing in the dispersion length leads to the earlier arrival times of the nuclides at the downstream end of the transport path. The release rate is not sensitive to this parameter, because two orders of magnitude difference in the dispersion length causes the maximum nuclide release rate to vary by a much smaller factor.

The matrix diffusion depth uncertainty analysis showed that, the change in the matrix diffusion depth from 0.1 m to either 0.03 m or 1.0 m affect the retardation. A decrease to 0.03 m results in an increase in release rate with respect to the reference case. Increasing of the diffusion depth to 1 m affect in reducing the release rate by almost two orders of magnitude with respect to the 0.03 m case as shown in Fig. 6.

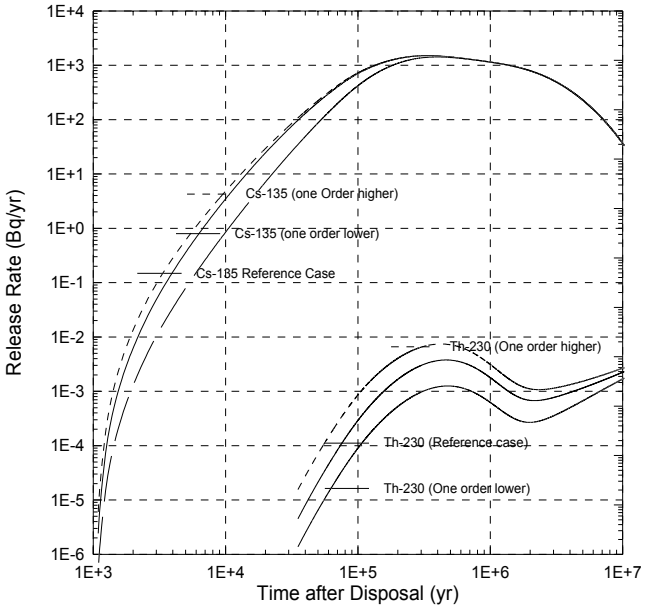

Fig.5. Transmissivity uncertainties for different isotopes

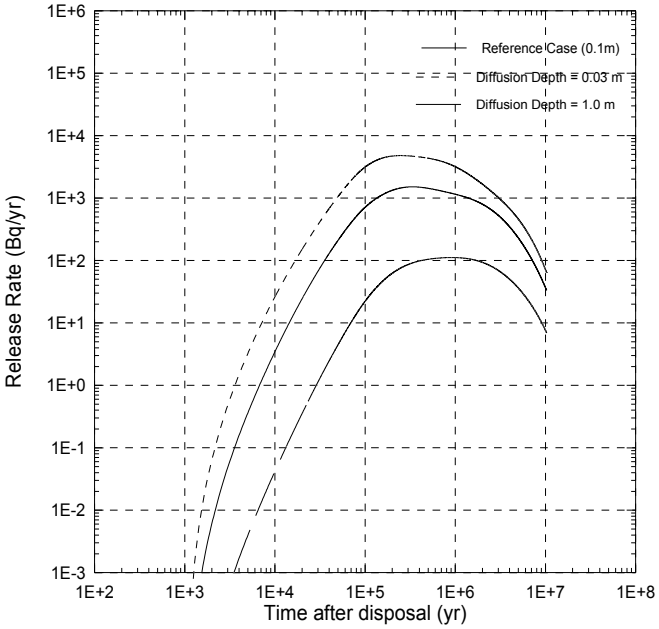

Fig. 6. Diffusion depth uncertainty analysis

4. Conclusion

The capability of Wavelet Galerkin discretization to calculate the radionuclides transport through a geospheric crystalline host rocks was shown by applying the model to the very complex transport situation on the repository system.

The flexibility of the model and it is quite moderated requirements of computing time when applying to this system allows us to make a series of calculations in an easy and rapid form. The discretization of the system is straightforward. It is easy to change the wavelet and dilation orders at any sensitive point to get the desired accuracy.

The analysis showed that the impact of uncertainties in the parameters on the release is different for different nuclides. In general, Transmissivity, diffusion depth, retardation coefficients, and aperture distribution and dispersion length are parameters that need to be well determined because any uncertainty in these parameters will have strong influence on the release. Also the choice of wavelet order and dilation order should be done carefully to ensure the desired accuracy of the solution. All the calculations are carried out with wavelet order of 5 and dilation order of 6 and they have the desired accuracy.

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