Chaotic Operation of a Colpitts Oscillator in the Presence of Parasitic Capacitances

O. TSAKIRIDIS*, D. SYVRIDIS†, E. ZERVAS‡ and J. STONHAM*

*Dept. of Computer and Electronics, †Dept. of Informatics and Telecommun., ‡Dept. of Electronics

†Brunel University, †University of Athens, ‡TEI of Athens

*Uxbridge, Middx UB8 3PH, †Ilisia 15784, Athens, ‡Egaleo 12210, Athens

*United Kingdom, †Greece, ‡Greece

Abstract: - A detailed description and analysis on the behaviour of a bipolar junction transistor in chaotic operation is presented. Bipolar Junction Transistors (BJTs) are basic ingredients of chaotic Colpitts oscillators, which aim at generating chaotic signals for secure communications. The high frequency operation of the chaotic oscillators is a challenging issue due to the parasitic capacitance of the junctions of a BJT. The presented analysis shows that the parasitic capacitances have adverse effects on the oscillator’s operation. They may generate conditions for chaotic oscillations or destroy them as well. Simulation results show that in order to determine more accurately the chaotic spectral signature and the performance of the system, the impact of the parasitic capacitances need to be taken into account during the design and analysis phase of the system.

Key-words: Chaos, Colpitts Oscillator, Parasitic Capacitances, Bipolar Transistor.

1 Introduction

In the last few years, dynamical chaos has enjoyed a tremendous attention, and it is considered today a major candidate for future secure communications. Chaotic oscillators, capable of generating chaotic oscillations from audio frequencies up to the optical band, are used as sources of chaotic carriers in a variety of applications including signal masking [1], chaos modulation [2], [3], and spectrum spreading [4]. These chaotic oscillators differ from each other in both structure and element set used. Among them is the classical Colpitts oscillator which, although commonly used to generate sinusoidal signals, with special settings of the circuit parameters it may exhibit chaotic behaviour.

Chaos in the Colpitts oscillator has been reported first by Kennedy [5]. Later, the chaotic behaviour of this oscillator has been investigated by several authors [6], [7], [8]. All these studies of the chaotic Colpitts oscillator have been limited in low frequencies, in the order of tens of KHz, where the BJT is modeled as a piecewise linear circuit. Although this approximation is satisfactory in low frequencies (close to the audio frequencies), it is poor in high frequencies (> 100 KHz) due to the transistor junctions’ capacitances. At high frequencies the values of these capacitances are not only comparable to those of other circuit elements but also vary with the bias of the transistor. Operating the chaotic sources in high frequencies is crucial in several telecommunication applications. Broadband communications, spread spectrum techniques and the cryptography of high entropy sources demand high frequency carriers. The design of such systems requires a more detailed description of its elements and a precise characterization of the sources’ spectral signature. There are several references, i.e. [6], to experimental configurations of chaotic oscillators operating in microwave frequencies but such efforts lack the required detailed modeling and analysis.

In this paper we expose the relations among the capacitances that rule the commercial transistors operating in real conditions and we model in more detail the chaotic oscillators, including the parasitic capacitances in the model. The nonlinear differential equations that govern the system are solved using the Dynamic Solver. Finally, a thorough study on the effects of the transistors’ parasitic capacitances on the chaotic behaviour of the Colpitts oscillators is presented.

The paper is organized as follows. Section 2. briefly
discusses prior work on the Colpitts oscillators in chaotic operation and BJT modeling. Section 3 present simulation results on the chaotic oscillator’s behaviour obtained using DS. Finally, Section 4 presents our conclusions.

2 Prior Work

The Colpitts oscillator is a combination of a transistor amplifier and an LC circuit used to feedback the output signal as it is depicted in Figure 1.

![Fig. 1. Circuit diagram of a classical Colpitts oscillator.](image)

The oscillation frequency is given by

\[ f = \frac{1}{2\pi \sqrt{L_1 C_2 / (C_1 + C_2)}} \]  

Kennedy [5] studied the circuit of Figure 1, consisting of a single bipolar junction transistor (BJT), and observed a complex dynamic behavior. For a set of element parameters, the system’s attractor is as in Figure 2 and a Lyapunov exponent of the dynamic system was calculated positive, an indication of the chaotic behavior of the Colpitts oscillator.

The relationship between the chaotic Colpitts oscillator and the Chua’s oscillator was illustrated by Kennedy [9]. As it is shown in [9], modeling the BJT as a two segment piecewise linear resistor, the dynamics of the continuous two region piecewise linear Colpitts oscillator may be mapped onto an equivalent asymmetrical Chua diode.

