New Method for Constructing Polyphase ZCZ Sequence sets

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Abstract: - In this paper, we propose a new method for constructing polyphase ZCZ sequences. The proposed sequences are composed of polyphase perfect sequences and polyphase unitary matrices. By this method, we can obtain polyphase ZCZ sequences which are not known yet. For example, as compared with the conventional method, quadriphase ZCZ sequences with wide zero correlation zones can be obtained by the proposed method.

Key-Words: - Zero correlation zone, Polyphase ZCZ sequences, Perfect sequences, Orthogonal sequences

1 Introduction

The set of sequences having low out-of-phase autocorrelation values and low cross-correlation values plays an important part in typical DS-CDMA systems. A periodic sequence with zero out-of-phase autocorrelation values is called a perfect sequence or an orthogonal sequence. That is, a perfect sequence is a periodic sequence with an ideal autocorrelation property. Similarly, a set of periodic sequences with zero cross-correlation values is called a set of uncorrelated sequences. That is, a set of uncorrelated sequences is a set of sequences with an ideal cross-correlation property. However, it is impossible to find a set of sequences with both the ideal autocorrelation and cross-correlation properties.

If periodic sequences have zero out-of-phase autocorrelation and cross-correlation values within the limits of $|\tau| \le T$, the sequences are called zero correlation zone (ZCZ) sequences, where τ is a time shift variable and T is an integer. ZCZ sequences were first studied by Suehiro. In [1], he proposed a signal design method for approximately synchronized CDMA (AS-CDMA) systems by using a kind of polyphase ZCZ sequence set. Later, the concept of zero correlation zone was definitely proposed by Fan et al., and then several classes of binary and nonbinary ZCZ sequences derived from complementary pairs or sets were proposed [2]-[3]. Moreover, Matsufuji et al. [5] proposed several classes of polyphase ZCZ sequences of period N_1N_2, where N_1 and N_2 are relatively prime.

In this paper, we propose a new method for constructing polyphase ZCZ sequences. By this method, we can obtain polyphase ZCZ sequences which are not known yet. For example, in comparison with the conventional method, the quadriphase ZCZ sequences obtained from the proposed method have wide zero correlation zones. To give an example, a quadriphase ZCZ sequence set with $(L,U,T)=(64,4,8)$ can be generated by the method based on quadriphase complementary pairs, where L is a period of sequences, U is the number of sequences, and T is a width of zero correlation zone. On the other hand, a quadriphase ZCZ sequence set with $(L,U,T)=(64,4,14)$ can be generated by the proposed method.

In section 2, the method for constructing polyphase ZCZ sequences is described in detail. In section 3, we give a proof that the proposed sequences are ZCZ sequences.

2 Polyphase ZCZ Sequences

In this section, we propose a new method for constructing polyphase ZCZ sequences. The polyphase ZCZ sequences are derived from polyphase perfect sequences and polyphase unitary matrices.

Let $A_0 = \left(a_0^0, a_1^0, \cdots, a_{l-1}^0\right)$ be a polyphase perfect sequence of period l, that is, A_0 is a perfect sequence whose elements are complex numbers of absolute value 1. Two integers l_0 and l_1 are defined as:

$$\begin{aligned} &l = l_0 \cdot l_1, \\ &1 \le l_0 < l, \\ &1 < l_1 \le l. \end{aligned} \tag{1}$$

A perfect sequence A_i is defined as:

$$\begin{aligned} A_i &= \left(a_0^i, a_1^i, \cdots, a_{l-1}^i\right) \\ &= \left(a_{i \cdot l_0}^0, a_{i \cdot l_0 + 1}^0, \cdots, a_{l-1}^0, a_0^0, \cdots, a_{i \cdot l_0 - 1}^0\right), \end{aligned} \tag{2}$$

that is, A_i is a perfect sequence obtained from shifting A_0 cyclically to the left by $i \cdot l_0$ places.

