An effective hybrid algorithm for mobile robot global path planning

FENG YUANJING, FENG ZUREN.
Systems Engineering Institute
Xi’an Jiaotong University
Xi’an, Shan’xi Province
PEOPLE REPUBLIC OF CHINA

Abstract: Ant Colony Optimization (ACO) exhibits parallelism, contains certain redundancy and historical information of the past solutions with pheromone trail, and is suitable for implementation on massively parallel architecture. But it is not easy to avoid local optima, especially for large-scale schedule problems. Simulated annealing (SA) is a naturally serial algorithm, but its behavior can be controlled by the cooling schedule. By reasonably combining these two global probabilistic search algorithms, we develop a general, parallel and easily implemented hybrid optimization framework, and apply it to mobile robot global path planning problems. This strategy combines the strongpoint of two algorithms. Local search algorithm of SA can further improve the solutions constructed by individuals of ant systems. Moreover, the ant system can provide effective initial solutions for neighborhood search algorithm. Simulate results shows that the hybrid strategy computation is simple, the convergence speed is fast, which significantly improve the computational efficiency of solving mobile robot’s global path planning problems.

Key-Words: Mobile robot path planning, Ant colony optimization, Simulated annealing, Hybrid optimization strategy

1. Introduction

Motion planning for mobile robots has long been an extensively studied field [1][7][11], which can be stated as follows. Given robots and environment information, found a free path between the start point and goal point in configure-space. According to the known information of environment, motion planning can be classified in two types: global path planning with full information of environment and local path planning with partial information of environment. In recent years, for global motion planning problems, a wide variety of techniques have been developed. Some research has focused on the use of methods such as potential fields [2][6], optimal methods [3], while others have taken different approaches such as differential geometry [11], and game theory[10].Among optimal methods, intelligent optimal approaches shows a great successful results , such as neural network (NN)[8][9], genetic algorithm (GA) [4][5], simulated annealing (SA)[6][7], and ant colony Optimization [12].

Among the probabilistic search algorithms, ACO is a cooperative search algorithm inspired by the behavior of real ants to found a nearest path form the nest to the food source. This is similar to the motion planning of mobile robotics. One basic idea of ant system is to use the counterpart of the pheromone trail used by real ants as a medium for communication and as an indirect form of memory of previously found solutions. The artificial ants are simple agents that have some basic capabilities, e.g. in case of the traveling salesman problem (TSP) they may use some heuristic information like the distance between cities to construct tours and maintain a tabu list in which they store the partial tour constructed so far. The ants build solutions constructively guided by heuristic information and the pheromone trails let by ants in previous iterations. After the solutions are constructed, the ants are allowed to update the pheromone trails. ACO is exhibits parallelism, contains certain redundancy and historical information of the past solutions, and is suitable for implementation on massively parallel architecture. But it is not easy to avoid local optima, especially for large schedule problems [13][14][15]. In contrast to ACO, SA employs certain probability to escape from local optima and the search process can be controlled by the cooling schedule [16]. But, SA maintains only one solution at a time, whenever it accepts a new solution, it must discard the old one, which often leads to low search efficiency. How to set optimal parameters and efficiently find global optima are still open problems. Based on the complementary strengths of ACO and SA, the hybrid framework of ACO and SA can
achieve more efficient optimization results. In this paper, reasonably combining ACO with SA from mechanism to structure, we develop an effective hybrid optimization strategy ACSA and investigate its potential on solving motion planning. The organization of remain contents is as follows. In Section 2 the hybrid framework of ACO and SA is presented, and the conversion from the hybrid strategy to ACO or SA is discussed. In Section 3 the hybrid strategy applied to global motion planning are described. In Section 4 the experiments for mobile robotics path planning is presented, and computational results and comparisons are presented and discussed. In Section 5 we end with some conclusion remarks.

2. The hybrid optimization strategy
Ant colony optimization [13] and simulated annealing [16] are naturally motivated, and represent global combinatorial optimization methods with complementary strengths and weaknesses. Because of the serial nature of SA, there is a single solution that is modified over time which often leads to low search efficiency. While ACO exhibits implicit parallelism through large mount of ants searching and can retain useful redundant information about what is every ant learned from previous searches linked by pheromone. However, ACO also may lead poor solutions because of the limited update capability of pheromone. By contrast, based on the suitable cooling schedule SA has good convergence property and the ability to probabilistically escape from local optima can be controlled [16]. Thus, a hybrid framework of ACO and SA, named ACSA, is presented as follows (see Fig. 1). Local search algorithm of SA can further improve the solutions constructed by individuals of ant systems. On the other hand, the ant system can provide effective initial solutions for domain search algorithm, because the guidance role of the pheromone trial which retain useful redundant information about previous search experience.

