

# Binary Distillation Column Control by Decoupling Controller

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*Abstract:* Distillation column is one of the most underestimated fields of chemical engineering and has been around for well over a hundred years. For high purity products, the dynamics of the column become highly nonlinear and coupled and the response are sensitive to external disturbances. Distillation column control systems are use to purify the mixture of methanol and water which shows strong interaction, nonlinear dynamics and a large number of possible control structures. In this paper, we consider a decoupling controller to eliminate the strong interactions. The decoupler cancels the effect of the distillate composition to the change of the bottom composition and the effect of the bottom composition by the distillate composition. Simulation result show good response for feed composition change, distillate composition change, bottom composition change, condenser holdup and reboiler holdup.

Keywords: distillation control, computer simulation, multivariable system, interactions, decoupling controller.

## 1. Introduction

Decoupling of input and output variables is one of the central control design problems that has attracted considerable attention since the early 70s, in which , the decoupling problems has solved for the case of non uncertain system by means of static measurement matrix is usually uncertain due to the measurement device error[1]. This is difficult problem and has not yet been solved. The objective of this paper is to design and to implement a decoupling controller that minimize the control loop interaction between input-output variables of column distillation process.

## 2. Decoupling Controller

Generally, the issue of selecting controlled variables is the first subtask in the control design problem (Foss, 1973); (Morari, 1982); (Skogestad and Postlethwaite, 1996); This consists of:

- a. Selection of controlled variables  $c$
- b. Selection of manipulated variables
- c. Selection of measurements
- d. Selection of control configuration

- e. Selection of controller type (control law specification, decoupler)

The decoupling structure control system developed by Boksenborm and Hood (1949) is shown in Figure 1. The decoupling matrix  $\mathbf{G}_c$  is of the [2]:

$$\mathbf{G}_c^* = \begin{bmatrix} G_{c,11}^* & G_{c,12}^* \\ G_{c,21}^* & G_{c,22}^* \end{bmatrix} \quad (1)$$

For the 2x2 MIMO process control can be written as:

$$\mathbf{G}\mathbf{u} = \mathbf{y} \quad (2)$$

$$\mathbf{u} = \mathbf{G}_c^* [\mathbf{w}-\mathbf{y}] \quad (3)$$

Where  $G_{ij}$  the transfer function,  $\mathbf{u}$  and  $\mathbf{y}$  denote the input and the output and  $\mathbf{w}=[w_1, w_2]^T$  is the setpoint. Substituting (3) into (2) yields

$$\mathbf{G} \mathbf{G}_c^* [\mathbf{w}-\mathbf{y}] = \mathbf{y} \quad (4)$$

Rearrange this equation leads to the closed loop expression:

$$\mathbf{y} = [\mathbf{I} + \mathbf{G} \mathbf{G}_c^*]^{-1} \mathbf{G} \mathbf{G}_c^* \mathbf{w} \quad (5)$$

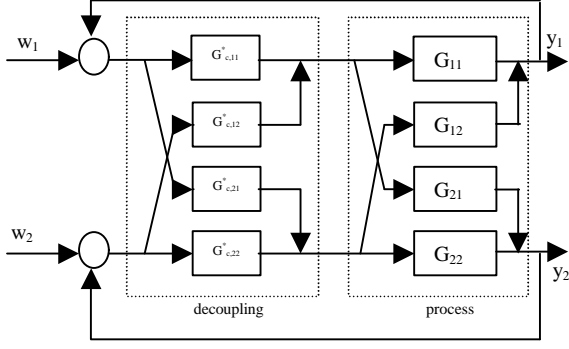


Figure 1. Decoupling control system (Boksenbom and Hood, 1949)

In order to make individual loops of the closed loop system (4) are independent each other, it is required that:

$$\mathbf{X} = [\mathbf{I} + \mathbf{G} \mathbf{G}_c^*]^{-1} \mathbf{G} \mathbf{G}_c^* = \text{diag}[x_1, x_2] \quad (5)$$

Where  $\mathbf{X}$  must be a diagonal matrix. Since the sum and product of two diagonal matrices are diagonal matrices, and the inverse of diagonal matrix is also diagonal matrix.