A nonlinear and a bifurcation analysis of the Colpitts oscillator, modeled as a piecewise linear system, was carried out in [10], [11], [12]. The results show the occurrences of broader-collision bifurcations and moreover, border bifurcations corresponding to singularities of the Jacobian of the system. The feedback loop in a Colpitts oscillator, which provides conditions for chaotic signal generation, can be designed so that the output of the oscillator has preassigned spectral characteristics [8], [13]. The feedback loop can be implemented by a series of \( n \) parallel-serial units, consisting of an inductor, a resistor and a voltage divider composed of capacitors, resulting in a chaotic oscillator whose output exhibits the desired spectral characteristics.

The two stage chaotic Colpitts oscillator was studied in [14]. It contains two bipolar junction transistors, coupled in series and a resonance loop consisting of an inductor and three capacitors. The two-stage oscillator enables the fundamental frequency of chaotic oscillations to be increased by a factor of three.

The synchronization capabilities of coupled Colpitts oscillators have been investigated numerically and experimentally in [15], [7], [16], [13]. Moreover the use of microwave chaotic Colpitts oscillators for direct chaotic communications has been reported in [6].

The BJT (in common-base configuration) in the Colpitts oscillators studied in the aforementioned papers is modeled as a nonlinear resistor and a linear current-controlled current source. This model is shown in Figure 3, where \( a_f \) is the common-base forward short-circuit current gain and \( R_E \) is the voltage-controlled nonlinear resistor with characteristic \( I_E = f(V_{BE}) = f(-V_{CB}) \) given by

\[ I_E = \begin{cases} 
I_0 \left( \frac{V_{BE} - V_{th}}{V_T} \right), & V_{BE} \geq V_{th} \\
0, & V_{BE} < V_{th} 
\end{cases} \]  

The threshold voltage is given by

\[ V_{th} = V_T \left[ \ln(a_f I_0 / I_S) - 1 \right] \]  

where \( I_S \) is the saturation current of the base-emitter junction and \( V_T \approx 26 \text{ mV} \) at room temperature.

![Fig. 2. System attractor of the chaotic Colpitts oscillator.](image)
At high frequencies the capacitive reactance attain values comparable to the ohmic elements of the circuit [17] and the BJT behaviour depends on the frequency. This case necessitates more complex and, in any case, non ohmic models. One such model is the hybrid II equivalent, which satisfactorily describes the operation of the elements for frequencies up to 100 MHz, and the $S$ parameter model for frequencies greater than 100 MHz. Figure 4 depicts the equivalent hybrid II model.

**Base-emitter depletion capacitance.** The base-emitter capacitance $C_{BE}$ comprises, in principle, the contributions of three factors: a depletion capacitance, an overlap capacitance and a diffusion capacitance. As long as the base-emitter voltage is not too forward we can neglect the last contribution. Moreover, assuming no currents flowing there is no voltage drop over any resistor, the capacitance $C_{BE}$ can be approximated by the (almost ideal) formula

$$C_{BE} \approx \frac{C_{jE}}{1 - V_{BE}/V_{de}} + C_{BEO}$$

where $C_{jE}$ is the capacitance of the base-emitter junction for zero bias, $V_{de}$ is the diffusion voltage of the junction (a reasonable value is 0.9 V), $p_E$ is a grading coefficient ($\sim 0.4$) and $C_{BEO}$ is the constant capacitance between the junctions. The constant capacitance $C_{BEO}$ describes any overlap (peripheral) capacitances between base and emitter, which will be taken equal to zero for the rest of the paper. We should note that the expression above is an approximation of the base-emitter capacitance. The full expression is [18]

$$C_{BE} = \frac{s_E C_{jE}}{1 - V_{jE}/V_{de}^{pe}} + \frac{(1 - s_E) C_{jE}}{1 - V_{FE}/V_{de}^{pe}} + C_{BEO}$$

where

$$V_{jE} = V_{BE} + 0.1V_{de} \ln s_E, \quad V_{FE} \approx V_{de}(1 - 3^{-1/p_E})$$

The first term describes a normal depletion capacitance ($s_E = 1$), the second term describes a constant (non dependent on $V_{BE}$) capacitance ($s_E = 0$) and the parameter $s_E$ rules the transition between these terms.

**Collector-emitter depletion capacitance.** The expression of the emitter-collector depletion capacitance formula is similar to that of the base-emitter capacitance, that is

$$C_{CE} \approx \frac{(1 - X_p) C_{jC}}{1 - V_{BC}/V_{dc}^{pe}} + X_p C_{jC} + C_{BEO}$$

where $C_{jC}$ is the capacitance of the emitter-collector junction, $V_{dc}$ is the diffusion voltage of the junction (a reasonable value is 1.2 V), $p_C$ ($\sim 0.4$) is a grading coefficient and $C_{BEO}$ is the peripheral capacitance between the junctions, which is taken equal to zero. The parameters $C_{jC}$ and $C_{jE}$ have been evaluated experimentally as it is described in Section 3.

