Periodic Sequences Derived from Self-Orthogonal Finite-Length Sequences

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Abstract: - Self-orthogonal finite-length sequence has impulsive autocorrelation function with no sidelobes except at left and right shift-ends. In this paper, some types of periodic sequences are derived from the self-orthogonal finite-length sequence. Periodic sequence with successive zero values, self-orthogonal periodic sequence and near-self-orthogonal periodic sequence have good properties of periodic even and odd autocorrelation functions. Derived sequences are expressed by DFT together with convolution of element sequences and applicable to fast convolution or correlation processing.

Key-Words: - Self-orthogonal, finite-length sequence, periodic sequence, DFT, convolution, aperiodic correlation, even autocorrelation, odd autocorrelation, fast signal processing.

1 Introduction

In the direct sequence spread spectrum systems of radar and communication, pseudonoise sequences are desired to have good properties of sequence values, auto-and crosscorrelation values. In a cellular CDMA communication system, finite-length sequences with good properties could suppress intersymbol, intracell and intercell interferences. In a pulse compression radar system, periodic sequence with good properties might suppress intersymbol interference. Self-orthogonal finite-length sequences which have impulsive aperiodic autocorrelation function with zero sidelobes except at left and right shift-ends are effective to suppress the above interferences in the CDMA system [1]-[3]. In this paper, some types of periodic sequences are derived from the self-orthogonal finite-length sequences. The derived sequences have the expressions of the DFT together with the convolution of element sequences and have the good properties of even and odd autocorrelation functions.

2 Self-Orthogonal Finite-Length Sequence

A finite-length sequence whose aperiodic autocorrelation function has no sidelobes except at left and right shift-ends, can be called self-orthogonal or shift-orthogonal finite-length sequence, because its shifted sequences are orthogonal within a limited shift range. This finite-length sequence has generally complex or real values. An aperiodic autocorrelation function $\{\rho_{M,\ell,\ell,i'}\}$ of a complex-valued self-orthogonal finite-length sequence $\{a_{M,\ell,i}\}$ of length M and distinct number ℓ is represented by

$$\rho_{M,\ell,\ell,i'} = \frac{1}{M}\sum_{i=0}^{M-1} a_{M,\ell,i} a^*_{M,\ell,i-i'} = \begin{cases} 1 & , i'=0 \\ \varepsilon_{M-1} & , i'=M-1 \\ \varepsilon^*_{M-1} & , i'=-(M-1) \\ 0 & , elsewhere \end{cases} \quad (1)$$

where $a_{M,\ell,i}=0$ for $i<0$ and $i>M-1$, and i' is shift, and $*$ stands for complex conjugate, and ε_{M-1} is shift-end value as

$$\varepsilon_{M-1} = |\varepsilon_{M-1}|\cdot e^{j\varphi_M}. \quad (2)$$

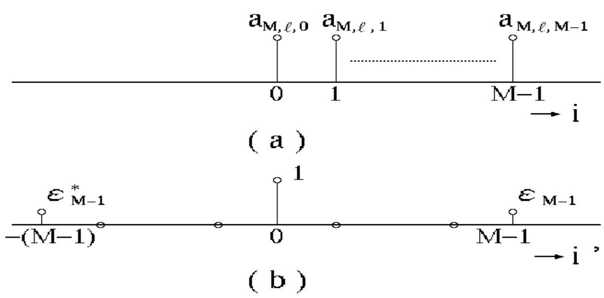

Fig. 1 (a)Self-orthogonal finite-length sequence and (b) its autocorrelation function .

Fig. 1 illustrates the self-orthgonal finite-length sequence and its autocorrelation function. From Eq. (1) the shifted sequences $\{a_{M,\ell,i-n}\}$ satisfy the orthogonality as follows:

$$\frac{1}{M}\sum_{i=0}^{M-1} a_{M,\ell,i}a^{*}_{M,\ell,i-n} = 0, \quad n = 1,2,\cdots,M-2 \tag{3}$$