The requirement can be ensured if  $\mathbf{G} \mathbf{G}_c^*$  is a diagonal matrix. From (1) and (2), we have:

$$\mathbf{G} \mathbf{G}_c^* = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} G_{c,11}^* & G_{c,12}^* \\ G_{c,21}^* & G_{c,22}^* \end{bmatrix} \quad (6)$$

$$= \begin{bmatrix} q_1 & 0 \\ 0 & q_2 \end{bmatrix}$$

so

$$\begin{bmatrix} G_{11}G_{c,11}^* + G_{12}G_{c,21}^* & G_{11}G_{c,12}^* + G_{12}G_{c,22}^* \\ G_{21}G_{c,21}^* + G_{22}G_{c,21}^* & G_{21}G_{c,22}^* + G_{22}G_{c,22}^* \end{bmatrix} = \begin{bmatrix} q_1 & 0 \\ 0 & q_2 \end{bmatrix} \quad (7)$$

Comparing each element of the matrices (7) results in a set of four equations:

$$\begin{aligned} q_1 &= G_{11}G_{c,11}^* + G_{12}G_{c,21}^* \\ 0 &= G_{11}G_{c,12}^* + G_{12}G_{c,22}^* \\ 0 &= G_{21}G_{c,11}^* + G_{22}G_{c,21}^* \\ q_2 &= G_{21}G_{c,12}^* + G_{22}G_{c,22}^* \end{aligned} \quad (8)$$

If the transfer function elements of  $\mathbf{G}$  are known, and having specified the diagonal element of  $\mathbf{G}_c^*$ , then the appropriate off-diagonal elements of  $\mathbf{G}_c^*$  to achieve decoupling control are calculated by solving the set of equations in (7). Simplest way is to set  $\mathbf{G} \mathbf{G}_c^*$  to a diagonal

matrix, and this gives the following relationships:

$$G_{c,12}^* = \frac{-G_{12}G_{c,22}^*}{G_{11}} \quad (9)$$

and

$$G_{c,21}^* = \frac{-G_{21}G_{c,11}^*}{G_{22}} \quad (10)$$

The decoupling elements  $G_{c,11}^*$  and  $G_{c,22}^*$  of the decoupling matrix  $\mathbf{G}_c$  can then be Proportional-Integral (PI) controller.

## 2.1 Decoupling Control non-interacting

The diagram block of the decoupling control non interacting is shown in Fig.2, where  $\mathbf{G}_c$  is control matrix,  $\mathbf{u}$  is output, and the output of the decoupling is denoted  $\mathbf{u}^*$ . The system is described by the following relationship [2]:

$$\mathbf{y} = \mathbf{G} \mathbf{u}^* \quad (11)$$

$$\mathbf{u}^* = \mathbf{G}_c^* \mathbf{u} \quad (12)$$

$$\mathbf{u} = \mathbf{G}_c[\mathbf{w} - \mathbf{y}] \quad (13)$$

Substituting (12) and (13) into (11) yields:

$$\mathbf{y} = \mathbf{G} \mathbf{G}_c^* \mathbf{G}_c[\mathbf{w} - \mathbf{y}] \quad (14)$$

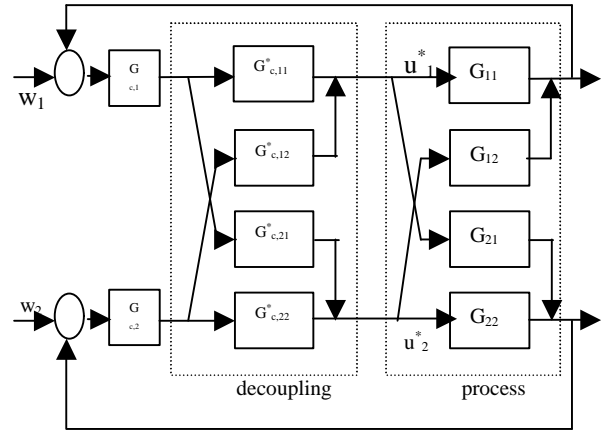


Figure 2. Decoupling Control non-interacting structure (Zalkind, 1967; Luyben, 1970)

