Abstract: - This paper investigates the Modeling and the Analysis of the electric field distribution in air gaps, stressed by the breakdown voltages, under different geometries and arrangements, with Finite Element Method. The influence on the electric field distribution, caused by the insertion of a dielectric insulated plate, called Barrier, between the electrodes is presented. The average, the maximum and the minimum value of the field strength along the axis of the gap, when the breakdown voltage stresses the air gap are recorded and analyzed. The results show that the distribution of the field along the gap axis is strongly affected by the geometry and the arrangement of the gap. The average and the minimum value of the field strength, decreases when the gap length increases, while the maximum value of the field strength or the field factor in the gap functions differently, strongly depending on the gap geometry. In a non-uniform field arrangement, such as rod - plane, a charged dielectric plate, placed between the electrodes of the air gap, influences the distribution of the electric field. The simulation results are compared with relative results of referent work.

Key Words: Breakdown Voltages, Air Gaps, Numerical Analysis, Field strength, Barrier Effect.

1 Introduction

The study of the field strength distribution in electric fields, and especially in non-uniform electric fields like the fields in rod - plane and rod - rod air gaps, is of great importance in high voltages, because one of the most determinant factors of the dielectric strength of the insulating materials is the field strength distribution inside the mass of the materials, when stressed by high voltages.

The experimental measurement of the field strength in small air gaps stressed by High Voltages is a very difficult process and not quite accurate. The insertion of a measuring arrangement (e.g. a probe), especially near the stressed electrode, is not an easy process and it affects the distribution of the field, influencing the results. The analysis of the proper models of the air gap electric fields using Laplace’s and Poisson’s equations for general 2 or 3-dimensional fields with a theoretical - mathematical way is more accurate but it is time consuming and in some cases it leads to difficulties. The most convenient way is to use numerical procedures.

This paper investigates the Modelling and Analysis of the electric field distribution in air gaps at breakdown, under different geometries and arrangements of the gaps, using the Finite Element Method. The influence on the electric field distribution, caused by the insertion of a dielectric insulated plate, called barrier, between the electrodes in a non-uniform electric field arrangement is also analysed and presented. The simulation results are compared with relative experimental or theoretical results of referent research works [5], [7], [9], [12].

2 Theoretical approach.

Software Quick field developed by Terra Analysis has been used in the present paper, which is based on the Finite Element Method in order to solve two-dimensional problems, with plane and axisymmetric models. [1], [2], [3], [4], [17]

The program is based on Gauss’s and Poisson’s equations:

\[ E = - \nabla U \]  \hspace{1cm} (1)

\[ \nabla D = \rho \]  \hspace{1cm} (2)

or

\[ \nabla^2 U = \rho / \varepsilon, \]  \hspace{1cm} (3)

where \( \rho \) is the space charge density in C/m\(^3\), \( \varepsilon \) is dielectric constant of the medium, \( U \) is voltage, and \( D = \varepsilon E \), is the dielectric displacement. The electric
charge density, and total Electric charge in a particular surface $S$, or volume included in surface $S$, is calculated by equations:

$$q = AD_n, \quad \text{and} \quad Q = \int D_n \, ds \quad (4)$$

The boundary conditions and especially the mesh density used for the analysis are of great importance for accurate results. Space charges have not been taken into account at present work.

### 3 The simulated models.

The models that have been drawn and analyzed are typical sphere –sphere, rod - plane and rod - rod air gap arrangements of different geometries. The sphere electrode is an almost full sphere or a hemisphere. The rod electrode is a cylinder long enough, with a small diameter, and a hemisphere tip. The plate electrode is a disk plate, with a relatively big diameter. One electrode of each arrangement is stressed by high voltage and the other is grounded. In rod-plate arrangements an insulating dielectric barrier is located between the electrodes. The insulating barrier is a dielectric disk plate perpendicular to the axis of the gap, with very small thickness (less than 1 mm), located in a distance $L$ from the stressed electrode. The dielectric constant of the barrier is $\varepsilon_{rb}$.