The self-orthogonal finite-length sequence is replaced by an impulse train

$$a_{M,\ell}(t) = \sum_{i=0}^{M-1} a_{M,\ell,i}\delta(t-i\Delta t) \tag{4}$$

and its Fourier transform

$$A_{M,\ell}(f) = \sum_{i=0}^{M-1} a_{M,\ell,i}Z^{-i} \tag{5}$$

where t is time, f is frequency, $\delta(t-i\Delta t)$ is the impulse at every time interval Δt, and $Z = e^{j2\pi f\Delta t}$, $j=\sqrt{-1}$. The energy spectrum of $a_{M,\ell}(t)$ becomes

$$\begin{aligned}|A_{M,\ell}(f)|^2 &= A_{M,\ell}(f)\cdot A^{*}_{M,\ell}(f)\\ &= M\sum_{i'=-(M-1)}^{M-1}\rho_{M,\ell,\ell,i'}Z^{-i'}\\ &= M\left\{\varepsilon^{*}_{M-1}Z^{M-1}+1+\varepsilon_{M-1}Z^{-(M-1)}\right\}\end{aligned} \tag{6}$$

which is factorized to

$$|A_{M,\ell}(f)|^2 = M\varepsilon_{M-1}Z^{M-1}\prod_{m=0}^{M-2}\left\{Z^{-1}-\alpha_M e^{-j\frac{\phi_M}{M-1}}e^{j\frac{(2m+1)\pi}{M-1}}\right\} \times\left\{Z^{-1}-\beta_M e^{-j\frac{\phi_M}{M-1}}e^{j\frac{(2m+1)\pi}{M-1}}\right\} \tag{7}$$

where

$$\left.\begin{aligned}\alpha_M &= \left(\frac{1+\sqrt{1-4|\varepsilon_{M-1}|^2}}{2|\varepsilon_{M-1}|}\right)^{\frac{1}{M-1}}\\ \alpha_M\beta_M &= 1\end{aligned}\right\} \tag{8}$$

From Eq. (6), the sequence spectrum $A_{M,\ell}(f)$ is constructed by combining the either prime element polynomial with α_M and β_M for each m, because the prime element polynomials with α_M and β_M of the same m correspond to the reverse prime element sequences of length 2 with the same autocorrelation function. The sequence spectrum is given by

$$A_{M,\ell}(f) = \sqrt{M|\varepsilon_{M-1}|}e^{j\varphi_M}K_{M,\ell} \times\prod_{m=0}^{M-2}\left\{Z^{-1}-\gamma_{M,m}e^{-j\frac{\varphi_M}{M-1}}e^{j\frac{(2m+1)\pi}{M-1}}\right\} \tag{9}$$

where

$$\left.\begin{aligned}K_{M,\ell} &= \frac{1}{\sqrt{\prod_{m=0}^{M-2}\gamma_{M,m}}}\\ \gamma_{M,m} &= \alpha_M \ \ or\ \ \beta_M\end{aligned}\right\}. \tag{10}$$

Eq. (9) explains that the sequence $\{a_{M,\ell,i}\}$ is constructed by the $(M-1)$-multiple convolution of prime element sequences of length 2. If the sequence has real values, then $\varphi_M = 0$ or π and the first order prime element polynomials with complex conjugate coefficients are combined to the second order compound element polynomial with real coefficient in Eq. (9). Since combing element sequences can make the longer compound element sequences zero-valued, the self-orthogonal finite-length sequence is effective to make digital convolution or correlation processing fast[2].

3 A Derived Finite-Length Sequence

We introduce another finite-length sequence derived from self-orthogonal finite-length sequence. Let the sequence $\{a'_{M,\ell',i}\}$ have the shift-end value ε'_{M-1}. Convolution between $\{a_{M,\ell,i}\}$ and $\{a'_{M,\ell',i}\}$ makes the sequence $\{b_{M',\lambda,i.}\}$ of length $M'=2M-1$. The autocorrelation function of $\{b_{M',\lambda,i.}\}$, which is derived from convolution between the autocorrelation functions of $\{a_{M,\ell,i}\}$ and $\{a'_{M,\ell',i}\}$ or derived from $|A_{M,\ell}(f)|^2\cdot|A'_{M,\ell'}(f)|^2$, is given by

$$\rho'_{M',\lambda,\lambda,i'} = \begin{cases}\delta_{2(M-1)} &, \ i'=2(M-1)\\ \delta_{M-1} &, \ i'=M-1\\ 1 &, \ i'=0\\ \delta^{*}_{M-1} &, \ i'=-(M-1)\\ \delta^{*}_{2(M-1)} &, \ i'=-2(M-1)\\ 0 &, \ elsewhere\end{cases} \tag{11}$$