Let B_n be an $l_1 \times l_1$ polyphase unitary matrix, that is,

$$B_n = \frac{1}{\sqrt{l_1}} \begin{bmatrix} b_{0,0}^n & b_{0,1}^n & \cdots & b_{0,l_1-1}^n \\ b_{1,0}^n & b_{1,1}^n & \cdots & b_{1,l_1-1}^n \\ \vdots & \vdots & \ddots & \vdots \\ b_{l_1-1,0}^n & b_{l_1-1,1}^n & \cdots & b_{l_1-1,l_1-1}^n \end{bmatrix}, \tag{3}$$

$$\sum_{k=0}^{l_1-1} b_{k_0,k}^n \cdot b_{k_1,k}^{n*} = \sum_{k=0}^{l_1-1} b_{k,k_0}^{n*} \cdot b_{k,k_1}^n = l_1 \cdot \delta_{k_0 k_1},$$

where * denotes a complex conjugate. Note that the elements in B_n are complex numbers of absolute value 1.

A sequence set C_0 is defined as:

$$\begin{aligned} C_0 &= \left\{C_0^0, C_1^0, \cdots, C_i^0, \cdots, C_{l_1-1}^0\right\} \\ &= \left\{A_0, A_1, \cdots, A_i, \cdots, A_{l_1-1}\right\}, \\ C_i^0 &= \left(c_0^{0,i}, c_1^{0,i}, \cdots, c_j^{0,i}, \cdots, c_{l-1}^{0,i}\right) \\ &= \left(a_0^i, a_1^i, \cdots, a_j^i, \cdots, a_{l-1}^i\right), \\ &0 \le i \le l_1 - 1, \\ &0 \le j \le l - 1. \end{aligned} \tag{4}$$

A sequence set C_n is also defined as:

$$\begin{aligned} C_n &= \left\{C_0^n, C_1^n, \cdots, C_i^n, \cdots, C_{l_1-1}^n\right\}, \\ C_i^n &= \left(c_0^{n,i}, c_1^{n,i}, \cdots, c_j^{n,i}, \cdots, c_{l \cdot l_1^n - 1}^{n,i}\right), \\ &0 \le i \le l_1 - 1, \\ &0 \le j \le l \cdot l_1^n - 1, \end{aligned} \tag{5}$$

where $c_j^{n,i}$ is defined by the following recursive procedure.

$$c_j^{n,i} = b_{i,j(\mathrm{mod}\ l_1)}^n \cdot c_{[j/l_1]}^{n-1,j(\mathrm{mod}\ l_1)}, \tag{6}$$

where $[j/l_1]$ denotes a maximum integer which is not larger than j/l_1.

Then, we can obtain the following theorem.

Theorem 1: The set C_n derived from the above formulas (1)-(6) is a polyphase ZCZ sequence set with $(L, U, T) = \left(l \cdot l_1^n, l_1, (l-2) \cdot l_1^{n-1}\right)$.

For example, suppose that $l = 16$, $l_0 = 4$, $l_1 = 4$, and $A_0 = (0000012302020321)$, then four quadriphase perfect sequences are derived from the formula (2). Those perfect sequences are represented as:

$$\begin{aligned} A_0 &= (0000012302020321), \\ A_1 &= (0123020203210000), \\ A_2 &= (0202032100000123), \\ A_3 &= (0321000001230202), \end{aligned}$$

where 0, 1, 2, 3 represents 1, j, -1, $-j$ respectively, and $j = \sqrt{-1}$. Moreover, let B_1 be

$$B_1 = \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix},$$

then we can generate the following quadriphase ZCZ sequence set with $(L, U, T) = (64, 4, 14)$ by using the above-mentioned method.