Since  $\mathbf{G}_c$  is a diagonal matrix, the objective will be achieved if:

$$\mathbf{X} = \mathbf{G} \mathbf{G}_c^* = \text{diag}[x_1, x_2] \quad (15)$$

To determine  $\mathbf{G}_c^*$ , we need to calculate the inverse of  $\mathbf{G}$  from (15):

$$\mathbf{G}_c^* = \mathbf{G}^{-1} \mathbf{X} \quad \mathbf{G}^{-1} = \frac{\text{adj}(\mathbf{G})}{\det(\mathbf{G})} \quad (16)$$

where  $\text{adj}(\mathbf{G})$  and  $\text{det}(\mathbf{G})$  denote the adjoint and the determinant of the matrix  $\mathbf{G}$  respectively, which are given by:

$$\text{det}(\mathbf{G}) = G_{11} G_{22} - G_{12} G_{21} \quad (17)$$

$$\text{Adj}(\mathbf{G}) = \begin{bmatrix} G_{22} & -G_{12} \\ -G_{21} & G_{11} \end{bmatrix} \quad (18)$$

Since  $\mathbf{X} = \text{diag}[x_1, x_2]$ , if than following

$$\mathbf{G}^*_{\mathbf{c}} = \mathbf{G}^{-1} \mathbf{X} = \begin{bmatrix} G_{22} x_1 & -G_{12} x_2 \\ -G_{21} x_1 & G_{11} x_2 \end{bmatrix} \frac{1}{\text{det}(\mathbf{G})} \quad (19)$$

The simplest form of this decoupling matrix has unity diagonal elements, so that:

$$\mathbf{G}^*_{\mathbf{c},12} = \mathbf{G}^*_{\mathbf{c},22} = 1 \quad (20)$$

This leads to the following off-diagonal elements:

$$\mathbf{G}^*_{\mathbf{c},12} = G_{12}/G_{11} \quad (21)$$

and

$$\mathbf{G}^*_{\mathbf{c},21} = -G_{21}/G_{22} \quad (22)$$

## 2.2 Dual Composition Control $x_B$ and $y_D$

The choice of the proper configuration for dual composition control is more challenging than a single composition control, because there are more viable approaches and the analysis of performance is more complex. In this case there are a many of choices for manipulated variables (L, D, L/D, V, B, V/B, B/L, D/V) that can be paired to the four control objectives (i.e.  $x$ ,  $y$ , reboiler level, and accumulator level) indicating that there are a large number of possible configurations. In this paper it is assumed that the choice for control configuration is L and V. The setpoint for reflux flow controller is set by the overhead composition controller and the setpoint for the flow controller on the reboiler duty is set by the bottom composition controller.

The L and V configuration is used since it provides good dynamic response, and in general, the configuration is least sensitive to feed composition disturbances. Moreover it is easy to implement, but it is highly susceptible to coupling. For this reason we design decoupling controller to anticipate this coupling.

In many cases, the control of one of the two products is more important than

control of the other. For such cases, when the overhead products are the most important, L is usually used as a manipulated variable. When the bottoms products are most important, V is the proper choice as the manipulated variable. For a low reflux column for which the bottom product is more important, the L and V configuration is preferred.

## 3. Application Decoupling Controller for Distillation Column

The Decoupling controller is based on assumptions that the process is linear and there is an exact cancellation of numerator and denominator dynamics of interaction term.

A more specific problem of reliability relates to the time delay associated with the elements of the process transfer function matrix. Notice that the above decoupling techniques involve the ratios  $G_{12}/G_{11}$  and  $G_{21}/G_{22}$ .

Now suppose:

$$G_{12} = \frac{K_{12} \exp(-\mathbf{q}_{12})}{1 + \mathbf{t}_{12}s} \quad (23)$$

and

$$G_{11} = \frac{K_{11} \exp(-\mathbf{q}_{11})}{1 + \mathbf{t}_{11}s} \quad (24)$$

where  $\mathbf{q}_{ij}$  and  $\mathbf{t}_{ij}$  denote time-delay and time constant respectively. It then follows

$$\frac{G_{12}}{G_{11}} = \frac{K_{12} (1 + \mathbf{t}_{11}s)}{K_{11} (1 + \mathbf{t}_{12}s)} \exp(\mathbf{q}_{11} - \mathbf{q}_{12}) \quad (25)$$

If it holds  $\mathbf{q}_{11} > \mathbf{q}_{12}$ , the exponential term in (25) will be positive. This implies that the future values of process variables are needed for the implementation.

### 3.1 An Example

Consider process distillation column with two inputs and two output shown in Fig. 3. The manipulated variables are L reflux flow rate, V boil-up flow rate, and the controlled variables are  $y_D$  distillate purity, and  $x_B$  bottom purity. The manipulated variables are L reflux flow rate, V boil-up flow rate, and the

controlled variables are  $y_D$  distillate purity, and  $x_B$  bottom purity.