where

$$\left.\begin{array}{l}\delta_{2(M-1)} = \dfrac{\varepsilon_{M-1}\varepsilon'_{M-1}}{1+\varepsilon_{M-1}\varepsilon'^{*}_{M-1}+\varepsilon^{*}_{M-1}\varepsilon'_{M-1}} \\ \delta_{M-1} \quad = \dfrac{\varepsilon_{M-1}+\varepsilon'_{M-1}}{1+\varepsilon_{M-1}\varepsilon'^{*}_{M-1}+\varepsilon^{*}_{M-1}\varepsilon'_{M-1}}\end{array}\right\} \quad (12)$$

The spectrum of $\{b_{M',\lambda,i}\}$ is given by

$$B_{M',\lambda}(f) = \sqrt{\frac{M'\left|\delta_{2(M-1)}\right|}{M^2\left|\varepsilon_{M-1}\right|\cdot\left|\varepsilon'_{M-1}\right|}}A_{M,\ell}(f)A'_{M,\ell'}(f) \quad (13)$$

since

$$\left|B_{M',\lambda}(f)\right|^2 = M'\left\{\delta^{*}_{2(M-1)}Z^{2(M-1)}+\delta^{*}_{M-1}Z^{M-1} +1+\delta_{M-1}Z^{-(M-1)}+\delta_{2(M-1)}Z^{-2(M-1)}\right\} \quad (14)$$

where $A_{M,\ell'}(f)$ is expressed by replacing ℓ, φ_M, $K_{M,\ell}$ and $\gamma_{M,m}$ of $A_{M,\ell}(f)$ by ℓ', φ'_M, $K'_{M,\ell'}$ and $\gamma'_{M,m}$, respectively. If the absolute values of δ_{M-1} and $\delta_{2(M-1)}$ are small, the sequence $\{b_{M',\lambda,i}\}$ is useful. When $\varepsilon_{M-1} = -\varepsilon'_{M-1}$, *then* $\delta_{M-1} = 0$: the sequence $\{b_{M',\lambda,i}\}$ becomes a self-orthogonal finite-length sequence of length $M' = 2M-1$. The sequence $\{b_{M',\lambda,i}\}$ is applied to the synthesis of some types of periodic sequences, afterward.

4 Periodic Sequences from Self-Orthogonal Finite-Length Sequence

We can derive periodic sequences from the above finite-length sequences. The derived periodic sequences have fine periodic autocorrelation functions, are effective to fast signal processing, and may have other properties of sequence values and correlation functions, etc.

4.1 Periodic Sequence with successive zero values

We introduce a periodic sequence $\{\overline{a}_{N,\ell,i}\}$ of period N, having zero values in half a period, as

$$\overline{a}_{N,\ell,i} = \begin{cases}\sqrt{2}a_{M,\ell,i} & ; i=0,1,\cdots M-1 \\ 0 & ; i = M, M+1,\cdots N-1\end{cases} \quad (15)$$

where $N = 2M$. The autocorrelation function of this periodic sequence is given by

$$\overline{\rho}_{N,\ell,\ell,i'} = \frac{1}{N}\sum_{i=0}^{N-1}\overline{a}_{N,\ell,i}\overline{a}^{*}_{N,\ell,i-i'} = \begin{cases}1 & , i'=0 \bmod N \\ \varepsilon_{M-1} & , i'=M-1 \bmod N \\ \varepsilon^{*}_{M-1} & , i'=M+1 \bmod N \\ 0 & , elsewhere.\end{cases} \quad (16)$$

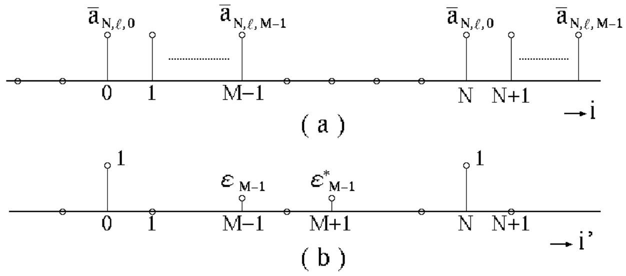

Fig. 2 (a) Periodic sequence $\{\overline{a}_{N,\ell,i}\}$ and
(b) autocorrelation function $\{\overline{\rho}_{N,\ell,\ell,i'}\}$.

