Stability and transient-behavioural assessment of power-electronics based dc-distribution systems.

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Abstract: - Power electronic systems are becoming increasingly important in marine engineering applications and their capability is stimulating new concepts and developments in electrical machines and distribution systems. A consequence is the increasing use of DC fed inverters. One problem with dc-based systems is that they are susceptible to instability, particularly when a constant-power regime is operating. This paper demonstrates that an approach to this problem, based on the root-locus and frequency domain techniques, is well worth consideration.

Key Words: DC Distribution, Stability, Root Locus, Nyquist Plot, Bode Diagram, Time & Frequency Domain

1 Introduction

Developments in power-electronic devices in the past couple of decades have created a renaissance of interest in researching, designing and testing novel electrical machines and power systems. Much of the new thinking, generated by these activities, is of considerable current and potential interest to the marine engineering community. The form of power distribution for propulsion and ships services is a key element of the overall architecture for any marine electrical power system and it is being explored at present whether this should be ac-ac, ac-dc, dc-ac or dc-dc. Much of the evolving thinking, so far as the marine industry is concerned, is chronicled in a series of papers, spreading over a decade now, on the Electric Warship [4,5,6,7,8,9]. Many conventional methods of analysis and design are not readily applicable to these proposed systems. One of the problems with dc systems is that of stability. They have a propensity to exhibit negative-impedance instability, particularly when seeking to supply constant power loads [10]. There has been some significant work over the last 15 years, or so, in this area by, for example, Middlebrook [10],and Sudhoff et al [11] which is based, essentially, on frequency-domain techniques. This paper presents a method for examining stability using root locus and frequency domain methods.

2 The negative impedance

It is easily appreciated why negative-impedance instability, in a dc system, might be of concern by considering a system delivering constant-power to a load. In this case we have, using the usual notation,

$$P = IV = \text{constant}, \qquad (1)$$

Geometrically this can be represented by the branch, in the first quadrant, of a rectangular hyperbola in the V-I plane as in Figure 1. Taking differentials of equation 1, we have.

$$\Delta P = V\Delta I + I\Delta V = 0,$$

or,

$$\frac{\Delta V}{\Delta I} = -\frac{V}{I} = -R, \text{say}, \qquad (2)$$

That is, the incremental resistance is negative and further varies in magnitude with the point of operation of the system.

3 Basic circuit arrangement

The basic circuit arrangement considered is that shown in Fig. 2, where, in effect, the voltages and currents shown are the incremental values. It is firstly necessary to establish an expression showing how the value of V, the voltage existing across the terminals where the source and load meet, behaves, using perturbational quantities about some given operating point. This is easily done, for example, using Millman's circuit theorem [14] whereby there results,

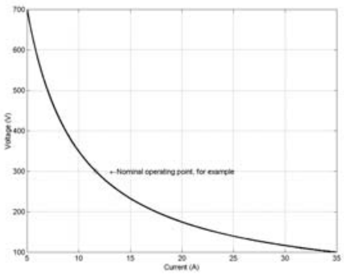

Figure 1: Constant Power Operating Characteristic

$$V = \frac{(V_s.Y_s + V_l.Y_l)}{(Y_l + Y_s)} \qquad (3)$$

where,

$Y_s = D_s/N_s$ and $Z_s = N_s/D_s$

$Y_l = D_l/N_l$ and $Z_l = N_l/D_l$

The N_i's and D_i's are polynomials in s, the Laplace variable. Substitution of these in equation 3, after some minor algebraic manipulation, leads to

$$V = \frac{N_\ell D_s V_s + N_s D_\ell V_\ell}{N_\ell D_s (1 + Z_s Y_\ell)}$$

Since neither N_l nor D_s have any roots in the right-half phase, stability is determined by the roots of,

$$1 + Zs\ Y_\ell = 0 \qquad (4)$$

For a stable system all these roots must be in the left-half of the complex plane. Determining the roots of such an equation, as one of the parameters is varied, is known as determining the root-locus [12].It is convenient to do this graphically (nowadays utilising computer graphics). This technique has been exploited in the main-stream of control engineering work for decades. It is, of course, known as the root-locus technique (13). However, applications are not restricted to control activities.