Begin
Initialize ant systems with $M$ individuals.
Do
For a circle tour
For (k =1; k<M; k++)
Compute probability $p^k_{ij}(t)$ and select the next location.
Set initial temperature $t_0$
For each circle tour path of ant $k$
Do {Generate a neighbor solution $s_j$ from $s_0$,
Compute energy change $\Delta E_{ij} = E_j - E_i$;
if $\min(1, \exp(-\Delta E_{ij}/t_N)) > \text{random}[0,1])$ {
  $i = j$
} while (no better solution generate);
Update $\Delta \tau^1_{ij}$;
$\tau^k_{ij}(N) = \rho \cdot \tau^k_{ij}(N-1) + \Delta \tau^k_{ij}(N-1)$;
Decrease temperature $t_{N+1} = \text{update}(t_N)$ and set $N = N + 1$
} Until (equilibrium condition satisfied);
Output optimization results;
End
Fig 1 a hybrid ACSA algorithm

It can be seen that during the hybrid search process, ACO provides a set of initial solutions for SA at each temperature to perform Metropolis sample for each solution until equilibrium condition is reached, and ACO uses the solutions found by SA to continue parallel search. Temperature is adjusted to control the behavior of SA, i.e., at a high temperature, SA performs a “course” search with high escaping probability from current solution; while at a low temperature, SA performs a “fine” search among the neighbor solutions of current solution. In hybrid framework, adjusted of temperature can decrease the stagnancy of ACO searching. In addition, such hybrid framework can convert to traditional ACO by omitting the SA unit, and it can convert to traditional SA omitting the ACO. Such hybrid strategy reserves the generality of ACO, SA and can easily be implemented and applied to any combinatorial or functional optimization problems. For different problems, the solution constructing methods, pheromone update scheme, algorithm criteria and parameters should be designed suitably. In the next section, we will explore the potential of such hybrid optimization strategy to global path planning of mobile robots.

3. Hybrid optimization strategy for path planning
3.1. Path planning model and Constructing of solution
Path planning problems of mobile robotics can be expressed as optimal problem: the cost function is planned free path, subject to avoiding the obstacles, its mathematic model is as follow:
\[
\min f(X) \quad X \in \mathbb{R}^n
\]
\[
s.t. \quad g_i(X) \leq 0 \quad i = 1, 2, \ldots, p
\]
Where \( f(X) \) is cost function, and \( g(X) \) are constraint conditions. This is nonlinear constrained programming problem, which can be changed into unconstrained optimization problem:
\[
\min E = \mu \cdot E_i + (1 - \mu)E_c
\]
Where \( E \) is the energy function, \( \mu \) is weight. \( E_i \) is square sum of all path lengths. \( E_c \) is sum of all path points collision punish functions.

Suppose that start point of mobile robot is \( S \), and the goal point is \( G \). Between \( S, G \), there are some obstacles \( O_k \) (\( k = 1, 2, \ldots, K \)), see Figure 2. Mobile robot should found an optimal free path. Start point \( S(x_s, y_s) \) is in coordinate \( o-xy \). Construct coordinate \( S-x'y' \), with origin \( S \) and abcissa \( SG \). The line \( SG \) is cut into \( m \) parts with bisectrix \( (L_1, L_2, \ldots, L_{m-1}) \). Length of each part is \( l \). In configure space, part line \( (L_1, L_2, \ldots, L_{m-1}) \) into \( 2n \) parts centered with \( SG \). Therefore, there are \((m-1) \times (2n+1)\) path points in configure search domain expressed as follow:
\[
L_i(x_1, y_1), L_2(x_2, y_1), \ldots, L_{m-1}(x_{m-1}, y_1), \ldots, L_4(x_n, y_{m-1}), \ldots, L_{m-2}(x_{2n-1}, y_{m-1})
\]
(3)
Where \( L_i(x_j, y_j) \) is \( j \)th point on line \( L_i \). Therefore, the path from start point \( S \) to goal point \( G \) can be expressed as
\[
Path = \{S, L_1(x_1, y_1), L_2(x_2, y_2), \ldots, L_{m-1}(x_{m-1}, y_{m-1}), G\}
\]
(4)
Transform between coordinate \( S-x'y' \) and \( o-xy \) is
\[
\begin{bmatrix}
  x \\
  y
\end{bmatrix} = R
\begin{bmatrix}
  x' \\
  y'
\end{bmatrix} + \begin{bmatrix}
  x_s \\
  y_s
\end{bmatrix}
\]
(5)
Where \( R = \begin{bmatrix}
  \cos\theta & -\sin\theta \\
  \sin\theta & \cos\theta
\end{bmatrix} \) is transform matrix, \( \theta \) is the angle between \( ox \) and \( Sx' \).