The periodic autocorrelation function $\{\overline{\rho}_{N,\ell,\ell,i'}\}$ takes the same values as those of the aperiodic autocorrelation function $\{\rho_{M,\ell,\ell,i'}\}$. This periodic sequence keeps the property of the self-orthogonal finite-length sequence. Fig. 2 shows the periodic sequence $\{\overline{a}_{N,\ell,i}\}$ and the autocorrelation function $\{\overline{\rho}_{N,\ell,\ell,i'}\}$.

The periodic sequence $\{\overline{a}_{N,\ell,i}\}$ is also represented by the discrete Fourier transform (DFT) as

$$\overline{a}_{N,\ell,i} = \frac{1}{\sqrt{N}}\sum_{k=0}^{N-1}\overline{A}_{N,\ell,k}e^{j\frac{2\pi ki}{N}} \quad (17)$$

where $\{\overline{A}_{N,\ell,k}\}$ is the DFT spectrum of $\{\overline{a}_{N,\ell,i}\}$. Introducing the impulse train $\overline{a}_{N,\ell}(t)$ of period T and divided interval $\Delta t = T/N$ similar to Eq. (4) and the discrete frequency $f = k/T$, $k = 0,1,\cdots,N-1$, we can obtain the DFT spectrum

$$\bar{A}_{N,\ell,k} = \frac{\sqrt{2}}{\sqrt{N}} A_{M,\ell}(f)\Big|_{f=k/T,\Delta t=T/N}$$
$$= \sqrt{|\varepsilon_{M-1}|}\, e^{j\varphi_M} K_{M,\ell} \prod_{m=0}^{M-2} \left\{ e^{-j\frac{2\pi}{N}k} - \gamma_{M,m} e^{-j\frac{\varphi_M}{M-1}} e^{j\frac{(2m+1)\pi}{M-1}} \right\} \tag{18}$$

This spectrum is the expression based on convolution. From Eq. (6) and Eq. (18) the absolute value of this spectrum is given by

$$\left|\bar{A}_{N,\ell,k}\right| = \sqrt{1 + 2|\varepsilon_{M-1}| \cos\left\{\frac{2\pi}{N} k(M-1) + \varphi_M\right\}} \tag{19}$$

and from Eq. (18) the phase angle of this spectrum is obtained. We can calculate the sequence values by the DFT of $\{\bar{A}_{N,\ell,k}\}$ with the absolute value and the phase angle, in addition to the convolution. We can derive the other sequences by varying the continuing zero values together with period N. In a particular case without zero value, we can use Eqs. (17),(18) and (19) for $N = M$.

4.2 Self-Orthogonal periodic Sequence

We introduce a periodic sequence $\{\tilde{a}_{N,\ell,i}\}$ of period N, with overlapping the first value and the last value in a self-orthogonal finite-length sequence $\{a_{M,\ell,i}\}$, as

$$\tilde{a}_{N,\ell,i} = \begin{cases} \tilde{K}_N (a_{M,\ell,0} + a_{M,\ell,M-1}) & ,i=0 \\ \tilde{K}_N a_{M,\ell,i} & ,i=1,2,\cdots N-1 \end{cases} \tag{20}$$

where $N = M - 1$ and

$$\tilde{K}_N = \sqrt{\frac{N}{M(1+\varepsilon_{M-1}+\varepsilon^*_{M-1})}} \tag{21}$$

The autocorrelation function of the periodic sequence $\{\tilde{a}_{N,\ell,i}\}$ is given by

$$\tilde{\rho}_{N,\ell,\ell,i'} = \frac{1}{N}\sum_{i=0}^{N-1} \tilde{a}_{N,\ell,i} \tilde{a}^*_{N,\ell,i-i'}$$
$$= \begin{cases} 1 & , i' = 0 \bmod N \\ 0 & , elsewhere. \end{cases} \tag{22}$$

Fig. 3 shows the periodic sequence $\{\tilde{a}_{N,\ell,i}\}$ and the autocorrelation function $\{\tilde{\rho}_{N,\ell,\ell,i'}\}$. Therefore, this sequence is a self-orthogonal periodic sequence, since the shifted sequences $\{a_{N,\ell,i-n}\}$, $n = 0,1,2,\cdots,N-1$ are orthogonal from Eq. (22).