4 The Circuit

The circuit discussed by Sudhoff et al is shown in Fig. 3 where the voltage across the load is easily established to be governed by the equation

$$V = \frac{RV_s}{CRLs^2 + (CRr - L)s + (R - r)} \qquad (5)$$

Note that, R,C,L, r are all positive quantities. Thus the stability can be examined by finding the position of the roots of the denominator of equation 5 in the complex plane. However, before this is done it is worthwhile doing a Routh-Hurwitz analysis [12] to determine the limits of stability as specified by relationships between the

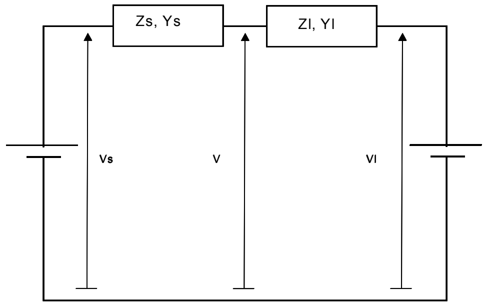

Figure 2: The basic circuit arrangement

parameter values. Consider the Routh-Hurwitz table for this denominator.

s^2	CRL	(R-r)
s	CRr-L	0
s^0	(R-r)	

According to the Routh-Hurwitz criterion the condition, for stability is that there should be no changes in sign of the items in the first column of the table. Since CRL must be positive then (CRr-L) and (R-r) must also be positive for stability

That is:

$$C > \frac{L}{Rr} \text{ and } R > r.$$

Critical conditions occur when these inequalities change to equalities. This result could be obtained from quadratic equation theory ; however, for higher-order systems the Routh-Hurwitz criterion can be a very effective and useful technique, hence the demonstration here. The above two inequalities are, of course, exactly as those specified by Sudhoff, [11]. Unfortunately the Routh-Hurwitz criterion

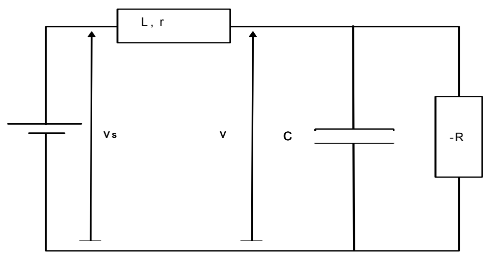

Figure 3: The Circuit

indicates only if a system is stable or not, it does not give any other notion of the system's behaviour. And this is exactly where the root-locus demonstrates not only under what conditions the system is stable, but gives information on the margin of stability and of the characteristic response to be expected from the system. Both of these, of course, depend on the root locations in the complex plane.

From the circuit arrangement in Fig. 3, the component values chosen were r = 300mOhms, L = 10mH and R = 24.3 Ω, where the system is delivering, under constant power conditions, 3.7 kW at 300V, fig. 1. The problem is to determine suitable values of the capacitor, C, to produce acceptable behaviour. We can see that, from the above inequalities, the R>r condition is satisfied, and that the critical value of C is given by,

$$C = \frac{L}{rR} = 1.37\text{mF}$$

For this example, in Sudhoff's paper [11], using a development of the Nyquist criterion, it is concluded that C = 40mF is a stable situation, and, C = 0.5mF, represents an unstable situation. This is exactly what would be expected from the above result.

Obviously, for stability of some acceptable degree, C, must be greater than 1.37mF. The question remains as to what values of, C, give desired or, at least, acceptable behaviour.

It would be convenient if there was a graphical way of displaying how the roots of the governing equation vary as C is varied. This is precisely what the root-locus plot achieves. These roots being, of course, the roots of the characteristic equation for the system.