$$\begin{aligned} C_0^1 = (&00000123020203210000012302020321 0 \\ &00002301020221030000301220201032), \\ C_1^1 = (&01230202032100000123131321033333 \\ &01232020032122220123313121031111), \\ C_2^1 = (&02020321000001230202103222223012 \\ &02022103000023010202321022221230), \\ C_3^1 = (&03210000012302020321111123013131 \\ &03212222012320200321333323011313). \end{aligned}$$

The autocorrelation function of C_0^1 is given by

$$\begin{aligned} \left|R_0(\tau)\right| = (&64,0,0,0,0,0,0,0,0,0,0,0,0,0,48,0,0,0,16,0,0, \\ &0,0,0,0,0,0,0,0,32,0,0,0,32,0,0,0,0,0,0,0,0, \\ &0,0,16,0,0,0,48,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0). \end{aligned}$$

The cross-correlation function between C_0^1 and C_2^1 is also given by

$$\begin{aligned} \left|R_{0,2}(\tau)\right| = (&0,0,0,0,0,0,0,0,0,0,0,0,0,0,16,0,0,0,16,0, \\ &0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0, \\ &0,16,0,0,0,16,0,0,0,0,0,0,0,0,0,0,0,0,0,0). \end{aligned}$$

Note that quadriphase perfect sequences of period 2, 4, 8, and 16 have been found. If you want to generate a quadriphase ZCZ sequence set with zero correlation zones wider than the conventional method, you have to select a quadriphase perfect sequence of period 16 or 8 as A_0.

We now give another example. Suppose that $l=9$, $l_0=3$, $l_1=3$, and $A_0=(000012021)$, then three perfect sequences are derived from the formula (2). Those perfect sequences are represented as:

$$A_0=(000012021),\quad A_1=(012021000),\quad A_2=(021000012),$$

where 0, 1, 2 represents 1, $\exp(2\pi j/3)$, $\exp(4\pi j/3)$ respectively. Furthermore, let B_1 and B_2 be

$$B_1=B_2=\frac{1}{\sqrt{3}}\begin{bmatrix}1 & 1 & 1\\ 1 & \exp(2\pi j/3) & \exp(4\pi j/3)\\ 1 & \exp(4\pi j/3) & \exp(2\pi j/3)\end{bmatrix},$$

then we can generate the following ZCZ sequence set with $(L,U,T)=(81,3,21)$ by using the above-mentioned method.

$$\begin{aligned}C_0^2&=(000012021000120210000201102\\&\quad 000012021111201021222120021\\&\quad 000012021222012102111012210),\\C_1^2&=(012021000012102222012210111\\&\quad 012021000120210000201102000\\&\quad 012021000201021111120021222),\\C_2^2&=(021000012021111201021222120\\&\quad 021000012102222012210111012\\&\quad 021000012210000120102000201).\end{aligned}$$

The autocorrelation function of C_0^2 is given by

$$\begin{aligned}|R_0(\tau)|=(&81,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\\&9,9,0,9,9,0,0,0,0,0,9,9,0,9,9,0,0,0,0,0,0,\\&0,0,0,0,0,9,9,0,9,9,0,0,0,0,0,9,9,0,9,9,\\&0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0)\end{aligned}$$

The cross-correlation function between C_0^2 and C_1^2 is also given by

$$\begin{aligned}|R_{0,1}(\tau)|=(&0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\\&9,18,54,18,9,0,0,0,0,9,18,27,18,9,0,0,0,\\&0,0,0,0,0,0,0,9,9,0,9,9,0,0,0,0,9,9,0,9,9,\\&0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0).\end{aligned}$$

3 Proof of Theorem 1

In this section, we give a proof of **Theorem 1**, that is, we prove that the proposed sequences are ZCZ sequences with $(L,U,T)=\left(l\cdot l_1^n, l_1, (l-2)\cdot l_1^{n-1}\right)$.