Distance between path point \( a(x_{i'}, y_{i'}) \) on line \( L_i \) and next path point \( a(x_{i+1}', y_{i+1}') \) is
\[
d_{ab} = \sqrt{l^2 + (y_{i+1}' - y_{i'})^2}, \quad \text{if path } ab \text{ intersect or tangency with obstacles, define } d_{ab} = \infty.
\]
For each ant \( \varphi \), the square sum of all path lengths \( E_{tip} \) is
\[
E_{tip} = l^2 + (y_{i+1}' - y_{i'})^2 + \sum_{k=1}^{m-1} \sum_{k=1}^{m-1} l^2 + (y_{ki+1}' - y_{ki}')^2
\]
(6)
The sum of all path points collision punish functions is
\[
E_{c} = \sum_{i=1}^{N} \sum_{k=1}^{K} C_{ik} = \sum_{i=1}^{N} \sum_{k=1}^{K} \frac{1}{d_{imin}}
\]
(7)
Where \( d_{imin} \) is the least distance between path point \( P(x_i, y_j) \) to Obstacle \( O_k \).

The energy function can be expressed according to (2) as follow
\[
E_{\varphi} = \mu \cdot E_{tip} + (1 - \mu) \cdot E_{c}
\]
(8)
In hybrid optimization, the pheromone trail of real ant is imitated by some real numbers \( \tau_{ab} \) associated with some solution attributes. As in path planning of mobile robot, \( \tau_{ab} \) represents the desire of moving from path point \( a \) to path point \( b \) in the path. To select the next path point \( s \) according to pseudo-random- proportional state transition rule [17]:
\[
s = \begin{cases}
\arg\max_{s' \in L_i} \left( \tau_{ss'}(t) \eta_{ss'}(t) \right) & q \leq q_0 \\
\left( s' \right) & q > q_0
\end{cases}
\]
(9)
Where \( L_i \) is set of path point on line \( L_i \), (see Figure
2). An ant makes with a probability \( q_0 \in [0,1] \) to select the best possible decision of \( \max_{l \in L} [\tau_{il}(t)]^\alpha [\eta_{il}(t)]^\beta \), and with probability \( 1-q_0 \) for the position \( s' \) chosen according to the following probability.

\[
p_{ab}^k(t) = \begin{cases} \frac{[\tau_{ab}(t)]^\alpha [\eta_{ab}(t)]^\beta}{\sum_{l \in L_j} [\tau_{al}(t)]^\alpha [\eta_{al}(t)]^\beta} & j \in L_i^k \\ 0 & j \notin L_i^k \end{cases}
\]

Parameter \( q_0 \) means the weight assignment for predetermined knowledge and exploring for new knowledge. \( \eta_{ab}(t) \) is heuristic information value corresponding to the cost function. \( \alpha, \beta \) are the weight parameters for pheromone and heuristic information value in \( p_{ab}^k(t) \), respectively.

### 3.2. Initial temperature and temperature decrement

A proper initial temperature should be high enough so that all states of the system have equal probability of being visited and, at the same time, it should not be rather high so that a lot of unnecessary searches in high temperature will be avoided. In this paper, the initial temperature is determined as follows.

After initialized ants system parameters, \( m \) individuals travel for a circle tour, we determine the best ant searched path with the makespan \( E_{\text{best}} \) and the worst ant searched path with the energy \( E_{\text{worst}} \). Then, at the initial temperature \( t_0 \) we set the probability to accept the worst individual with respect to the best individual \( p_0 \in (0,1) \), i.e.,

\[
p_0 = \exp\left(-\frac{(E_{\text{worst}}-E_{\text{best}})}{t_0}\right)
\]

So, the initial temperature is determined by

\[
t_0 = \frac{-(E_{\text{worst}}-E_{\text{best}})}{\ln(p_0)}.
\]

Here, we use exponential cooling schedule, \( t_k = \lambda \cdot t_{k-1} \) (where \( \lambda \in (0, 1) \) is temperature decrease rate), which is often believed to be an excellent cooling recipe [18], since it provides a rather good compromise between a computationally fast schedule and the ability to reach low-energy state.

### 3.3. New solution generator of SA

In ant colony optimization, the solutions are probability constructed. Here we combine with neighbor search algorithm SA. Swap operator is devised as the new solution generator of SA. According the construct of path solution (4), the swap operator is not same as in TSP, but only two distinct elements on the line \( L_i \) are randomly selected and swapped. Combining above different optimization operators, the hybrid strategy can explore the solution space with a wide range to get high efficiency and good quality.

