Synchronous Generator-Rectifier Average-Value Modeling for a Naval Electric Power System

JURI JATSKEVICH
Electrical and Computer Engineering
University of British Columbia
Vancouver, BC V6T 1Z4
CANADA

ERIC A. WALTERS and CHARLES E. LUCAS
PC Krause and Associates, Inc.
West Lafayette, Indiana 47906
USA

Abstract: - A prototype Integrated Power System that is representative of advanced power systems of future warships is considered. This system is comprised of an AC Generation and Propulsion System as well as a DC Zonal Electrical Distribution System. Obtaining accurate average-value models of individual subsystems that can be used for extracting the input/output impedance as well as for increasing the simulation speed of the respective models. In this paper, a parametric approach for developing a dynamic average-value model of a generator-rectifier subsystem is presented. The method initially requires a detailed switched model from which the rectifier/dc-link dynamics are captured using numerical averaging; however, the resulting model is continuous and computationally efficient. The developed average-value model is compared against measured and detailed simulation results and is shown to be very accurate in both the time- and frequency-domains. A 74-fold increase in simulation speed is achieved.

Key-Words: - naval electric systems, synchronous generator, average value model, impedance, simulation.

1 Introduction

Power electronics play a vital role in the performance, reliability, and survivability of modern ships. The Office of Naval Research (ONR) reference system is a reduced-scale benchmark system, representative of future generation surface warships, and is designed to motivate interdisciplinary research [1]. This benchmark system is based on reduced scale Integrated Power System (IPS) testbeds developed under the Naval Combat Survivability initiative [2] and installed at Purdue University and the University of Missouri-Rolla. The overall IPS is comprised of an AC Generation and Propulsion System and a DC Zonal Electrical Distribution System (ZEDS) as shown in Fig. 1. The DC ZEDS, in turn, includes a Ship Service Inverter Module (SSIM) that supplies a pulsed Load Block (LB), a Motor Controller (MC), a Constant Power Load (CPL), three diode or'ing networks, and six Ship Service Converter Modules (SSCM's) that are supplied from the starboard- and port-side dc buses. Each dc bus is fed from a dedicated Power Supply (PS). For increased reliability and redundancy, the loads are divided into three zones as shown in Fig. 1. The AC Generation and Propulsion System has starboard- and port-side subsystems that are each comprised of a generator, an ac bus, and a propulsion subsystem. Both propulsion systems are comprised of a three-phase bridge rectifier, a dc filter, an inverter, and an induction motor. The details of each module, including the controllers, can be found in [3-7]. This reference system allows researchers to investigate the controls, survivability, and functionality of finite-inertia, coupled electro-mechanical systems that are representative of future warships. Specific interests include: (i) stability and dynamic performance under pulsed loads, (ii) power quality, (iii) efficiency, and (iv) survivability, which is the ultimate goal.

Computer simulation represents a powerful mechanism for predicting the dynamic performance of power electronic components and subsystems prior to their actual realization in hardware. In particular, there are numerous techniques for investigating the stability and the design of controllers that are based upon frequency-domain characteristics that can be extracted from the computer models of the respective subsystems [8-9]. However, because the detailed models are
computationally intensive, determining the impedance over a wide range of frequencies, using a frequency sweep technique for example, is a very time consuming procedure, particularly when obtaining data points at very low frequencies (<1Hz). As a result, it can be advantageous to develop average-value models for the system components wherein the effects of fast switching are neglected over the prototypical switching interval leaving the respective state variables constant, i.e. "averaged", in the steady-state. Although such models only approximate the slower dynamics of the original systems, they are continuous and can be linearized about a desired operating point. Thereafter, obtaining a local transfer-function and/or frequency-domain characteristics becomes a straightforward and rapid procedure. For example, many simulation programs offer automatic linearization and subsequent state-space and/or frequency-domain analysis tools [10-11]. In addition, average-value models can be very advantageous when performing transient and dynamic studies of large systems due to the improved computational performance as compared to switch-level models.

