Generation of Binary Sequences with good AIP using the Logistic Map - Application to DS-CDMA Decorrelating Receiver

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Abstract — Recently, sequences generated by nonlinear dynamical systems have been considered in CDMA spread spectrum application and it has been shown that they can behave better than classical ones. In almost all cases, classical criterium have been considered to design good spreading sequences, they are based on periodic or aperiodic correlation functions, like Figure Of Merit or Maximal Correlation Pick. Moreover, conventional receiver has been considered in evaluating spreading sequence performances. In this contribution we will consider the Average Interference Parameter (AIP) as a criterium to select good spreading sequences. We will exploit the wealth of the logistic map and propose quantized Super Stable Orbits (SSOs) as binary sequences. We will show that it is possible to find new binary sequences with AIP better than those of classical Gold sequences. As an application, we will consider the performances of decorrelating receiver using proposed binary sequences.

Keywords — DS-CDMA, Spreading Sequences, Decorrelating receiver, Chaos Super Stable Orbits.

I. INTRODUCTION

It is well known that the choice of spreading sequences is very important in the design of a CDMA system. In the case where the symbols to be transmitted are binary, it was shown [1-2] that aperiodic, even and odd correlation functions have the same importance in mitigating multi-user interference, on the other hand, sequences with good periodic auto-correlation function allow fast code synchronisation. But, in general, it is difficult to find a tradeoff in resolving these problems and every classical type of sequences presents a gap in at least one type of correlation function, so works are still conducted to find better types of sequences. In the last years, sequences generated by nonlinear dynamical systems have been proposed as a candidate to DS-CDMA system. In [4,5], assuming that spreading sequences are second order stationary, the optimal signal to interference ratio was determined and it was shown that this optimum is achieved by considering sequences with exponentially aperiodic auto-correlation functions. Based on this result it was shown that truncated and quantized chaotic sequences generated by the tailed shift map allow average performances that are very close to the optimum. However, finding the staring points of considered chaotic spread spectrum sequences making performances close to the optimum remains hard to deal with. This is due to the fact that chaotic sequences are sensitive to initial conditions, two sequences beginning by two very close points are in general decorrelated and have different statistical properties. Since a chaotic sequence is always close, for a short period of time, to some Unstable Periodic Orbit (UPO) merged in the strange attractor of the considered chaotic system, these orbits have been considered instead of quantized chaotic sequences in spread spectrum applications [8,11,12]. But the task of finding UPOs with predefined correlation properties is not less difficult than finding initial conditions for truncated chaotic sequences, especially when the period is large. This is due to the fact that the set of UPOs is dense in the strange attractor and is of measure zero; thus huge precision is required to locate them. In [14], we showed that for the logistic map to every existing UPO corresponds a Super Stable Orbit (SSO) having almost the same correlation properties and that the quantization of the two gives two binary sequences that differ at most in one symbol. Moreover, using symbolic dynamic, SSOs are much easier to find and to handle than UPOs. In this paper, we considered quantized SSOs as binary sequences. We will consider the AIP criteria to select good binary sequences, we found a set of sequences presenting better performances than classical Gold ones with respect to AIP criteria. In [9], it has been shown that the performances of a decorrelating receiver used in an asynchronous DS-CDMA depend on the AIP of the used spreading sequences. We simulated such receiver and determined its error rate using the two types of sequences. We found that the performances in term of error rate are improved when proposed sequences are used with respect to Gold sequences.

This paper is organized as follows. In section II we will give the DS-CDMA model and give the relation between decorrelating receiver performances and correlation properties of spreading sequences. Section III will deal with SSOs of the logistic map and they essential properties. In section IV we will give a comparison of performances allowed by quantized SSOs and Gold sequences used in DS-CDMA application when the period is \( N = 31 \) and \( N = 63 \). We conclude and give some perspectives in the last section.
II. SEQUENCE PERFORMANCES IN DS-CDMA APPLICATION