Now the denominator of equation 5, when equated to zero, is the Laplace transformation of the characteristic equation, viz,

$$RCLs^2 + (CRr - L)s + (R - r) = 0 \qquad (6)$$

This equation is not presented in a form suitable for the application of the root-locus technique, but simple manipulation renders it into such a form; .i.e

$$C[RLs^2 + Rrs] - sL - r + R = 0$$

or,

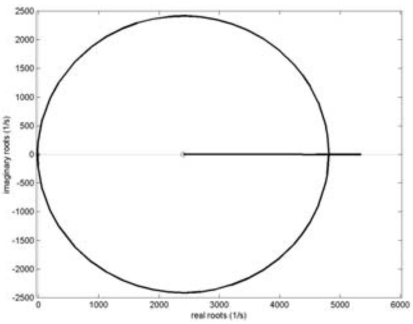

Figure 4a: Root Locus for varying C

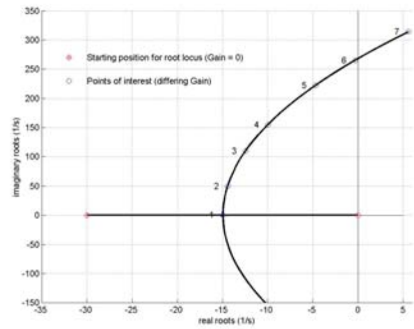

Figure 4b: Detail of Figure 4a

$$1 + \frac{(-Ls + R - r)}{CRs(Ls + r)} = 0, \qquad (7)$$

which has the form used in control engineering, shown previously, as,

$$1 + k\frac{N(s)}{D(s)} = 0,$$

with the 1/C term playing the role of the k term. This is also the form required in exploiting MATLAB. At this stage the root-locus commands for the computer programme MATLAB (version 5.3) were utilised in displaying this root-loci on a VDU. The plot resulting can be interrogated for specific value of k (i.e.1/C) and the corresponding roots. This whole process can be done with less than ten lines of computer code.

So entering the specific values of the coefficients of the numerator and denominator polynomials of equation (7), for the problem in hand, in to the **rlocus (n,p)** command of MATLAB, leads to Fig. 4a. Because of the values of the coefficients, this plot is interesting but not particularly useful for the purpose required. The values of the two poles of the equation 7 are actually situated on the real axis at -30 and 0 but they look to be coincident in Fig. 4a. It can be seen, however, that only a small portion of the root-locus, that to the left of the imaginary axis, indicates a stable region. By using the zoom facility provided by MATLAB the area of interest, as shown in Fig. 4b, can be examined in detail. Since roots of the equations, if complex, occur in complex pairs the part of the root-locus below the real axis is shown notionally.

The **rlocfind** command provided by MATLAB prints out the value of any roots specified by the cursor on the computer screen, as well as the value of k (or 1/C in our case). Fig. 4b shows seven specific points on the graph - the figure legend specifies the values of C found for these particular points and the corresponding damping factor [12].

What is seen immediately from this plot is that increasing the capacitor value increases the damping effect and decreases the frequency of the oscillation. As will become apparent, roots along the real axes are achievable only with very large, and unrealistic, values of capacitance.

Although one can glean from the position of the roots the sort of response expected from different values of C, MATLAB has a facility for displaying the response to a unit-step and unit-impulse disturbing a system. In the present application we wish to disturb the system in some way to observe what happens. As an illustration this contribution shows what happens when a step-disturbance is introduced in the supply voltage. Because of the MATLAB facilities this can be simulated with very little effort on the designer's part.