Thereafter, obtaining a local transfer-function and/or frequency-domain analysis tools [10-11]. In addition, average-value models can be very advantageous when performing transient and dynamic studies of large systems due to the improved computational performance as compared to switch-level models.

In this paper, a subsystem composed of the ac generator connected to the PS rectifier is considered in detail. The analytical derivation of average-value models for machine-converter systems is challenging. Initial steps in this direction for a fixed reactance behind a voltage source can be found in [12]. Approximate reduced order models have been presented in [13-14] and a dynamic model was derived in [15], wherein a very good match in the time- and frequency-domains to the detailed simulation is reported. An approach similar to [12] has also been recently used with synchronous generators [16], wherein the parameters of the averaged model were obtained from a detailed simulation. However, in addition to requiring a non-proper generator model, the rectifier model parameters were not dependent on operating conditions in [16].

The method presented in this paper extends the work of [12] and [16]. In particular, the relationship between the averaged dc rectifier variables and the averaged ac generator variables viewed in the synchronous reference frame is represented using nonlinear functions for the operating conditions. The data points for the numerical approximation of these functions are obtained by simulating the detailed model for a wide range of loading conditions. The advantages of this method are that it does not require extensive analytical derivations and that it is general and can be readily applied to other machine-converter configurations.

\[ \begin{align*}
\mathbf{V}_{abc} &= \begin{bmatrix} v_a & v_b & v_c \end{bmatrix} \\
\mathbf{V}_{qdr} &= \begin{bmatrix} 0 & r_s & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix} \mathbf{i}_{abc} + \frac{d}{dt} \mathbf{\lambda}_{abc} \\
\mathbf{\lambda}_{qdr} &= \begin{bmatrix} L_{s}\left(\theta_r\right) & L_{sr}\left(\theta_r\right) & i_{abc} \\
L_{qr}\left(\theta_r\right) & L_r & i_{qdr}
\end{bmatrix}
\end{align*} \]

where the flux linkages are defined as

\[ \begin{align*}
\mathbf{\lambda}_{abc} &= \begin{bmatrix} \lambda_{abc} \\
\lambda_{qdr}
\end{bmatrix} \\
\mathbf{i}_{abc} &= \begin{bmatrix} i_a \\
i_b \\
i_c
\end{bmatrix}
\end{align*} \]

The expressions for the self- and mutual-inductance matrices, which are rotor-position dependent, can be found in [19].

2 Detailed Model

In detailed simulations of power electronic systems [17], it is often more convenient to represent the electrical machine in terms of physical variables, rather than transformed q-d variables. Based on physical variables, an electrical machine can be represented using simple circuit elements; voltage sources, resistors, and coupled inductors. A circuit diagram of the synchronous generator–rectifier subsystem considered in this paper is depicted in Fig. 2. This subsystem has been studied previously [18]. For consistency, the system parameters are summarized in Appendices (a)-(b). The stator windings together with the rectifier represent a switched network. The rotor windings are magnetically coupled with the stator. The corresponding voltage equations in physical machine variables can be written as

![Fig. 1: Integrated Power System.](image-url)
For each switching topology of the system depicted in Fig. 2, the corresponding state equation can be assembled by partitioning the overall circuit graph into a spanning tree and link branches, and selecting the inductor link currents and capacitor tree voltages as the state variables. A basic algorithm for generating the corresponding state equations has been set forth in [20] and is generalized in [21]. The final state equation for the \( i \)-th topological state has the following implicit form

\[
\mathbf{M}'(\theta_r) \frac{\mathbf{d}\mathbf{x}'}{dt} = \mathbf{F}'(\mathbf{x}', \theta_r) + \mathbf{g}'(\mathbf{u})
\]  

(3)

where \( \mathbf{M}(\theta_r) \) is a positive-definite mass matrix, \( \mathbf{F}(\mathbf{x}, \theta_r) \) is a term that contains state self-dynamics, and the forcing term \( \mathbf{g}(\mathbf{u}) \) accounts for external inputs, e.g. excitation voltage. In order to establish an overall transient response, the initial condition for a subsequent topology is established in such a way that the currents through inductors and voltages across capacitors are continuous according to circuits laws [21].

