OFDM System for Frequency-Selective Radio Channel with Maximum Time-Delay Estimation

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Abstract: - The frequency domain channel model for Rician fading channels has been applied. It allows performance results to be shown as a function of the channel parameters \( \{ P_0, K, \tau_{rms} \} \), the normalized receiver power (NRP), the Rician \( K \)-factor, and root mean square (rms) delay spread (RDS). Assuming perfect channel estimation or, if differential schemes are applied, complete channel correlation, the performance is determined by \( P_0, \rho \) and \( \sigma^2_f \). These parameters specify the average signal power and the depth of fading. Better performance is thus achieved over channels having a higher \( K \)-factor because the fades are shallower. By extending the OFDM system model, it becomes possible to analyze the imperfections of OFDM systems. Frequency synchronization offsets, gives rise to Inter carrier interference (ICI) and Intersymbol interference (ISI), which can be accounted for by an additional noise term. The impact of Doppler spreads, phase noise, or channel-estimation errors can be incorporated.

Key-Words: - Frequency Domain (FD), NRP, Rician, OFDM, BER, ICI, ISI.

1 Introduction

The Orthogonal Frequency Division Modulation (OFDM) is a special case of multicarrier transmission, where a single data stream is transmitted over a number of low-rate subcarriers (SCs). It worth mentioned here that OFDM can be seen as either a modulation technique or a multiplexing technique. One of the main reasons to use OFDM is to increase robustness against frequency-selective fading or narrowband interference. In a single-carrier system, a single fade or interferer can cause the entire link to fail, but in a multicarrier system, only a small percentage of the SCs will be affected. Error-correction coding can then be used to correct for the few erroneous SCs.

In OFDM, the channel’s variability in the frequency domain (FD) has a similar role as the time variance in a flat-fading narrowband system. Multiple reflections are sometimes a line-of-sight (LOS) component of the transmitted signal arrive at the receiver via different propagation paths and, therefore, with different amplitudes and delay times. The OFDM system for Frequency-Selective Radio Channel with Maximum Time-Delay Estimation is getting more and more important due to three major reasons: 1) system robustness, 2) ICI and ISI cancellation and 3) broadband.

The rest of the paper is organized as follows. We introduce the system model in section 2. The relation to channel parameters, spaced-frequency correlation function and maximum excess delay are formulated in section 2. In section 3, we propose the analytical evaluation of the bit-error rate (BER) for OFDM system and discuss the time-dispersion channel model and coherent detection with perfect channel estimation. In section 4, we will compare the BER with some of simulation results. In section 5, it contains our concluding remarks.

2 System Model

Let us define the channel correlation functions assuming that those functions are wide-sense stationary (WSS). This means that the autocorrelation function

\[ \phi_p(f_1,f_2,t_1,t_2) = E\{H^*(f_1,t_1)H(f_2,t_2)\} \quad (1) \]

The channel is thus characterized for all times and all frequencies by the so-called spaced-frequency, spaced-time correlated function

\[ \phi_p(\Delta f, \Delta t) = E\{H^*(f,t)H(f + \Delta f,t + \Delta t)\} \quad (2) \]
Similarly, the A[dB] + to channel parameters }ˆ,,{ 0 Π τ Table 2.1 [3] gives an overview of to the channel parameters defined in [1], the shape of DPS was defined as shown in Fig. 2 and (3).

\[
\phi_h(\tau) (dB) = \begin{cases} 
0 & \tau < 0 \\ 
\rho^2 \delta(\tau) & \tau = 0 \\ 
\Pi & 0 < \tau \leq \tau_1 \\ 
\Pi e^{-\gamma(\tau-\tau_1)} & \tau > \tau_1 
\end{cases}
\] (3)

where \( \gamma = A \cdot \ln(10)/10 \) [1/ns]. We can derive from this DPS, other channel parameters like Normalized Received Power (NRP), Rician K-factor, RDS and coherent bandwidth, as well as higher-order statistical parameters, such as the level crossing rate and the average bandwidth of fades [2]. The proposed simulation system is shown in Fig. 3. The amplitude distribution of the Transfer Function (TF) \( H(f, t) \) is Rayleigh fading because \( H(f, t) \) is complex Gaussian noise process. A Rician fading channel may be simulated by adding a phasor given by

\[
H(f, t) = \int_{-\infty}^{\infty} h(\tau, t)e^{-j2\pi f\tau}d\tau = \sum_i \beta_i(t)e^{-j[2\pi f(t) + \phi_i(t)]}
\] (4)

where \( H(f, t) \) denotes the channel TF.

2.1 Relation to Channel parameters
In the application of the Frequency Domain (FD) channel model, it is important to relate its parameters \{\rho^2, \Pi, \gamma, \tau_1\} to the channel parameters defined in last section: the NRP \( P_0 \), the Rician K-factor \( K \), and the RDS \( \tau_{rms} \). Table 2.1 [3] gives an overview of expressions relating the model parameters \{\rho^2, \Pi, \gamma, \tau_1\} to channel parameters \{\rho^0, K, \tau_{rms} \} spread around the means.