Fig. 5a predicts this response when C = 0.44F. The response for this value is critically-damped, ie. it has two equal real roots. However, this capacitor value is very large. Although the root-locus has branches along the real axis, the roots here can only be obtained with even larger capacitor values, and so are of very limited interest in practice. Fig 5b predicts the response when employing a capacitor of 8.1mFvalue. The response is now oscillatory, over-shooting by about 60%but the transient has virtually died out in 0.45 s.. Fig 5.c is drawn for a value of C equal to 2mF and the frequency of the oscillation has increased markedly from that of 8.1mF case, Fig 5.b and it is seen to have a much more oscillatory character. The overshoot is some 95%. Fig. 5d shows the result obtained using a capacitor of value C=4.1 mF. The overshoot is about 82%, but again the transient is effectively over in 0.5s. The value of the capacitor at which sustained oscillation is predicted is C = 1.37 mF. Fig. 5d indicates the response for a value of capacitor of1.4mF, slightly above this critical value, but continuous oscillation is seen to be very close indeed.

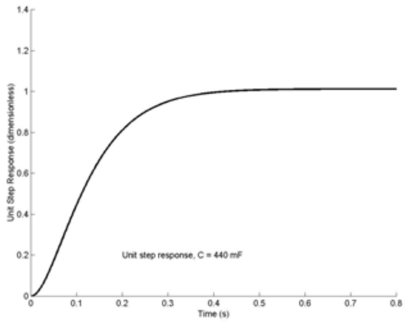

Figure 5a: Time response. C = 0.44 F

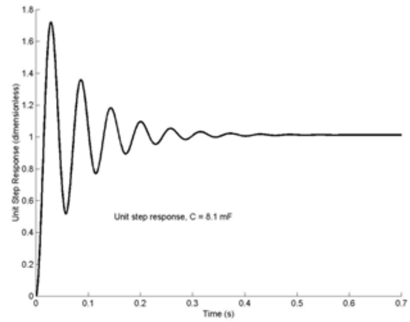

Figure 5b: Time response. C = 8.1 mF

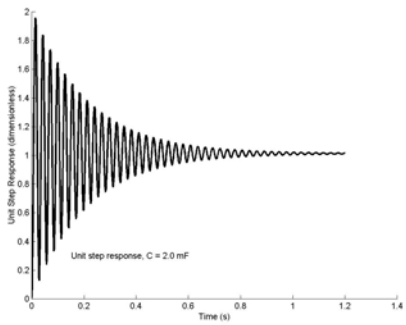

Figure 5c: Time response. C = 2 mF

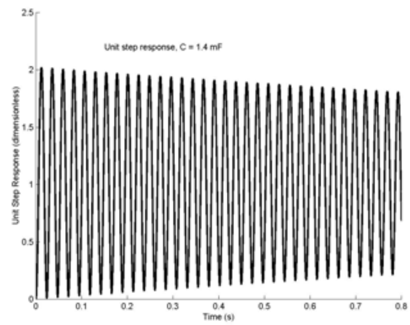

Figure 5d: Time response. C = 1.4 mF

This series of graphs showing the behavioural changes with decreasing capacitance is what one could roughly predict from a preliminary viewing of the root-locus curve. The root-locus not only indicates critical values of components to ensure stability, but indicates at a glance, the main characteristic features of the responses to be expected. Two features are particularly interesting. The first is that a change of capacitance from 0.44F to 20mF changes the real-part of the characteristic roots from $15s^{-1}$ to $14s^{-1}$. However, the imaginary parts change markedly. The second feature is that in the stable region where the locus is moving rapidly towards unstable behaviour, for example, from 20mF to1.4mF, the decay decreases very markedly, but the oscillatory nature of the responses do not change much in frequency.