All the models are axisymmetric with a spherical boundary surface. The radius of the boundary sphere is large enough (at least 10 times the arrangement dimensions) in order to eliminate errors. The boundary Dirichlet condition is $V=0$. [1], [17]

![Figure 1. Sphere - sphere air gap.](image1)

![Figure 2. Rod - plate air gap.](image2)

![Figure 3. Rod - plate air gap with barrier.](image3)

![Figure 4. Rod - rod air gap.](image4)

The average value of field strength, along the axis of an air gap is defined as the applied voltage $V$, divided by the length of the gap $G$.

$$Eav = \frac{V}{G} \quad (V/m) \quad (5)$$

The field factor (or efficiency factor) $n$ is a net number, produced by the division of the maximum field strength $E_{max}$ along the axis of the air gap, divided by the average field strength $E_{av}$ of the gap.

$$n = \frac{E_{max}}{E_{av}} \quad (6)$$

For a sphere-sphere air gap the field factor is calculated from equation

$$n = \frac{G}{(R+1)+\sqrt{(G/R+1)^2+8}/4}, \quad (7)$$

or $$n = \frac{G}{2R}, \quad \text{if} \ G >> R \quad (8)$$

and for a rod – plate gap from equation

$$n = 2*G/(r*\ln(4G/r)), \quad \text{if} \ G >> r \quad (9)$$

where $G$ is the gap length, $R$ is the radius of the sphere, and $r$ is the radius of the tip of the rod. [4]

### 3.1 Models of air gaps without barrier

Sphere – sphere, rod – plate and rod - rod arrangements, with different dimensions of the
sphere, the plate and the rod, different length of the gap and no barrier between the electrodes have been modeled and analyzed. The field distribution along the axis of the gap is demonstrated in figures below, in relation to the breakdown voltages. Most of the values of the breakdown voltages are measurement data from experimental referent work. [5], [6], [7], [8], [9], [11], [12]

From figures 5 and 6, it results that in the rod-plate arrangement the field is much more inhomogeneous than in the rod-rod and sphere-sphere arrangements. With the same value of average field strength, the maximum value of the field strength is much more higher in the rod-plate air gap, and especially in the arrangement with grounded plate. In the rod-plate air gap with grounded rod, the field is less inhomogeneous. The inhomogeneity of the field is one of the basic reasons for the breakdown voltage to have different values for arrangements with the same gap length, but different geometry. This is shown in figure 7, for 3 different air gap arrangements.

3.1.1 The sphere-sphere air gaps.
In the sphere – sphere air gaps the field along the axis is almost homogeneous. The very small inhomogeneity of the field depends mainly on the gap length, and secondary on the sphere diameter. The models, which have been analyzed, are models with sphere diameters of 100 and 250 mm, for which the values of the breakdown voltage are known. It results from the analysis that the breakdown voltage of the gap increases almost linearly with the gap length, while the field strength distribution depends mainly on the gap length (figure 8).

From figure 9, it is obvious that the maximum value of field strength at breakdown voltage changes slightly with the gap length.
The values of the breakdown voltage are from referent work.
Figure 8. Field strength distribution along the axis of a sphere-sphere air gap for different lengths of the gap, at the breakdown voltage is shown.

Figure 9. The maximum, the minimum, and the average value of the field strength along the axis of a sphere-sphere air gap are shown, in relation to the breakdown voltage of the gap (Vb).

The maximum value of field strength (between 30 and 33 KV/cm) appears on both electrodes. It depends mainly on the sphere diameter and changes slightly with the gap length. The average field strength decreases almost linearly with the gap length.

Figure 10. The maximum, the minimum, and the average value of the field strength along the axis of a sphere-sphere air gap are shown, in relation to the breakdown voltage of the gap (Vb).

The values of breakdown voltages are measurement data from experimental referent work. The values of the field strength along the axis of the gap are from the Finite Element Analysis of models like the one in figure 11.