A six-pulse rectifier with non-zero inductances on the source and/or the dc side may operate with a wide range of loads. In either case, a complete prototypical switching interval is \( \pi/3 \) radians and under 60-Hz base frequency operation, the switching interval is \( 1/360 \) sec [19]. Moreover, the simulation method considered in this paper automatically implements all operating modes [21].

### 3 Average Value Model

In contrast to the detailed model, the proposed average-value-model is developed using the traditional synchronous machine equations expressed in the rotor frame of reference. Specifically, using Park’s transformation

\[
\mathbf{K}_r = \begin{bmatrix}
\frac{2}{3} & \cos \theta & \cos(\theta - 2\pi/3) & \cos(\theta + 2\pi/3) \\
\sin \theta & \sin(\theta - 2\pi/3) & \sin(\theta + 2\pi/3) \\
1/2 & 1/2 & 1/2
\end{bmatrix}
\]

(4)

equations (1) and (2) are transformed to the rotor reference frame. The resulting equations with flux linkages per-second as the state variables are

\[
d\psi_{qs} = \omega_b \left( -r_s i_{qs} - \frac{\omega}{\omega_b} \psi_{ds} + v_{qs} \right)
\]

(5)

\[
d\psi_{qs} = \omega_b \left( -r_s i_{qs} + \frac{\omega}{\omega_b} \psi_{qs} + v_{ds} \right)
\]

(6)

\[
d\psi_{kij} = -\omega_b r_s \psi_{kij} \cdot i_{kij} \cdot \delta_{ij} \quad \text{for } j = 1,2,3
\]

(7)

\[
d\psi_{klij} = -\omega_b r_s \psi_{klij} \cdot i_{klij} \cdot \delta_{ij} \quad \text{for } j = 1,2,3
\]

(8)

\[
d\psi_{sl} = \omega_b \left( -r_s i_{sl} + \frac{\omega}{\omega_b} \psi_{sl} + \epsilon_{sl} \right)
\]

(9)

The current in each winding is computed as

\[
i_{qs} = \left( \psi_{qs} - \psi_{mq} \right) / x_{ls}
\]

(10)

\[
i_{ds} = \left( \psi_{ds} - \psi_{md} \right) / x_{ls}
\]

(11)

\[
i_{fjl} = \left( \psi_{fjl} - \psi_{md} \right) / x_{fjl}
\]

(12)

\[
i_{kij} = \left( \psi_{kij} - \psi_{mq} \right) / x_{klij} \quad \text{for } j = 1,2,3
\]

(13)

\[
i_{klij} = \left( \psi_{klij} - \psi_{md} \right) / x_{klij} \quad \text{for } j = 1,2,3
\]

(14)

where the mutual fluxes are given by

\[
\psi_{mq} = x_{aq} \left( \psi_{qs} + \frac{3}{x_{ls}} \sum_{j=1}^{3} \frac{\psi_{klij}}{x_{klij}} \right)
\]

(15)

\[
\psi_{md} = x_{ad} \left( \psi_{ds} + \frac{3}{x_{ls}} \sum_{j=1}^{3} \frac{\psi_{fjl}}{x_{fjl}} \right)
\]

(16)

and the reactances are defined as

\[
x_{aq} = \left( \frac{1}{x_{mq}} + \frac{1}{x_{ls}} + \frac{3}{\sum_{j=1}^{3} x_{klij}} \right)^{-1}
\]

(17)

\[
x_{ad} = \left( \frac{1}{x_{mq}} + \frac{1}{x_{ls}} + \frac{3}{\sum_{j=1}^{3} x_{fjl}} \right)^{-1}
\]

(18)

and \( x_i = \omega_b L_i \) for all reactances.