A. DS-CDMA Model

The Aperiodic Auto Correlation function of a sequence $a_k$ is defined by

$$C_k(l) = \sum_{k-1}^{N-l} a_k[j]a_k[j+l]^* \text{ if } 0 \leq l \leq N-1$$

$$C_k(l) = \sum_{N+l}^{N-l} a_k[j-l]a_k[j]^* \text{ if } 1 - N \leq l \leq -1$$

$$else$$

$$C_k(l)$$

The Aperiodic Auto Correlation function of a sequence $a_k$ is defined by

$$C_k(l) = C_k,k(l).$$

B. Average Interference Parameter

The AIP of the sequence set

$$a_k = a_k[0]a_k[1]...a_k[N-1]a_k[0]... k = 1, 2, ..., M$$

is defined by

$$AIP = \frac{1}{K^2} \sum_{k, i}^{K} (2\mu_{k,i}(0) + \mu_{k,i}(1))$$

Where

$$\mu_{k,i}(n) = \sum_{l=-N}^{N-1} C_k,l(l)C_k,l(l+n)^*$$

C. Decorrelator Receiver Performances

The received signal is fed into a bank of matched filters corresponding to active users. The output of each filter $k$ is [7]

$$Z_k[i] = \sum_{l=1}^{K} \sum_{m=-M}^{M} A_l b_l[m] a_k(t - mT - \tau_l) + n(t) a_k(t - iT - \tau_k)^* dt$$

where $b_l[m]$ is the symbol transmitted by user $l$ at time $m$. $A_l$ is the attenuation introduced by the channel between base station and user $l$. Let

$$R_k[l] = \int a_l(t - mT - \tau_l) a_k(t - \tau_k)^* dt \forall l \leq k$$

$$\eta_k[i] = \int n(t) a_k(t - iT - \tau_k)^* dt$$

We obtain

$$Z[i] = R[-1]Ab[i] +$$


where $b$, $Z$, $N$, and $\eta$ are vectors with components equal to $Z_k$, $b_k$, and $\eta_k$ respectively and $A$ is a diagonal matrix with components equal to $A_l$. The autocorrelation function of $N$ is given by

$$E[N[i]N[i+j]] = \sigma^2R[j]$$
In chip synchronization case where \( \tau_i = m_iT_c \) the above formula becomes

\[
R_{kl}[0] = C_{kl}(N + m_k - m_l) \\
R_{kl}[-1] = C_{kl}(m_l - m_k) \* \\
R_{kk}[1] = 0
\]

The channel is supposed to be an Additive White Gaussian Process with two sided spectral density \( \frac{N_0}{2} \); the bit energy is \( E_b \). In frequency domain formula (5) becomes

\[
Z(f) = S(f)AB(f) + \eta(f)
\]

Where

\[
S(f) = R[1]^{H} e^{i2\pi f} + R[0] + R[1] e^{-i2\pi f}
\]

\( R[1]^{H} \) is the complex conjugate of \( R[1] \). The decorrelating receiver model is presented in figure 2.

Theoretically, the decorrelating receiver consists of inverting the matrix \( S(f) \). The signal after the decorrelator and before decision stage is given in frequency domain by

\[
X(f) = S(f)^{-1}Z(f) = AB(f) + \eta_d(f)
\]

\( \eta_d(f) \) is the response of the decorrelator to \( \eta(f) \). Equivalently, in time domain

\[
X(i) = Ab(i) + \eta_d(i)
\]

The power of the \( k^{th} \) component of the noise is given by

\[
\sigma_k^2 = E[\eta_d(m)\eta_d(m)]_{kk} = \sigma^2 \int_0^{1} (S(f)^{-1})_{kk}df > \sigma^2
\]

Based on formulae above it has been found in [5] that performances correlating receiver in term of error probability depends on Average Interference Parameter (AIP) of considered spreading sequences. In our simulation work we considered the following algorithm describing a serial interference canceller

\[
x_k^{i+1}(m) = \frac{1}{D_{kk}} [y_k(m) - \sum_{l=1}^{k-1} R_{kl}(0)x_l^{i+1}(m) - \sum_{l=k+1}^{K} R_{kl}(0)x_l^{i}(m) - \sum_{l \neq k} R_{kl}(-1)x_l^{i}(m + 1) + R_{kl}(1)x_l^{i+1}(m - 1)]
\]

III. Binary sequences generated by the Logistic map

In this work we considered the logistic map given by

\[
x_{k+1} = f_a(x_k)
\]

where

\[
f_a(x) = 1 - ax^2, 0 \leq a \leq 2
\]

\( a \) is the bifurcation parameter.

A. Periodic Orbits

Let \( x_k, k = 0, 1, \ldots \) be the sequence defined by the initial condition \( x_0 \) and the recurrence 13; a \( p \) periodic orbit is a set of \( p \) ordered points \( X = x_0x_1 \ldots x_{p-1} \) satisfying \( f_a(x_i) = x_{i+1} \) for all \( 0 \leq i \leq p-2 \) and \( f_a(x_{p-1}) = x_0 \). So, beginning by some point \( x_i \), the sequence defined by 13 is the repetition of the periodic orbit \( X \); it is noted by \( x_0x_1 \ldots x_{p-1} \).