Let $R_{i_0,i_1}^A(\tau),(0\le\tau\le l-1)$ be the correlation function between A_{i_0} and A_{i_1}. If $i_0=i_1$, $R_{i_0,i_1}^A(\tau)$ denotes the autocorrelation function, and if $i_0\ne i_1$, $R_{i_0,i_1}^A(\tau)$ denotes the cross-correlation function. Since A_i is derived from shifting the perfect sequence A_0 cyclically, the correlation function $R_{i_0,i_1}^A(\tau)$ is represented as:

$$\begin{aligned}R_{i_0,i_1}^A(\tau)&=\sum_{k=0}^{l-1}a_k^{i_0}\cdot a_{k+\tau\ (\mathrm{mod}\ l)}^{i_1*}\\&=\sum_{k=0}^{l-1}a_{i_0\cdot l_0+k\ (\mathrm{mod}\ l)}^{0}\cdot a_{i_1\cdot l_0+k+\tau\ (\mathrm{mod}\ l)}^{0}\\&=\begin{cases}l & (\tau=(i_0-i_1)\cdot l_0\ (\mathrm{mod}\ l))\\0 & \text{otherwise}\end{cases}.\end{aligned}\tag{7}$$

Similarly, let $R_{i_0,i_1}^n(\tau),(0\le\tau\le l\cdot l_1^n-1)$ be the correlation function between $C_{i_0}^n$ and $C_{i_1}^n$. The correlation function $R_{i_0,i_1}^n(\tau)$ is also represented as:

$$R_{i_0,i_1}^n(\tau)=\sum_{k=0}^{l\cdot l_1^n-1}c_k^{n,i_0}\cdot c_{k+\tau\ (\mathrm{mod}\ l\cdot l_1^n)}^{n,i_1*}.\tag{8}$$

Now we prove **Theorem 1** by means of mathematical induction.

When $n=1$, the correlation function $R_{i_0,i_1}^1(\tau)$ is calculated as:

$$\begin{aligned}R_{i_0,i_1}^1(\tau)&=\sum_{k=0}^{l\cdot l_1-1}c_k^{1,i_0}\cdot c_{k+\tau\ (\mathrm{mod}\ l\cdot l_1)}^{1,i_1*}\\&=\sum_{k=0}^{l\cdot l_1-1}b_{i_0,k\ (\mathrm{mod}\ l_1)}^{1}\cdot c_{[k/l_1]}^{0,k\ (\mathrm{mod}\ l_1)}\\&\quad\cdot b_{i_1,k+\tau\ (\mathrm{mod}\ l_1)}^{1*}\cdot c_{[(k+\tau)/l_1]}^{0,k+\tau\ (\mathrm{mod}\ l_1)*}\\&=\sum_{k=0}^{l\cdot l_1-1}b_{i_0,k\ (\mathrm{mod}\ l_1)}^{1}\cdot a_{[k/l_1]}^{k\ (\mathrm{mod}\ l_1)}\\&\quad\cdot b_{i_1,k+\tau\ (\mathrm{mod}\ l_1)}^{1*}\cdot a_{[(k+\tau)/l_1]}^{k+\tau\ (\mathrm{mod}\ l_1)*}.\end{aligned}\tag{9}$$

The variables k and τ can be described as:

$$\begin{aligned}k&=k_0\cdot l_1+k_1,\ 0\le k_0\le l-1,\ 0\le k_1\le l_1-1,\\\tau&=\tau_0\cdot l_1+\tau_1,\ 0\le\tau_0\le l-1,\ 0\le\tau_1\le l_1-1.\end{aligned}\tag{10}$$

From the formulas (7), (9), and (10),

$$R^1_{i_0,i_1}(\tau)=\sum_{k_1=0}^{l_1-1} b^1_{i_0,k_1}\cdot b^{1*}_{i_1,k_1+\tau_1 \pmod{l_1}} \sum_{k_0=0}^{l-1} a^{k_1}_{k_0}\cdot a^{k_1+\tau_1 \pmod{l_1} *}_{k_0+\tau_0+[(k_1+\tau_1)/l_1]},$$
$$=\sum_{k_1=0}^{l_1-1} b^1_{i_0,k_1}\cdot b^{1*}_{i_1,k_1+\tau_1 \pmod{l_1}} \cdot R^A_{k_1,k_1+\tau_1 \pmod{l_1}}\left(\tau_0+[(k_1+\tau_1)/l_1]\right). \quad (11)$$