The mathematical model is derived from the fundamental principles as follow:

Overall material balance:

Tray feed,  $i = N_F$

$$\frac{dM_i}{dt} = L_{\{i+1\}} - L_i + V_{\{i-1\}} - V_i + F \quad (26)$$

$$\frac{d(M_i x_i)}{dt} = L_{\{i+1\}} x_{\{i+1\}} + V_{\{i-1\}} y_{\{i-1\}} - L_i x_i - V_i y_i + F z_F \quad (27)$$

Total condenser,  $i = NT (M_{NT} = M_D, L_{NT} = L_T)$

$$\frac{dM_i}{dt} = V_{\{i-1\}} - L_i - D \quad (28)$$

$$\frac{d(M_i x_i)}{dt} = V_{\{i-1\}} y_{\{i-1\}} - L_i x_i - V_i y_i - D x_i \quad (29)$$

Reboiler,  $i = 1 (M_i = M_B, V_i = V_B = V)$

$$\frac{dM_i}{dt} = L_{\{i+1\}} - V_i - B \quad (30)$$

$$\frac{d(M_i x_i)}{dt} = L_{\{i+1\}} x_{\{i+1\}} - V_i y_i - V_i y_i - B x_i$$

At steady state condition relation between control variable and manipulated variable are:

$$\begin{aligned} \Delta y_D &= \left. \frac{dy_D}{dL} \right|_V \Delta L + \left. \frac{dy_D}{dV} \right|_L \Delta V = K_{11} \Delta L + K_{12} \Delta V \\ \Delta x_B &= \left. \frac{dx_B}{dL} \right|_V \Delta L + \left. \frac{dx_B}{dV} \right|_L \Delta V = K_{21} \Delta L + K_{22} \Delta V \end{aligned} \quad (31)$$

The  $K$ 's are steady state gains that can be determined from mathematical models or from experimental test. They describe how, say,  $L$  affects  $y_D$  when  $x_B$  is not controlled. A second gain may be defined that gives a measure of how, say  $L$  would affect  $y_D$  if  $x_B$  were under closed loop control by the relationship:

$$K_{11} = \left. \frac{\Delta y_D}{\Delta L} \right|_{V-\text{const } t (\Delta V=0)} \quad (32)$$

and

$$K_{21} = \left. \frac{\Delta x_B}{\Delta L} \right|_{V-\text{const } t (\Delta V=0)} \quad (33)$$

There is another gain between  $y_D$  and  $L$ :

$$a_{11} = \left. \frac{\Delta y_D}{\Delta L} \right|_{x_B-\text{const } t} \quad (34)$$

$a_{11}$  is called the relative gain which indicate that how much  $L$  will be affect  $y_D$ , if all control variable in closed loop, or constant.