The dynamic average-value model is formed by establishing a relationship between the dc-link variables and the ac variables transformed to a suitable reference frame. In particular, it is convenient to consider a synchronous reference frame in which the averaged \( d \)-axis component of
the rectifier input ac voltage is identically zero [12]. The resulting synchronous reference frame and relationship of the corresponding variables is shown in Fig. 3, wherein the transformation angle $\Theta_\alpha$ is selected to ensure that $v_{ds}^e = 0$. By setting $v_{ds}^e = 0$, the $q^e$-axis is synchronized with the peak of the phase a rectifier input $(abc)$ voltages. Here, the superscripts “$e$” and “$r$” denote the synchronous and rotor reference frames, respectively; whereas, the bar symbol denotes the so-called fast average evaluated over a prototypical switching interval. The averaged generator voltages expressed in the rotor reference frame are $v_{qs}^r$ and $v_{ds}^r$. Based on Fig. 3, the relationship between the voltages in the rotor and synchronous reference frames is

$$
\begin{bmatrix}
v_{qs}^e \\
v_{ds}^e 
\end{bmatrix} =
\begin{bmatrix}
\cos(\delta) & \sin(\delta) \\
-\sin(\delta) & \cos(\delta)
\end{bmatrix}
\begin{bmatrix}
v_{qs}^r \\
v_{ds}^r
\end{bmatrix}
$$

(19)

It is assumed that the fundamental component of the generator currents has a lagging power factor and that the current vector $\bar{i}_{qds}$ lags the voltage $v_{qds}^e$ by an angle $\phi$.

![Fig. 3: Generator-rectifier variables in the rotor and synchronous reference frames.](image)

The next step is to relate the averaged rectifier dc current $\bar{i}_{dc}$ to the current vector $\bar{i}_{qds}^e$, and the averaged output voltage $\bar{v}_{dc}$ to the rectifier voltage vector $v_{qds}^e$. Assuming that the rectifier does not contain energy-storing components, it is reasonable to approximate these relationships as

$$
\|v_{qds}^e\| = \alpha \bar{v}_{dc}
$$

(20)

$$
\bar{i}_{dc} = \beta \|\bar{i}_{qds}^e\|
$$

(21)

were $\alpha$ and $\beta$ are algebraic functions of the loading conditions. The angle between the vectors $v_{qds}^e$ and $\bar{i}_{qds}^e$ can be expressed as

$$
\phi = \arctan\left(\frac{\bar{i}_{qds}^e}{\bar{i}_{qds}^e}\right) - \delta
$$

(22)

The functions $\alpha$, $\beta$, and $\phi$ may be specified in terms of impedance defined as an operation point

$$
z = \frac{\bar{v}_{dc}}{\|\bar{i}_{qds}^e\|}
$$

(23)

Obtaining analytical expressions for $\alpha(z)$, $\beta(z)$, and $\phi(z)$ is extremely difficult. The approach taken here utilizes a detailed simulation as described in the previous section. In particular, the system of Fig. 2 has been connected to a resistive load that was varied across a wide range of values in order to obtain $\alpha(z)$, $\beta(z)$, and $\phi(z)$ that are valid for various operating conditions; wherein, variables in (20)-(23) were obtained by averaging the respective steady-state currents and voltages over the rectifier switching interval. The resulting functions are plotted in Fig. 4, wherein, it can be observed that the functions are nonlinear, particularly at heavy loads. These functions may be stored in a look-up table or fitted into a spline, for example.