Figure 11. Field strength distribution in a sphere–sphere air gap model. The continuous lines are equipotentials, and the arrows lengths are proportional to the field strength values. Colors are in connection with the density of the field.
3.1.2 The rod-rod air gap.

Although the rod-rod air gap is a symmetric arrangement, the electric field between the rods is much more inhomogenous than in the sphere-sphere arrangement. The grade of inhomogeniety depends mainly on the rod dimensions and secondary on the gap length. The experimental results for breakdown DC and AC voltages and the F.E. Analysis results for the field strength values are shown in figure 12.

Figure 12. The maximum, the minimum, and the average value of the field strength along the axis of a rod-rod air gap at breakdown. Vₜₕ is the breakdown voltage.

The average and the maximum value of field strength at breakdown decrease when the gap length increases up to 8 cm, and then tend to achieve a steady value.

The model, which has been analyzed with the Finite Element Analysis, is shown in figure 13.

Figure 13. Field strength distribution in a rod –rod air gap model.

3.1.3 The rod-plate air gap.

A rod-plate air gap is a non-symmetric arrangement. The electric field between the electrodes is more inhomogenous than in other air gaps. The grade of inhomogeniety depends on the rod and plate dimensions, as well as on the gap length. The F.E. Analysis results, in relation to the experimental results for the values of breakdown voltages in the small rod – plate air gaps stressed by AC voltages are shown in figure 14.

Figure 14. The average, the maximum, and the minimum value of the field strength along the axis of a rod – plate air gap, at breakdown AC voltages.

It is obvious from the above figure that in the small rod-plate air gaps, the average and the minimum field strength values at breakdown decrease strongly when the air gap length increases, while the maximum field strength has an almost steady value.

Figure 15. Field strength distribution in a rod – plate air gap model.
3.1.4 The plate-plate air gap.  
A plate-plate air gap is a symmetric arrangement with an almost homogeneous electric field along the axis of the gap.

![Figure 16](image)

Figure 16. The average (Eav), the maximum (Emax), and the minimum (Emin) value of the field strength along the axis of a plate–plate air gap at breakdown A.C. voltages.

At the breakdown voltage the maximum value of the Field strength is nearly constant, while the average value of the field strength decreases, since the Field factor increases.

3.2 Models of air gaps with Barriers.

The electrical insulation of power apparatus is generally made of composite dielectrics. A basic composite insulation arrangement is that of a dielectric plate, called barrier, placed between a high-potential usually rod electrode and a grounded usually plane or rod electrode in gas. The influence of the Barrier to the distribution of the electric field and thus to the discharge and the breakdown voltage of the gap is called the Barrier Effect. The experimental study of the Barrier effect dates back to 1920’s.

The dielectric barrier, when placed between the electrodes separates the air gap in two different parts. The first part between the stressed electrode and the barrier behaves like a rod - plate arrangement and the second part between the barrier and the grounded electrode behaves like a plate – plate, or a plate – rod arrangement.

The surface of the barrier facing the rod electrode, which is usually the stressed electrode, is called front surface of the barrier. The rear surface of the barrier is the surface that faces the other electrode, which is usually the grounded electrode.

When an air gap with a barrier is stressed by high voltage, space charges drift through the air to the barrier and accumulate on the surface facing the stressed rod electrode, due to the low conductivity of the barrier (figure 17). If there is enough time, the charge is distributed firstly on the front surface of the barrier linearly, and then uniformly on the surface and the whole body of the barrier. The distribution of the electric charge on the barrier influences greatly the field distribution in the gap. The influence depends on the magnitude and the duration of the voltage. In case of an impulse voltage of a very short duration the accumulated charge produces a high voltage (nearly equal to the rod voltage) at the center of the barrier, thus resulting to a quick breakdown of the gap. The field strength distribution in the gap changes, thus influencing the value of the breakdown voltage of the gap. The influence depends on the gap length and the position of the barrier in the gap. This is called the Barrier Effect.