### 3 Simulation Results

In this section, we present simulation results for a Colpitts oscillator using the Dynamic Solver (D.S.). The model of the oscillator, that includes the parasitic capacitances, is developed in parallel with the classical chaotic Colpitts oscillator in order to easy the comparisons and emerge their differences. The equivalent circuit of the classical chaotic Colpitts oscillator is given in Figure 5. The system is described by the following set of equations (E1)

$$C_2 \frac{dV_{CE}}{dt} = I_L - I_C$$
$$C_1 \frac{dV_{BE}}{dt} = -\frac{V_{EE} + V_{BE}}{R_{EE}} - I_L - I_B$$
$$L \frac{dI_L}{dt} = V_{CC} - V_{CE} + V_{BE} - I_L R_L$$
$$I_B = \begin{cases} 0 & V_{BE} < V_{th} \\ (V_{BE} - V_{th})/R_{on} & V_{BE} > V_{th} \end{cases}$$
$$I_C = \frac{\alpha_I I_B}{1 - \alpha_I}$$

Figure 6 depicts the equivalent circuit of a Colpitts oscillator with the parasitic capacitances present.
Fig. 5. Equivalent circuit of the Colpitts oscillator in absence of the parasitic capacitances.

Fig. 6. Equivalent circuit of the Colpitts oscillator with the parasitic capacitances present.

The set of differential equations (E2) that describe the operation of the system in Figure 6 is

\[
\begin{align*}
L \frac{dI_L}{dt} &= V_{CC} - V_{CE} + V_{BE} - I_L R_L \\
I_B &= \begin{cases} \\
0 & V_{BE} < V_{th} \\
\frac{(V_{BE} - V_{th}) / R_{en}} & V_{BE} > V_{th} \\
\end{cases} \\
I_C &= \beta_I I_B \\
C_{11} &= C_1 + C_{BE} \approx \frac{C_{JE}}{(1 - V_{BE} / V_{BE})^p} \\
C_{22} &= C_2 + C_{CE} \\
&\approx C_2 + \frac{(1 - X_p) C_{JC}}{(1 - V_{BC} / V_{BC})^p} + X_p C_{JC} \\
\end{align*}
\]

The sets of equations (E1) and (E2) were solved numerically using the parameter values of Table 1. The parasitic capacitances present in any bipolar junction transistor can be best modeled as capacitors connected between each of the three ports of the transistor. The capacitances were measured using a capacitance bridge meter (Hameg HM 8018). DC voltages can be applied to the ports of the capacitance meter in order to bias the transistor. The capacitance \( C_{BE} \) is measured by grounding base and measuring between collector and emitter whereas \( C_{BC} \) is measured by grounding collector and measuring between emitter and base.

Table 2 contains the measured static capacitances for the 2N2222 BJT.

<table>
<thead>
<tr>
<th>Capacitance</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_{BE} )</td>
<td>21.5 pF</td>
</tr>
<tr>
<td>( C_{BC} )</td>
<td>7.9 pF</td>
</tr>
</tbody>
</table>

Solving (E1) using the set of parameters in Table 1, results in a chaotic output, whose spectrum is depicted in Figure 7. The corresponding system’s attractor is drawn in Figure 8. However, the system of equations (E2), which includes the effect of the parasitic capacitances, under the same set of parameter values, results in an harmonic
output as it is observed from the corresponding spectrum and system’s attractor depicted in Figures 9, and 10 respectively. It is interesting to note that the opposite behaviour has been also observed. That is, (E1) may result in an harmonic output under the values $R_2 = 25.5 \ \Omega$, $L_1 = 3 \ \text{e-06} \ \text{H}$, $C_1 = 5 \ \text{e-10} \ \text{F}$ and $C_2 = 5\text{e-10} \ \text{F}$, whereas (E2) for the same parameters shows chaotic behaviour.

![Fig. 7. Output spectrum of the Colpitts oscillator in absence of the parasitic capacitances.](image)

![Fig. 8. Colpitts oscillator’s attractor in absence of the parasitic capacitances.](image)

**4 Conclusions**

A study on the effects of a BJT’s parasitic capacitances on the operation of a Colpitts oscillator was performed. The parasitic capacitances influence adversely the harmonic or chaotic operation of the oscillator, a result that must taken into consideration during the design phase of high frequency circuits. At high frequencies, the values of the parasitic capacitances are comparable to those of other circuit elements and thus the resulted behaviour of the circuit is questionable. As it was shown, by solving the system equations that describe the Colpitts oscillator, the circuit with the same external elements but with or without including the parasitic capacitances in the model may operate chaotically or harmonically and vice-versa. These results strengthen the need for further analysis and research efforts.

**References:**