### 3.4 Pheromone trial update

After all ants have finished local search and completed their tour, each ant \( \phi \) deposits a quantity of pheromone \( \Delta \tau_{ab}^\phi = Q/E_\phi \), on each connection that it has used, where \( E_\phi \) is the path energy of path done by ant \( k \) at iteration \( N \). Thus, the pheromone trial intensities are updated according to the formula:

\[
\tau_{ab}^\phi(N) = \rho \cdot \tau_{ab}^\phi(N-1) + \Delta \tau_{ab}^\phi(N-1)
\]

Where \( \rho \in (0,1) \) is the persistence of the trail, thus \( 1-\rho \) represents the evaporation. The lower \( \rho \), the faster information gathered in previous iterations is forgotten.

### 3.5. Stop and equilibrium conditions

Theoretically, to guarantee convergence of SA, the equilibrium condition of sample process should be satisfied and we should terminate the algorithm as the temperature approaches zero [16], which may cause huge computation. To provide a rather good compromise between solution quality and search efficiency, we devise these two conditions as follows.

Because the length of the solution based on our encoding scheme is determined by the number \( m, n \) simultaneously, we set the step of Metropolis sample process to \( n/m \) to be adapted to the scale of path planning. Since there is no practicable rule to set final temperature, a very low final temperature may cause lots of useless search, and a high final temperature cannot achieve good solution quality. So, in our procedure a proximity condition based on search information is applied, in detail, if the best solution found so far keeps fixed at consecutive temperatures, the algorithm will stop.

### 4. Computational results and comparisons

To test the performances of our hybrid optimization
strategy for path planning, we simulate the environment with several polygons denote obstacles, and with the same start point \( S \) and goal point \( G \), see Table 1. Here suppose path planning with certain environment information.

The hybrid strategy for global path planning of mobile robots mentioned above can be easily implemented. The program is realized in C and run on Pentium IV 2.4G with 256 RAM. Moreover, we set ant number \( M \) to 10, \( m \) to 100, \( n \) to 50, \( p_0 \) to 0.1, \( \lambda \) to 0.9, \( \rho \) to 0.9, \( \mu \) to 0.6, \( Q \) to 1. We test the performances of the hybrid strategy ACSA and ACO, SA with the same parameters and stop criteria designed above, see Figure 3 and Table 1. In Figure 2, the polygons represent the obstacles, the dashed line is the planned path with SA (Figure 3-a), ACO (Figure 3-b), ACSA (Figure 3-c) with the same simulate environment. Table 1 shows that the results obtained by ACSA are much better than those obtained by SA and ACO applied alone. Among three algorithms, ACO used least iterations and CPU run time, but the cost energy is the largest one with 78.34547, it is a local optima. SA can successfully generate solution, but with too much iteration number. The hybrid optimization strategy ACSA we mentioned can robustly given global solution with energy 48.53322, although the CPU run time is little more than ACO. The superiority of the best optimization quality demonstrates the effectiveness and the global search property of the hybrid search.

### 5. Conclusion

We reasonably combined ant colony optimization with simulated annealing to develop a hybrid framework in this paper, in which ACO was introduced to present a parallel search architecture, and SA was introduced to increase escaping probability from local optima at high temperatures and perform “fine” neighbor search at low temperatures. The hybrid strategy is general, easy to implement, and explicit parallelism. Using effective operation-based encoding scheme and some specific operators, we applied the hybrid strategy to global path planning of mobile robots. Computer simulation results showed that, the hybrid strategy was very effective and robust compared to ACO, SA applied alone. In order to achieve better solution quality, time performance and robustness, any advanced operators, search mechanism and improvements on global optimization techniques can be introduced. Moreover, because of the generality of our hybrid strategy, it can be applied to any optimization problems, i.e. the uncertainty environment path planning of mobile robots. Theoretically, analyzing the convergence property and search efficiency (e.g., by Markov chains) to provide some guidance for application and incorporating some problem-specific information in our hybrid search.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Iterations</th>
<th>Time (s)</th>
<th>Path energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACSA</td>
<td>167</td>
<td>0.354</td>
<td>48.53322</td>
</tr>
<tr>
<td>ACO</td>
<td>95</td>
<td>0.245</td>
<td>78.34547</td>
</tr>
<tr>
<td>SA</td>
<td>1478</td>
<td>26.43</td>
<td>49.45678</td>
</tr>
</tbody>
</table>

Table 1 Comparisons of results

![](image)

Figure 3 Simulate results
process to be adapted to the features of the problems are our future work.

References