From the formulas (7) and (11), if $\tau_1=\tau_0=0$,

$$R^1_{i_0,i_1}(\tau)=\sum_{k_1=0}^{l_1-1} b^1_{i_0,k_1}\cdot b^{1*}_{i_1,k_1}\cdot R^A_{k_1,k_1}(0) = l\sum_{k_1=0}^{l_1-1} b^1_{i_0,k_1}\cdot b^{1*}_{i_1,k_1} = \begin{cases} l\cdot l_1 & (i_0=i_1) \\ 0 & \text{otherwise} \end{cases}. \quad (12)$$

Similarly, if $\tau_1=0$ and $\tau_0\neq 0$,

$$R^1_{i_0,i_1}(\tau)=\sum_{k_1=0}^{l_1-1} b^1_{i_0,k_1}\cdot b^{1*}_{i_1,k_1}\cdot R^A_{k_1,k_1}(\tau_0)=0. \quad (13)$$

From the formulas (12) and (13), if $\tau_1=0$,

$$R^1_{i_0,i_1}(\tau)=\begin{cases} l\cdot l_1 & (\tau_0=0 \text{ and } i_0=i_1) \\ 0 & \text{otherwise} \end{cases}. \quad (14)$$

On the other hand, suppose that $\tau_1\neq 0$, we can obtain the following formula.

$$l_0\le (l_1-\tau_1)\cdot l_0 \pmod{l} \le l-l_0. \quad (15)$$

If $0\le\tau_0\le l_0-2$,

$$0\le \tau_0+[(k_1+\tau_1)/l_1]\le l_0-1, \quad (16)$$

since $0\le[(k_1+\tau_1)/l_1]\le 1$. From the formulas (7), (11), (15), and (16), if $\tau_1\neq 0$ and $0\le\tau_0\le l_0-2$,

$$R^1_{i_0,i_1}(\tau)=0. \quad (17)$$

Similarly, if $l-l_0+1\le\tau_0\le l-1$,

$$l-l_0+1\le \tau_0+[(k_1+\tau_1)/l_1]\le l. \quad (18)$$

From the formulas (7), (11), (15), and (18), if $\tau_1\neq 0$ and $l-l_0+1\le\tau_0\le l-1$,

$$R^1_{i_0,i_1}(\tau)=0. \quad (19)$$

Furthermore, suppose that $\tau_0=l_0-1$, then

$$l_0-1\le \tau_0+[(k_1+\tau_1)/l_1]\le l_0. \quad (20)$$

Therefore, if and only if $\tau_1=l_1-1$ and $k_1\ge 1$, the following equation is satisfied.

$$\tau_0+[(k_1+\tau_1)/l_1]=(l_1-\tau_1)\cdot l_0 \pmod{l}=l_0. \quad (21)$$

From the formulas (7) (11), and (21), if $\tau_1\neq 0$ and $\tau_0=l_0-1$,

$$R^1_{i_0,i_1}(\tau)=\begin{cases} l\sum_{k_1=1}^{l_1-1} b^1_{i_0,k_1}\cdot b^{1*}_{i_1,k_1-1} & (\tau_1=l_1-1) \\ 0 & \text{otherwise} \end{cases}. \quad (22)$$

Similarly, suppose that $\tau_0=l-l_0$, then

$$l-l_0\le \tau_0+[(k_1+\tau_1)/l_1]\le l-l_0+1. \quad (23)$$

Therefore, if and only if $\tau_1=1$ and $k_1\le l_1-2$, the following equation is satisfied.