The periodic sequence $\{\tilde{a}_{N,\ell,i}\}$ is also represented by the DFT as

$$\tilde{a}_{N,\ell,i} = \frac{1}{\sqrt{N}}\sum_{k=0}^{N-1} \tilde{A}_{N,\ell,k} e^{j\frac{2\pi}{N}ki} \tag{23}$$

where $\{\tilde{A}_{N,\ell,k}\}$ is the DFT spectrum of $\{\tilde{a}_{N,\ell,i}\}$, given by

$$\tilde{A}_{N,\ell,k} = \frac{\tilde{K}_N}{\sqrt{N}} A_{M,\ell}(f)\Big|_{f=\frac{k}{T},\,\Delta t=\frac{T}{N}}$$
$$= \frac{\sqrt{|\varepsilon_{M-1}|}}{\sqrt{1+2|\varepsilon_{M-1}|\cos\varphi_M}} e^{j\varphi_M} K_{M,\ell}$$
$$\times \prod_{m=0}^{M-2} \left\{ e^{-j\frac{2\pi}{N}k} - \gamma_{M,m} e^{-j\frac{\varphi_M}{M-1}} e^{j\frac{(2m+1)\pi}{M-1}} \right\} \tag{24}$$

The absolute value of the spectrum $\{A_{N,\ell,k}\}$ is apparently

$$\left|\tilde{A}_{N,\ell,k}\right| = 1 \tag{25}$$

and the phase angle of the spectrum $\{A_{N,\ell,k}\}$ is obtained from Eq. (24).

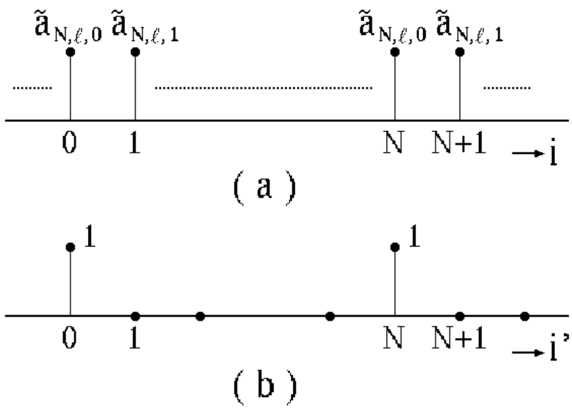

Fig. 3 (a) Periodic sequence $\{\tilde{a}_{N,\ell,i}\}$ and (b) autocorrelation function $\{\tilde{\rho}_{N,\ell,\ell,i'}\}$.

A self-orthogonal periodic sequence is effective to the application to radar. If we apply the sequence to data transmission, the sequence of one period must be treated as a finite-length sequence. The aperiodic autocorrelation function of the finitized sequence from the self-orthogonal periodic sequence $\{\tilde{a}_{N,\ell,i}\}$ is obtained as follows :

$$\left.\begin{array}{l} \tilde{\rho}'_{N,\ell,\ell,i'}=1 \quad ,\ i'=0 \\ \tilde{\rho}'_{N,\ell,\ell,i'}=\dfrac{\tilde{K}_N^2}{N}\Big(a_{M,\ell,i'}a^*_{M,\ell,M-1} \\ \qquad\qquad -a^*_{M,\ell,M-1-i'}a_{M,\ell,M-1}\Big) \\ \qquad ,\ i'=1,2,\cdots,N-1 \\ \tilde{\rho}'_{N,\ell,\ell,i'}=\tilde{\rho}'^*_{N,\ell,\ell,-i'} \quad ,\ i'=-1,-2,\cdots,-(N-1) \end{array}\right\} \quad (26)$$

Fig. 4 shows the repeated sequence of this finitized sequence and its decomposed sequences. The autocorrelation functions of the decomposed even and odd sequences correspond to even and odd autocorrelation functions, respectively.