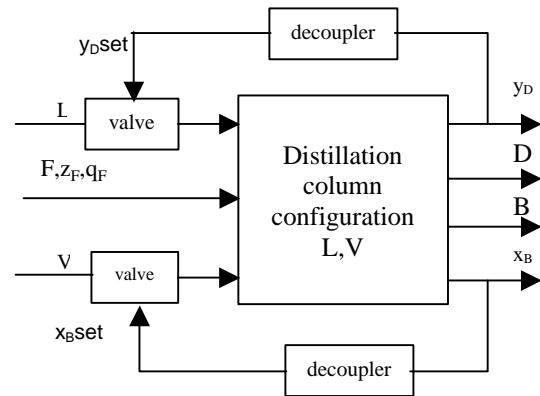


Figure 3. Block diagram distillation column configuration L,V with decoupler

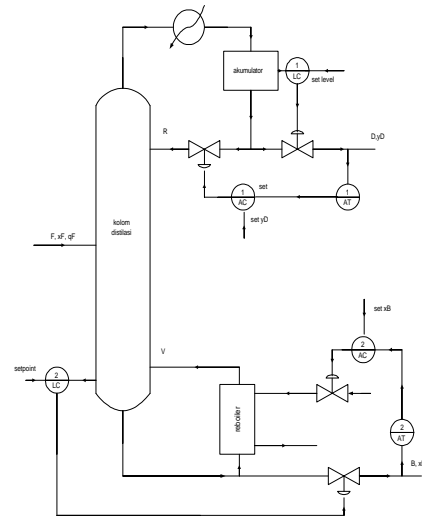


Figure 4. Distillation Column Configuration L,V

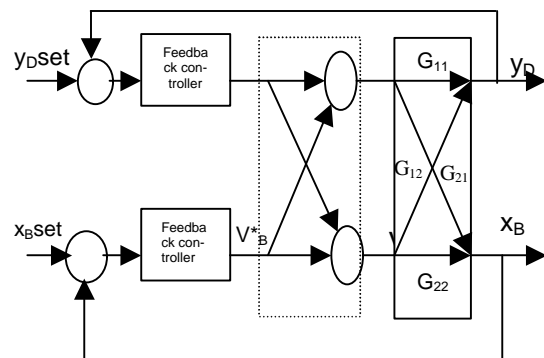


Figure 5. Block diagram control system with decoupler

From fig.5 we can write the decoupler expression:

$$\Delta L(s) = D_{11}(s)\Delta L^*(s) + D_{12}(s)V_B^*(s) \quad (35)$$

$$\Delta V_B(s) = D_{21}(s)\Delta L^*(s) + D_{22}(s)V_B^*(s) \quad (36)$$

For  $D_{11}(s) = D_{22}(s) = 1$ , therefore the decoupler expression became:

$$\Delta L(s) = \Delta L^*(s) + D_{12}(s)V_B^*(s) \quad (37)$$

$$\Delta V_B(s) = D_{21}(s)\Delta L^*(s) + V_B^*(s) \quad (38)$$

Finally we find combination equation decoupler-process:

$$\Delta y_D = G_{11}(\Delta L + D_{12}V_B^*) + G_{12}(D_{21}L^* + V_B^*) = (G_{11} + G_{12}D_{21})\Delta L^* + (G_{11}D_{12} + G_{12})\Delta V_B^* \quad (39)$$

$$\Delta x_B = G_{21}(\Delta L + D_{12}V_B^*) + G_{22}(D_{21}L^* + V_B^*) = (G_{21} + G_{22}D_{21})\Delta L^* + (G_{21}D_{12} + G_{22})\Delta V_B^* \quad (40)$$

Therefore:

$$D_{12}(s) = -\frac{G_{12}(s)}{G_{11}(s)} \quad (41)$$

$$D_{21}(s) = -\frac{G_{21}(s)}{G_{22}(s)} \quad (42)$$

$$H_1(s) = G_{11}(s) - \frac{G_{12}(s)G_{21}(s)}{G_{22}(s)} \quad (43)$$

$$H_2(s) = G_{22}(s) - \frac{G_{12}(s)G_{21}(s)}{G_{11}(s)} \quad (44)$$

To simulate the proposed decoupling controller for the distillation column, we use the distillation data [4]:

1. number of tray= 41 (include reboiler and condenser)
2. feed location  $i = 20$
3. relative volatility =  $\alpha = 1.5$
4. nominal reboiler holdup=  $M_O(1) = 0.5$  [kmol]
5. nominal holdup condenser =  $M_O(NT) = 0.5$  [kmol]
6. nominal holdup tray =  $M_O(i) = 0.5 * \text{ones}(1, NT-2)$ ;  $i=2:NT-1$
7. nominal feed rate =  $F_O = 1$  [kmol/min]
8. nominal fraction liquid in feed=  $qF_O = 1$ .
9. nominal reflux flow =  $L_O = 2.70629/0.5$
10. nominal liquid flow below fed=  $L_{Ob} = L_O + qF_O * F_O$
11. affect flow vapor in liquid flow =  $\lambda = 0$
12. nominal vapor flow =  $V_O = 3.20629/0.5$ ;  $V_{Ot} = V_O + (1-qF_O) * F_O$

Thermodynamic data:

- a. boiling point light component=272.65 °K
- b. Boiling point heavy component= 309.25 °K
- c. Heat capacity light component = 96 kJ/kmol°K
- d. Heat capacity heavy component= 121 kJ/mol°K
- e. Hvap for light component = 19575 kJ/kmol
- f. Hvap for heavy component = 28350 kJ/kmol
- g. Vapor pressure of pure liquid component =  $1.013e5$
- h. Vapor pressure of pure heavy liquid component =  $1.013e5$
- i. Universal gas constant = 8.314 kJ/kmol °K

## 4. Simulation Result and Conclusion

Simulation result show that the response of the process control with decoupler has smaller offset than the process control without decoupler. It can be seen in Fig. 6 and Fig 3 that there is no oscillation for the process control with the decoupler. Fig. 8 and Fig. 9 show the respons for process with large time delay.