5 Frequency Domain Techniques

The root locus is essentially a time domain analysis technique and indeed one of its main advantages is in being able to develop some understanding of the likely time domain behaviour. The root locus assumes that an approximate knowledge of the system's governing equations are known, which may not be the case, and, even if these are known, then for higher order systems the technique becomes unwieldy and therefore less attractive. However, these disadvantages can be overcome by using other control engineering techniques such as Bode diagrams or Nyquist Plots. These frequency domain techniques can again be used in the non-control context of electrical power systems. As an illustration, this section will use the same circuit given in Fig 3. The celebrated Nyquist stability criterion deals with determining whether, or not, a polynomial equation written in the form,

$$1+G(s)=0,$$

has roots situated in the right-half plane of the complex plane [12]. The criterion was originally developed in the 1930's as a means of predicting the stability of amplifiers employing feedback. It subsequently became established as a major topic in feedback control studies, where G(s) generally represents the open-loop transfer function of the system. Nyquist theory development involves the application of complex-variable theory – in particular contour integration. Mathematically it is a topic in polynomial theory, and feedback systems analysis may be viewed as an application of this theory. The practical significance of the Nyquist theory is that by plotting on the complex plane, the frequency-response function locus $G(j\omega)$, as ω goes from 0 to ∞, and noting this locus's disposition, relative to the $(-1, j0)$ point, enables the stability to be assessed. Simple rules have been established in the control literature for actually doing this [13]. These rules differ depending on whether, G(s), represent a minimum phase or non-minimum phase system. In addition, if the system is stable, then the closeness of approach of the $G(j\omega)$ contour to the $(-1, j0)$ point gives a notional indication of the system's response, to a disturbance, that is to be expected, e.g. oscillatory, sluggish, etc. Whilst the Nyquist diagram is excellent for qualitative discussion of system behaviour, it is not particularly convenient for quantitative studies. For these, Bode diagrams are superior normally (12).

Note that $G(j\omega)$ can be written in polar form as,

$$G(j\omega)=\mathrm{H}.exp(j\theta),$$

where, H and θ are both functions of ω, the frequency. Also note that

$$\ln G(j\omega)=\ln \mathrm{H}+j\theta.$$

Bode diagrams are no more than a plot of H (in dBs) against the log ω, and a second plot of θ (usually in degrees) against log ω. Normally these two plots are plotted one above the other, Fig. 6 shows the Bode Plot for a case of C = 1.4mF which matches the root locus plot shown in Fig 5.d. The advantages of going to this seemingly complex plotting procedure are discussed, in detail, in almost every elementary control text book [12]. Suffice it to say that it is just an equivalent way of drawing a Nyquist plot. If consideration is given only to the condition where the denominator of, G(s), has no positive real-part roots, a common condition, then there are two points on the Bode diagrams of particular significance. The first is the value of the gain at the frequency where the phase has reached –

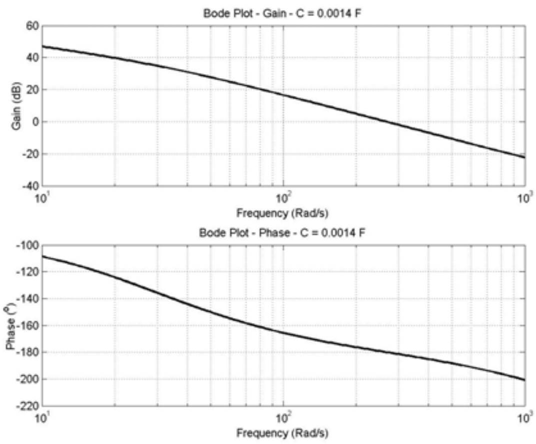

Figure 6: Bode Plots for C = 1.4 mF

180^0. For stability, this gain must be less than 0dBs, at this frequency. This value is known as the **gain-margin**, i.e. it is the increase in gain required to make the system go unstable. In a similar fashion the **phase-margin** is measured at the frequency where the gain crosses the 0dB point, and it is the increase in negative phase that would be required to make the phase -180^0, at this cross-over frequency. Thus for stable systems both the gain- and phase-margins must be positive. As can be seen from Fig 6, the value of C = 1.4mF is close to instability, both phase and gain margins are close to zero. While, generally, it is difficult to provide a quantitative correlation between these two margins and the time behaviour of the system, various rules-of-thumb have been devised. For example, some given values of gain- and phase-margins combinations are known to produce, usually, acceptable time responses. Indeed, specifications for control system performance are often drawn up in terms of these two parameters. Further if the system response is dominated by 2 complex conjugate roots it is possible to estimate the damping factor and ω_n from ζ and ϕ_m [15].