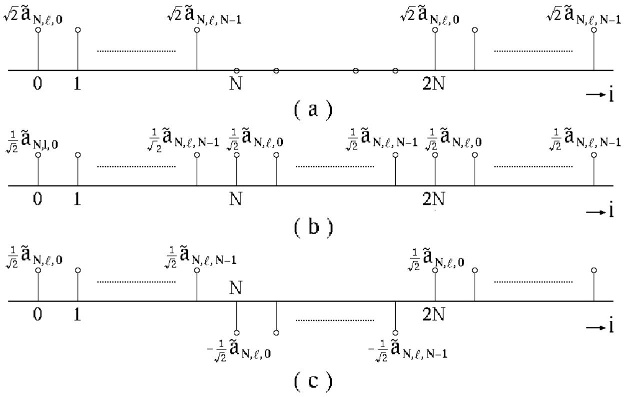

Fig. 4 (a) Repeated sequences of finitized sequence, (b) even sequence and (c) odd sequence.

From Eqs. (20),(26) and Fig. 4 ,we can obtain the following : the normalized even autocorrelation function is the same as the periodic autocorrelation function $\{\tilde{\rho}_{N,\ell,\ell,i'}\}$, and the normalized odd autocorrelation function takes 1 at $i'=0 \bmod 2N$, -1 at $i'=N \bmod 2N$, and that of $\{\tilde{\rho}'_{N,\ell,\ell,i'}\}$ at other $i' \bmod 2N$. For the longer N, the absolute values of odd autocorrelation function out of phase decrease.

4.3 Near-Self-Orthogonal Periodic Sequence

We introduce a periodic sequence $\{\hat{b}_{N,\lambda,i}\}$ of period N, with overlapping the first value and the last value in a derived finite-length sequence $\{b_{M,\lambda,i}\}$, as

$$\hat{b}_{N,\lambda,i}=\begin{cases}\hat{K}_N(b_{M',\lambda,0}+b_{M',\lambda,M'-1}) & ,i=0\\ \hat{K}_N b_{M',\lambda,i} & ,i=1,2,\cdots,N-1\end{cases} \quad (27)$$

where $N=M'-1=2(M-1)$ and

$$\hat{K}_N=\sqrt{\frac{N}{M'\{1+\delta_{2(M-1)}+\delta^*_{2(M-1)}\}}} \quad . \quad (28)$$

The autocorrelation function of the periodic sequence $\{\hat{b}_{N,\lambda,i}\}$ is given by

$$\hat{\rho}_{N,\lambda,\lambda,i'}=\frac{1}{N}\sum_{i=0}^{N-1}\hat{b}_{N,\lambda,i}\hat{b}^*_{N,\lambda,i-i'}$$
$$=\begin{cases}1 & ,\ i'=0 \bmod N\\ \hat{\delta}_{\frac{N}{2}} & ,\ i'=\dfrac{N}{2} \bmod N\\ 0 & ,\ elesewhere\end{cases} \quad (29)$$

where

$$\hat{\delta}_{\frac{N}{2}}=\frac{\delta_{M-1}+\delta^*_{M-1}}{1+\delta_{2(M-1)}+\delta^*_{2(M-1)}} \quad (30)$$

Fig. 5 shows the periodic sequence $\{\hat{b}_{N,\lambda,i}\}$ and the autocorrelation function $\{\hat{\rho}_{N,\lambda,\lambda,i'}\}$.

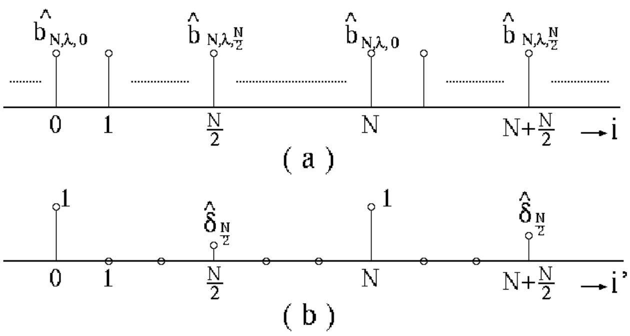

Fig. 5 (a) Periodic sequence $\{\hat{b}_{N,\lambda,i}\}$ and (b) autocorrelation function $\{\hat{\rho}_{N,\lambda,\lambda,i'}\}$.