![Figure 17](image)

Figure 17. Rod–plate air gap stressed by high voltage. Electric charge is accumulated and distributed on the barrier.

![Figure 18](image)

Figure 18. Values of the field strength on the rod and behind the barrier along the axis of the gap for different positions of the barrier in a 50 mm long Rod – plate air gap.
It follows from figure 18 that the values of the field strength on the rod and on the barrier increase as the barrier is coming closer to the plate, while the breakdown voltage of the arrangement decreases. Values of the breakdown voltages are from referent work, [12], [14], and values of the field strength from F.E. Analysis.

Figure 19. Rod–plate air gap 100 mm long with a barrier, stressed by impulse voltage of very short duration. The electric charge is linearly distributed on the front surface of the barrier, [8], [17].

From figure 19 it results that the field strength value on the rod increases significantly when the charged barrier is moved from the rod to the plate. Practically the space charge and the electric current through the air limit its value. When the barrier is near the rod, the electric field between the barrier and the plate is much inhomogenous and the maximum value of the field strength behind the barrier along the axis is very high at the breakdown voltage. When the barrier is near the plate the electric field between the barrier and the plate is less inhomogeneous, and the maximum value of the field strength behind the barrier along the axis at the breakdown voltage is almost the same to the corresponding value for a plate-plate air gap. The amount of the electric charge accumulated on the barrier at breakdown voltage increases as the barrier is moved from the rod to the middle of the gap, and then it decreases significantly. This happens because the length of the part between the barrier and the plate becomes smaller and the breakdown voltage decreases.

When the gap is stressed with a DC voltage the phenomenon is more complicated. If the rate of voltage increase is small, the duration of the applied voltage is long enough, and there is plenty of time for the electric charge to accumulate on the barrier uniformly. Of course the maximum value of the voltage on the barrier cannot exceed the voltage of the rod. From the analysis it follows that the breakdown voltage takes place when the value of the field strength behind the barrier along the axis is the same with that of the correspondent plate - plate air gap, and rather smaller than it is expected to be for a homogenous field. Since the value of the field strength along a direction perpendicular to the axis of the gap increases from the center to the periphery, it must be concluded that the breakdown spark does not follow the axis of the gap.

Figure 20. Rod – plate air gap 100 mm long. The gap is stressed by DC voltage, [7], [17].

The analysis also showed that the field strength on the rod increases very much when the barrier is
moved from the rod to the middle of the gap, but it
starts to decrease when the barrier is coming near the
plate.

4 Conclusions
From the analysis of the different arrangements of
the air gap models in connection to the values of the
breakdown voltages, the following conclusions arise.
1) The distribution of the electric field in a rod –
plate or rod – rod air gap is strongly affected by
the geometry (shape and dimensions) of the
electrodes and the arrangement of the air gap. It is
worth observing that there is a significant
difference in the graphs of voltage drop between
the two different arrangements, the one with the
plate grounded and the other with the rod
grounded.
2) The breakdown voltage of the plane gap
arrangements increases with the gap length,
especially in the arrangements of rod-rod and rod-
plate air gaps, in a non-linear way. The values of
the breakdown voltage of the air gap arrangements
with a barrier depend mainly on the position of the
barrier in the gap.
3) The average value of field strength along
the axis of the gap, at breakdown voltage,
decreases with the gap length. The maximum
value of the field strength along the axis of the gap
at breakdown voltage tends to get a steady value,
more or less independent of the gap length, in the
symmetric arrangements of sphere-sphere, rod-rod
and plate-plate arrangements. In the rod-plate
arrangements it seems that the maximum value of
the field strength of the gap increases with the gap
length.
4) In a rod plate air gap with a barrier the most
determinant factor for the breakdown voltage is
the value of the field strength on the rod and on
the barrier. The analysis showed that the value of
the field strength on the rod increases with the
distance between the rod and the charged barrier,
while the value on the barrier decreases.

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