$$\tau_0+[(k_1+\tau_1)/l_1]=(l_1-\tau_1)\cdot l_0 \pmod{l}=l-l_0. \quad (24)$$

From the formulas (7), (11), and (24), if $\tau_1\neq 0$ and $\tau_0=l-l_0$,

$$R^1_{i_0,i_1}(\tau)=\begin{cases} l\sum_{k_1=0}^{l_1-2} b^1_{i_0,k_1}\cdot b^{1*}_{i_1,k_1+1} & (\tau_1=1) \\ 0 & \text{otherwise} \end{cases}. \quad (25)$$

From the formulas (14), (17), (19), (22), and (25), if $0\le\tau\le l-2$ or $l\cdot l_1-l+2\le\tau\le l\cdot l_1-1$,

$$R^1_{i_0,i_1}(\tau)=\begin{cases} l\cdot l_1 & (\tau=0 \text{ and } i_0=i_1) \\ 0 & \text{otherwise} \end{cases}. \quad (26)$$

Note that $\tau=\tau_0\cdot l_1+\tau_1$. The formula (26) shows that the set C_1 is a ZCZ sequence set with $(L,U,T)=(l\cdot l_1, l_1, l-2)$.

Now, suppose that the set $C_{n-1}, (n\ge 2)$ is a ZCZ sequence set with $(L,U,T)=\left(l\cdot l_1^{n-1}, l_1, (l-2)\cdot l_1^{n-2}\right)$, i.e., if $0\le\tau\le(l-2)\cdot l_1^{n-2}$ or $l\cdot l_1^{n-1}-(l-2)\cdot l_1^{n-2}\le\tau\le l\cdot l_1^{n-1}-1$,

$$R^{n-1}_{i_0,i_1}(\tau)=\begin{cases} l\cdot l_1^{n-1} & (\tau=0 \text{ and } i_0=i_1) \\ 0 & \text{otherwise} \end{cases}. \quad (27)$$

Then, the correlation function $R^n_{i_0,i_1}(\tau)$ is calculated as:

$$R^n_{i_0,i_1}(\tau)=\sum_{k=0}^{l\cdot l_1^n-1} c^{n,i_0}_k\cdot c^{n,i_1 *}_{k+\tau \pmod{l\cdot l_1}} = \sum_{k=0}^{l\cdot l_1^n-1} b^n_{i_0,k \pmod{l_1}}\cdot c^{n-1,k \pmod{l_1}}_{[k/l_1]} \cdot b^{n*}_{i_1,k+\tau \pmod{l_1}}\cdot c^{n-1,k+\tau \pmod{l_1} *}_{[(k+\tau)/l_1]}. \quad (28)$$

The variables k and τ can be described as:

$$k=k_0\cdot l_1+k_1,\ 0\le k_0\le l\cdot l_1^{n-1}-1,\ 0\le k_1\le l_1-1,$$
$$\tau=\tau_0\cdot l_1+\tau_1,\ 0\le\tau_0\le l\cdot l_1^{n-1}-1,\ 0\le\tau_1\le l_1-1. \quad (29)$$

From the formulas (28) and (29),

$$R^n_{i_0,i_1}(\tau)=\sum_{k_1=0}^{l_1-1} b^n_{i_0,k_1}\cdot b^{n*}_{i_1,k_1+\tau_1\ (\mathrm{mod}\ l_1)} \sum_{k_0=0}^{l\cdot l_1^{n-1}-1} c^{n-1,k_1}_{k_0}\cdot c^{n-1,k_1+\tau_1\ (\mathrm{mod}\ l_1)*}_{k_0+\tau_0+[(k_1+\tau_1)/l_1]},$$

$$=\sum_{k_1=0}^{l_1-1} b^n_{i_0,k_1}\cdot b^{n*}_{i_1,k_1+\tau_1\ (\mathrm{mod}\ l_1)} \cdot R^{n-1}_{k_1,k_1+\tau_1\ (\mathrm{mod}\ l_1)}(\tau_0+[(k_1+\tau_1)/l_1]). \quad (30)$$