6 Other Control Engineering Techniques

This paper has provided a practical demonstration of the application of traditional control engineering techniques to an electrical power system and used a simple dc circuit with an inherent constant power instability as an example. It is to be recognized that the techniques illustrated here can be applied to other forms of electrical power system. It should also be noted that the remaining control engineering tools can also be applied. This work is continuing and is being more fully reported in the Proceedings of the Institute of Marine Engineering, Science & Technology . Other developments yet to be reported include parameter uncertainty or variation, compensation and non-linearity.

7 Conclusion

It has been demonstrated that the root-locus technique can be a useful tool in dealing with design and stability assessment of dc distribution systems. This has been verified using a particular example, but is generally applicable to any such system whose behaviour may be assumed linear (at least around some operating point), and whose dynamical equation is known. The availability of computer programs, such as MATLAB, enables the process to be carried out without any calculation being done by the designer. Information concerning suitable parameter values and the corresponding characteristic-root values are immediately available by interrogating the displayed root-locus. The response of the system to step changes in, say, the supply voltage is also immediately available using the MATLAB software. It has also been demonstrated that Frequency Domain techniques such as Nyquist Plots and Bode Diagrams can also be applied to an electrical power system and that the Bode Plot in particular can be used to gain quantified and physically meaningful insights into the conditions of stability of the circuit in question.

A further advantage of the frequency response approach is that, if measured frequency response data is available, the design may be done using these directly i.e. the mathematical form governing the behaviour may be unnecessary.

References:

[1] Voyce, J. and Norton, P. "The advancement for the AIM system", IMarEST INEC., 2002.

[2] Hodge, C G., Mattick, D.J., and Voyce, J., "The transverse-flux machine", 3rd Naval Symposium or Electrical Machines, Philadelphia, USA, 2000

[3] Pollock, C., 'Low-cost switched-reluctance drives", Trans IMarE, Vol. 110, part 4, 1998

[4] Hodge, C.G. and Mattick, D.J. "The electric warship", Trans IMarE, Vol 108, part 2, 1996

[5] Hodge, C.G. and Mattick, D.J. "The electric warship II", Trans IMarE, Vo1109, part 2, 1997

[6] Hodge, C.G. and Mattick, D.J. "The electric warship III", Trans IMarE, Vol 110, part 2, 1998

[7] Newell, J, Mattick, D J & Hodge, C.G. and Mattick, D.J. "The electric warship IV", Trans IMarE, Vol 112, part 2, 1999.

[8] Hodge, C.G. and Mattick, D.J. "The electric warship V", Trans IMarE, Vol 113, part 2, 2000.

[9] Hodge, C.G. and Mattick, D.J. "The electric warship VI" Trans IMarE, Vo1 114, part 2, 2001

[10] Middlebrook, RD. "Input filter considerations in design and application of switching regulations", IEEE Proc., IASAM, 1976.

[11] Sudhoff, S.D., Glover, S.F., Lamm, P.T, Schmucker, D.H., and Delisle, D.E. "Stability analysis of power electronics distribution systems using admittance space constraints", IEEE Trans. On Aero. and Elect. Sys., Vol 36, No 3, pp 965-973, 2000

[12] Dorf, R C., and Bishop, RH. "Modern Control Systems", 7th edition, Addison Wesley, 1995

[13] Evans, W.R "Graphical analysis of control systems", Trans .AIEE, Vo1 67, pp 547-551, 1948

[14] Bogart, TF., "Electrical Circuits", Glencoe, 1992.

[15] Flower J O & Hodge C G, "Stability and transient-behavioural assessment of power-electronics based dc-distribution systems. Part2: the frequency response approach" to be published, IMarEST Proceedings Part A, 2004.