This autocorrelation function $\{\hat{\rho}_{N,\lambda,\lambda,i'}\}$ is alike to the autocorrelation function $\{\overline{\rho}_{N,\ell,\ell,i'}\}$ of the periodic sequence $\{\overline{a}_{N,\ell,i}\}$. A sequence which is rejected a zero value from $\{\overline{a}_{N,\ell,i}\}$ has just the same type of the autocorrelation function as the type of $\{\hat{\rho}_{N,\lambda,\lambda,i'}\}$. This is a notice that the different-type sequences can have the same-type autocorrelation function.

The periodic sequence $\{\hat{b}_{N,\lambda,i}\}$ is represented by the DFT as

$$\hat{b}_{N,\lambda,i} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \hat{B}_{N,\lambda,k} e^{j\frac{2\pi}{N}ki} \tag{31}$$

where $\{\hat{B}_{N,\lambda,k}\}$ is the DFT spectrum of $\{\hat{b}_{N,\lambda,i}\}$, given by

$$\begin{aligned}\hat{B}_{N,\lambda,k} &= \frac{\hat{K}_N}{\sqrt{N}} B_{M',\lambda}(f)\Big|_{f=\frac{k}{T},\ \Delta t=\frac{T}{N}} \\ &= \frac{\sqrt{|\varepsilon_{M-1}||\varepsilon'_{M-1}|}}{\sqrt{1+4|\varepsilon_{M-1}||\varepsilon'_{M-1}|\cos\varphi_M \cos\varphi'_M}} \\ &\quad \times e^{j(\varphi_M+\varphi'_M)} K_{M,\ell} K'_{M,\ell'} \\ &\quad \times \prod_{m=0}^{M-2} \left\{ e^{-j\frac{2\pi}{N}k} - \gamma_{M,m} e^{-j\frac{\varphi_M}{M-1}} e^{j\frac{(2m+1)\pi}{M-1}} \right\} \\ &\quad \times \prod_{m=0}^{M-2} \left\{ e^{-j\frac{2\pi}{N}k} - \gamma'_{M,m} e^{-j\frac{\varphi'_M}{M-1}} e^{j\frac{(2m+1)\pi}{M-1}} \right\} \end{aligned} \tag{32}$$

From Eqs. (15),(30) and (32) the absolute value of this spectrum is given by

$$\left|\hat{B}_{N,\lambda,k}\right| = \sqrt{1+\hat{\delta}_{\frac{N}{2}}(-1)^k} \tag{33}$$

and from Eq. (32) the phase angle of this spectrum is obtained. If $|\varepsilon_{M-1}| = |\varepsilon'_{M-1}|$ and $\varphi_M + \varphi'_M = \pi$, then $\hat{\delta}_{\frac{N}{2}} = 0$: the periodic sequence $\{\hat{b}_{N,\lambda,i}\}$ becomes a self-orthogonal periodic sequence of length $N = 2(M-1)$.

The absolute values of even and odd autocorrelation functions from the sequence $\{\hat{b}_{N,\lambda,i}\}$ are estimated to decrease for the longer N.

5 Conclusion

Periodic sequences are derived from self-orthogonal finite-length sequences. These sequences have good properties of periodic even and odd autocorrelation functions, because the self-orthogonal finite-length sequence has an aperiodic autocorrelation function with no sidelobes except at both shift-ends. The derived self-orthogonal periodic sequence is effective to the application to radar as well as spread spectrum data transmission. All the derived sequences have the expressions of the DFT together with the convolution of element sequences, and are applicable to fast convolution or correlation processing. Sequence values and crosscorrelation values of these sequences are to be considered for the good properties hereafter.

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References:

[1]Yoshihiro Tanada and Takahiro Matsumoto, Real-Valued Convolutional PN Sequences and Their Application to Interference-Free Quasi- Synchronous CDMA, Proc. of the World Multiconference of Systemics, Cybernetics and Informatics, vol.7, pp.129-134, July 2000.

[2]Yoshihiro Tanada, Synthesis of a Set of Real-Valued Shift-Orthogonal Finite-Length PN Sequences, Proc. of the IEEE 4th International Symposium on Spread Spectrum Techniques and Applications, pp.58-62, Sept.1996.

[3]Yoshihiro Tanada, Self-Orthogonal Finite-Length Sequence and Its Derived Sequences, Proc. of the First International Workshop on Sequence Design and Application for CDMA Systems, Sept.2001.