![Fig. 4: Functions $\alpha$, $\beta$, and $\phi$.](image)

If a filter inductor $L_f$ is present in the system, the average-value model must incorporate the effect of this inductor. The steady-state effect due to resistive voltage drop across $L_f$ is incorporated into $\alpha(\cdot)$ and $\beta(\cdot)$. The dynamic effect of $L_f$ does have an impact on the output impedance, particularly in the higher frequency range. In order to account for this effect, the rectifier dc voltage is related to the capacitor voltage in the frequency domain as

$$
v_{dc} = v_c + H_L(s)\bar{i}_{dc}
$$

(24)
In order to avoid the numerical differentiation when implementing (24) in the time domain, $H_L(s)$ must be proper. Therefore, in order to represent the dynamics of the inductor $L_f$ in a range up to the rectifier switching frequency, it is assumed that

$$H_L(s) = \frac{L_f s}{s^2 + \tau + 1}$$

(25)

where $\tau$ is a time constant selected small enough to ensure that its effect at the switching frequency is negligible ($\tau = 10^{-5}$ has been used herein).

In the proposed model, the variables are evaluated as follows. The impedance $z$ is computed first and the functions $\alpha$, $\beta$, and $\phi$ are evaluated. Based on $\phi$, the rotor angle is computed using

$$\delta = \arctan \left( \frac{i_d}{i_q} \right) - \phi(z)$$

(26)

The dc-link current is computed using

$$i_{dc} = \beta(z) \| i_{qdc} \|$$

(27)

The generator voltages are expressed, using (20) and the vector relationships depicted in Fig. 3, as

$$v_{qg} = \alpha(z) v_{dc} \cos(\delta)$$

(28)

$$v_{dg} = \alpha(z) v_{dc} \sin(\delta)$$

(29)

where $v_{dc}$ is computed using (24).

4 Computer Studies

The detailed state model of the synchronous generator-rectifier subsystem described in Section 2 has been implemented in MATLAB/Simulink\textsuperscript{TM} as a component of the overall IPS [17]. The details of the implementation as well as the user interface are described in detail in [17], [21]. The system shown in Fig. 2 is defined in terms of branches with rotor-position-dependent inductances, whereas the appropriate switching logic is implemented to model the rectifier circuit in valve-by-valve detail assuming idealized on/off switching characteristics. The resulting detailed model was used as a benchmark in subsequent studies.

The proposed average-value model has also been implemented in Simulink using standard library blocks. In order to fully compare the average-value model to the detailed simulation, the respective models were compared first in the time-domain and then in the frequency-domain. In all cases, a constant excitation and generator speed were assumed.

4.1 Time-Domain Transient Study

In the following study, the system starts with zero initial conditions and the dc load is a constant resistor. After a steady-state condition is reached at time $t = 1.5$ s, the load resistor is stepped to 1/5 of its original value. The computer generated transient response observed in the dc current and the generator field current for the detailed and average-value models is shown in Fig. 5. As expected, the average-value model does not portray the ripple due to rectifier switching; however, the response produced by the average-value model matches the detailed transient in the “averaged” sense very well including the step-load transient.

In order to compare the computational complexity of the two models, the numerical statistics obtained for the same transient study run on a PC with AMD Athlon\textsuperscript{TM} 2200 processor is summarized in Table 1. The integration parameters were adjusted in order to achieve the maximum simulation speed for both models without noticeable deviation from the reference solution shown in Fig. 5. In both cases, the variable-step solver *ode15s* was found to be a preferred choice due to the system stiffness. Although both models have full order and similar dynamic properties, the average-value model is continuous, thereby allowing the solver to take much fewer integration steps. This explains the 74-fold increase in simulation speed achieved by the average-value model (0.13s) as compared to the detailed model (9.67s).

![Fig. 5: Simulated transient for the time-domain study.](image)

Table 1: Numerical statistics for the reference study.

<table>
<thead>
<tr>
<th>Model</th>
<th>Relative tolerance</th>
<th>Absolute tolerance</th>
<th>$t_{max}$, sec</th>
<th>Number of steps</th>
<th>CPU time, sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Detailed</td>
<td>1e-3</td>
<td>1e-3</td>
<td>2e-3</td>
<td>51,623</td>
<td>9.67</td>
</tr>
<tr>
<td>Average</td>
<td>1e-3</td>
<td>1e-3</td>
<td>2e-3</td>
<td>1,159</td>
<td>0.13</td>
</tr>
</tbody>
</table>
A fragment of the measured dc and field currents are plotted in detail in Fig. 6. The corresponding fragment of the simulated response of Fig. 5 is shown in Fig. 7 on a smaller time scale. From these plots, the very close match between the measured and simulated responses can be observed. Moreover, the accuracy of the average-value model in portraying the system dynamics is demonstrated.