B. Stability of periodic orbits

The stability of a \( p \) periodic orbit \( X = x_0x_1 \ldots x_{p-1} \) is defined by its Floquet multiplier:

\[
\lambda(X) = \frac{df_a^p}{dx} (x_i) = \prod_{k=0}^{p-1} f_a'(x_k)
\]

We have the following three possible situations:

- \(|\lambda(X)| < 1 \), the orbit \( X \) is stable
- \(|\lambda(X)| > 1 \), the orbit \( X \) is unstable
- \(|\lambda(X)| = 1 \), the orbit \( X \) can be stable or unstable

When \( \lambda(X) = 0 \), \( X \) is said to be super-stable. It is clear that a periodic orbit is super-stable if and only if it contains the critical point 0. So a SSO is always of the form \( 1x_1 \ldots x_{p-2}0 \).

C. Proposed binary sequences

Let \( X = x_0x_1 \ldots x_n \ldots \) be a sequence generated by the logistic map, i.e. \( x_0 \) is the initial condition and \( x_{k+1} = f_a(x_k) \); to \( X \) corresponds a symbolic sequence:

\[
S = S_0S_1 \ldots S_n \ldots
\]

\[
\begin{align*}
S_i = 1 & \text{ if } x_i \geq 0 \\
S_i = -1 & \text{ if } x_i < 0
\end{align*}
\]

It is clear that if the sequence \( X \) is periodic with a period equal to \( N \) it is the same thing for the corresponding symbolic sequence \( S \).

The proposed binary sequences are the quantization of SSOs according to the rule 15.
IV. Performances of selected Quantized SSOs in DS-CDMA application

In this section, will be presented a performance comparison between selected quantized SSOs and Gold sequences for the periods 31 and 63. For shirt period, we found that when the parameters corresponding to two binary sequences are close to each other, these sequences are similar and so they are correlated. Taking into account this idea, we followed the procedure to find good binary sequences that consists of choosing sequences with good autocorrelation functions in different subintervals of parameter set \([0, 2]\); then keeping only sequences that are less correlated. Among all found sequences we keep only those presenting good AIP.

A. AIP of selected quantized SSOs

The AIP of selected quantized SSOs is compared to AIP allowed by Gold sequences; we considered the two cases of period \(N = 31\) and \(N = 63\). In figure 3 is plotted the AIP of the two types of sequences. We can see that considered sequences allow better AIP with respect to the one allowed by Gold sequences. Moreover, quantized SSOs with good AIP exist with a number that is important compared to Gold sequences. For example, for the period 31 case, the number of Gold sequences is 33 and the number of quantized SSOs with good AIP is 60.

B. Application to Asynchronous DS-CDMA System using a decorrelating receiver

As an application of the previous results we consider in this section the BER of the correlating receiver using the two types of sequences quantized SSOs and Gold ones, versus signal to noise ratio \(E_s/N_0\). We considered the algorithm given in 12 to simulate the decorrelating receiver. We considered the cases of one and two stages, in the two cases we considered 10 users. Found results are plotted in figure 4, we can see that selected quantized SSOs improve the performances of the system in term of error rate.

V. Conclusion

In this contribution we exploited the wealth of the logistic map to determine binary sequences with good AIP. The considered binary sequences are the quantization of selected Super Stable Orbits. We considered the periods 31 and 63 and compared the AIP of considered sequences and the one allowed by classical Gold sequences. We showed the superiority of the former with respect to the later. As an application, we simulated the decorrelating receiver and determined its performances, in term of error rate, using the two types of sequences. We found that using quantized selected SSOs improve the error rate with respect to Gold sequences. In this work we supposed that the channel is an AWGN one, in real cases the channel is much less favorable due essentially to the fading phenomena and receivers are more complicated. Since spread spectrum sequence have to be chosen according to such parameters it is interesting to consider found quantized SSOs in other context.

References

Fig. 3. AIP versus sequence number for Quantized SSOs and Gold Sequences in the two cases a- $N = 31$ and b- $N = 63$

Fig. 4. BER versus $E_b/N_0$ for Quantized SSOs and Gold Sequences in the two cases a- $N = 31$ and b- $N = 63$