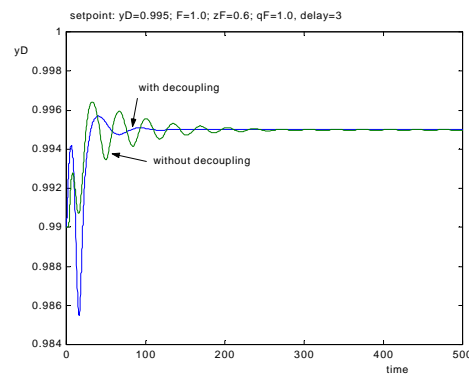


Figure 6. Comparation between control system with decoupler and without decoupler for setpoint  $y_D=0.995$ ;  $z_F=0.5$ ;  $F=1.0$ ;  $q_F=1$ ; delay=3.

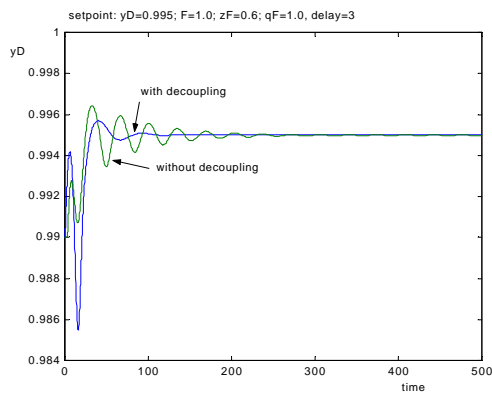


Figure 7. Comparison between control system with decoupler and without decoupler for setpoint  $y_D=0.995$ ;  $z_F=0.6$ ;  $F=0.1$ ;  $q_F=1$ ; delay=3.

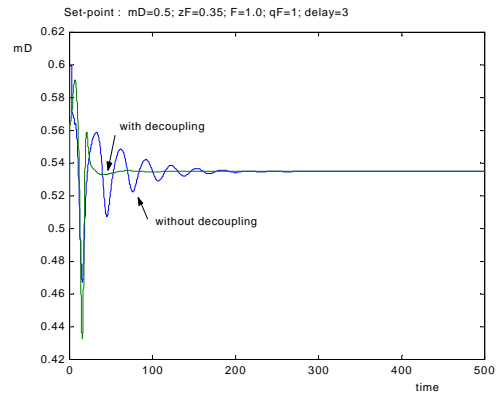


Figure 10. Comparison between control system with decoupler and without decoupler for setpoint  $m_D=0.5$ ;  $z_F=0.35$ ;  $F=0.1$ ;  $q_F=1$ ; delay=3.

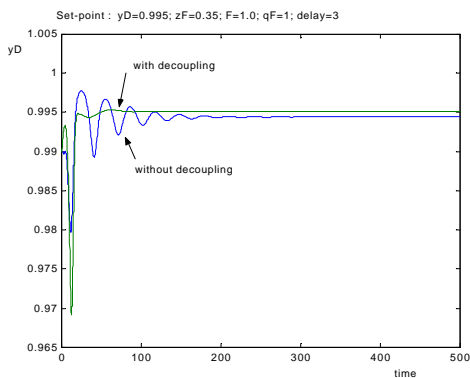


Figure 8. Comparison between control system with decoupler and without decoupler for setpoint  $y_D=0.995$ ;  $z_F=0.35$ ;  $F=1.0$ ;  $q_F=1$ ; delay=3

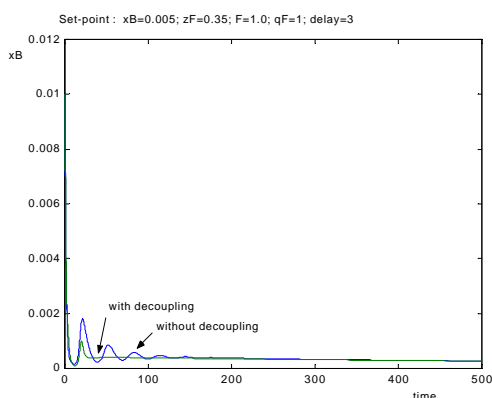


Figure 9. Comparison between control system with decoupler and without decoupler for setpoint  $x_B=0.005$ ;  $z_F=0.35$ ;  $F=1.0$ ;  $q_F=1$ ; delay=3

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