4.2 Impedance Characterization

The developed average-value model should exhibit the same frequency-domain characteristics as the original system. The output impedance of this generator-rectifier subsystem has been considered for verification of the analytically derived averaged model [15], wherein, an excellent agreement among the impedance curves obtained from the measured data, the detailed model, and the analytically derived averaged model was reported. For consistency, the detailed and average-value models utilized in this paper are compared against the measured results. The measured output impedance, phase and magnitude, are plotted in Fig. 8 (‘+’ points). Since the detailed model contains switching and is discontinuous, a small-signal injection and subsequent frequency sweep method has been implemented to extract the required impedance data. As can be seen in Fig. 8, the measured (‘+’) and detailed simulation (‘o’) impedance curves are very close. The impedances are evaluated in the frequency range from 1 to 200 Hz. For frequencies closer to the rectifier switching frequency (360 Hz), the results become distorted due to the interaction of the injected signal with the rectifier switching. In general, considering frequencies close to and above the switching frequency has limited use for the average-value model since the basic assumptions of the averaging are no longer valid [22].

Since the average-value model is continuous, the required output impedance can be extracted using linearization techniques as well as the frequency sweep (both yielding identical results). Because the average-value model has been developed based upon the detailed model, the impedance curves produced by the detailed model (‘o’) are considered as a reference. As can be seen in Fig. 9, the frequency response extracted from the average-value model very closely approaches that of the detailed model.

5 Conclusion

In this paper, an average-value model of a synchronous generator-rectifier subsystem that is a part of a reduced-scale naval integrated power system testbed has been presented. In the proposed method, the synchronous machine is implemented using a proper classical dq state-model form. The functions defining the relationship between the averaged dc-link variables and the generator currents and voltages viewed in the synchronous reference frame were represented as nonlinear functions of the operating conditions. Although establishing the correct averaged model requires simulating the detailed model over a wide range of loading conditions, the resulting model, once established, is continuous and valid for linearization
and small-signal impedance characterization as well as for large-signal time-domain studies. The simulation of the average-value model is shown to be 74-times faster than that of the original model. The reduced simulation time can be especially useful for dynamic studies at the system level wherein the size of the system can result in a simulation that is computationally intense and extremely slow.

**Appendix**

a) Parameters of the synchronous machine

\[ p = 4 \text{ poles}; \quad r_s = 0.382 \Omega; \quad L_{ds} = 1.12 \text{ mH} \]

\[ L_{mq} = 24.9 \text{ mH}; \quad L_{md} = 39.3 \text{ mH} \]

\[ r'_{kq1} = 5.07 \Omega; \quad L'_{kq1} = 4.21 \text{ mH} \]

\[ r'_{kq2} = 1.06 \Omega; \quad L'_{kq2} = 3.50 \text{ mH} \]

\[ r'_{kq3} = 0.447 \Omega; \quad L'_{kq3} = 26.2 \text{ mH} \]

\[ r'_{kd1} = 140 \Omega; \quad L'_{kd1} = 9.87 \text{ mH} \]

\[ r'_{kd2} = 1.19 \Omega; \quad L'_{kd2} = 4.91 \text{ mH} \]

\[ r'_{kd3} = 1.58 \Omega; \quad L'_{kd3} = 4.52 \text{ mH} \]

\[ r'_{ld} = 0.112 \Omega; \quad L'_{ld} = 1.53 \text{ mH} \]

stator-to-field turns ration \( N_s/N_f = 0.0269 \)

b) DC link filter parameters

\[ L_f = 1.19 \text{ mH}; \quad r_f = 0.32 \Omega; \quad C_f = 2.28 \mu\text{F} \]

**References:**