From the formulas (27) and (30), if $\tau_1=\tau_0=0$,

$$R^n_{i_0,i_1}(\tau)=\sum_{k_1=0}^{l_1-1} b^n_{i_0,k_1}\cdot b^{n*}_{i_1,k_1}\cdot R^{n-1}_{k_1,k_1}(0)$$

$$=l\cdot l_1^{n-1}\sum_{k_1=0}^{l_1-1} b^n_{i_0,k_1}\cdot b^{n*}_{i_1,k_1}=\begin{cases} l\cdot l_1^n & (i_0=i_1)\\ 0 & \text{otherwise}\end{cases}. \quad (31)$$

If $\tau_1=0$ and $0<\tau_0\le(l-2)\cdot l_1^{n-2}$,

$$R^n_{i_0,i_1}(\tau)=\sum_{k_1=0}^{l_1-1} b^n_{i_0,k_1}\cdot b^{n*}_{i_1,k_1}\cdot R^{n-1}_{k_1,k_1}(\tau_0)=0. \quad (32)$$

Similarly, if $\tau_1=0$ and $l\cdot l_1^{n-1}-(l-2)\cdot l_1^{n-2}\le\tau_0\le l\cdot l_1^{n-1}-1$, $R^n_{i_0,i_1}(\tau)=0$. On the other hand, if $\tau_1\ne0$ and $0\le\tau_0\le(l-2)\cdot l_1^{n-2}-1$,

$$R^n_{i_0,i_1}(\tau)=\sum_{k_1=0}^{l_1-1} b^n_{i_0,k_1}\cdot b^{n*}_{i_1,k_1+\tau_1\ (\mathrm{mod}\ l_1)} \cdot R^{n-1}_{k_1,k_1+\tau_1\ (\mathrm{mod}\ l_1)}(\tau_0+[(k_1+\tau_1)/l_1]) \quad (33)$$
$$=0,$$

since $0\le\tau_0+[(k_1+\tau_1)/l_1]\le(l-2)\cdot l_1^{n-2}$. Note that $0\le[(k_1+\tau_1)/l_1]\le1$. Similarly, if $\tau_1\ne0$ and $l\cdot l_1^{n-1}-(l-2)\cdot l_1^{n-2}\le\tau_0\le l\cdot l_1^{n-1}-1$, $R^n_{i_0,i_1}(\tau)=0$, since $l\cdot l_1^{n-1}-(l-2)\cdot l_1^{n-2}\le\tau_0+[(k_1+\tau_1)/l_1]\le l\cdot l_1^{n-1}$ and $k_1\ne k_1+\tau_1\ (\mathrm{mod}\ l_1)$. Therefore, if $0\le\tau\le(l-2)\cdot l_1^{n-1}$ or $l\cdot l_1^n-(l-2)\cdot l_1^{n-1}\le\tau\le l\cdot l_1^n-1$,

$$R^n_{i_0,i_1}(\tau)=\begin{cases} l\cdot l_1^n & (\tau=0 \text{ and } i_0=i_1)\\ 0 & \text{otherwise}\end{cases}. \quad (34)$$

Note that $\tau=\tau_0\cdot l_1+\tau_1$. The formula (34) shows that the set C_n is a ZCZ sequence set with $(L,U,T)=(l\cdot l_1^n, l_1, (l-2)\cdot l_1^{n-1})$, therefore **Theorem 1** has been proved by mathematical induction.

Note that this proof can be applied even when A_0 is any perfect sequence and B_n is any unitary matrix.

4 Conclusion

In this paper, we have proposed a new method for constructing polyphase ZCZ sequences. Also we have given a proof that the proposed sequences are ZCZ sequences. By the proposed method, we can obtain polyphase ZCZ sequences which are not known yet.

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