2.1.1 Spaced-frequency Correlation Function
The spaced-frequency correlation function is used and derived from the DPS via the Fourier Transform [3]

\[
\phi_h(\Delta f) = E[H'(f, \tau)H(f + \Delta f, \tau)] = F[\phi_h(\tau)] = \rho^2 + \Pi \cdot \tau_1 \text{sinc} (\tau_1, \Delta f) e^{-j2\pi f\Delta f} + \Pi \cdot \frac{1}{\gamma^2 + j2\pi f\Delta f} e^{-j2\pi f\Delta f} \] (5)

For \( \tau_1 = 0 \), that is, for the special case of an exponentially decaying DPS, the spaced-frequency correlation function can be written as

\[
\phi_h(\Delta f) = \frac{P_0}{K+1} \left( K + 1 \right) (K+1) \] (6)

where \( K_t = (K+1)/\sqrt{2K+1} \).

2.1.2 Maximum Excess Delay
The shape factor \( u \) introduces another degree of freedom into the channel model, which allows the variation of the maximum excess delay \( \tau_{max} \) by a certain factor for a given RDS \( \tau_{rms} \). Strictly speaking; the maximum delay spread is infinite due to the exponentially decaying part of the DPS, which never becomes zero. Therefore, we define the maximum excess delay as the delay time, where the exponentially
decaying part has decreased by about 43 dB. Such attenuation is reached if the duration of exponentially decaying part is exactly \( \tau_{\text{exp}} = 10/\gamma \), leading to the maximum delay spread \( \tau_{\text{max}} = \tau_1 = \tau_{\text{exp}} = \tau_1 + 10\gamma \).

Expressed in terms of channel parameters, this is:

<table>
<thead>
<tr>
<th>Mode</th>
<th>Channel</th>
</tr>
</thead>
<tbody>
<tr>
<td>( u = \tau_1^{(1)} \in [0, \infty) )</td>
<td>( u = 0 )</td>
</tr>
<tr>
<td>( P_0 = \rho^2 + \Pi )</td>
<td>( P_0 = \rho^2 + \Pi )</td>
</tr>
<tr>
<td>( k = \rho^2 \frac{\gamma}{\Pi u} )</td>
<td>( k = \rho^2 \frac{\gamma}{\Pi u} )</td>
</tr>
<tr>
<td>( t_\gamma = \frac{1}{\gamma} \left( \frac{1}{u} \right) \frac{u_1}{u_1 + (K+1) u_1} )</td>
<td>( t_\gamma = \frac{1}{\gamma} \left( \frac{1}{u} \right) \frac{u_1}{u_1 + (K+1) u_1} )</td>
</tr>
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</table>

It seems that \( \tau_{\text{max}} \) and \( \tau_{\text{rms}} \) are related by a factor \( K \) and \( u \).

### 3 Analytical Evaluation of the BER

Evaluating the above expressions for simultaneous timing \( (\delta \tau) \), frequency \( (\delta \phi, \delta \Phi) = \text{round}(\delta \Phi/F) \) and phase \( (\theta) \) offsets, the system model for the generalized case is obtained. It is written as

\[
Y_{1,i,k} = x_{i,k} \cdot h_{i,k} \cdot \text{sinc}(\delta \cdot T_{\text{FFT}}) e^{i \theta_{i,k}} + n_{i,k}
\]

where the phase distortion due to synchronization errors is expressed by

\[
\Psi_{i,k} = \theta_0 + 2 \pi \delta (k T_s + T_{\text{FFT}}/2 + \delta t) + 2 \pi \delta t \text{ } i T_{\text{FFT}}
\]

where \( \theta_0 \) is the carrier phase offset, \( T_s \) is the symbol time, and \( i \) is the subcarrier number. Note that the noise variable \( n_{i,k} \) in (9) includes the noise caused by ICI and ISI, or both. Often, the timing offset is expressed in samples, that is, \( \delta t' = t/T_s \), and the frequency offset is normalized to the single carrier spacing \( \delta \Phi' = \delta \Phi/F \).

Using these symbols, the phase distortions are expressed by

\[
\Psi_{i,k} = \theta + 2 \pi \delta (k T_s + T_{\text{FFT}}/2 + \delta t) + 2 \pi \delta t \text{ } i T_{\text{FFT}}
\]

The symbol periods in Fig. 4 are given at times. Since the implementation is usually done on digital hardware, those periods are often in terms of samples. \( N, N_{\text{guard}} \) and \( N_{\text{win}} \) then define the number of samples in the effective part, guard, and windowing interval, respectively. The effective part is also referred to as the FFT part because this part of the OFDM symbol is applied to the FFT to recover the data at the receiver. Note that this additional cyclic prefix extends the Guard Interval (GI) to some extent; that is, the delay-spread robustness is slightly enhanced. On the other hand, the efficiency is further reduced, as the window part is also discarded by the receiver.

The algorithm uses the correlation function \( G(d) \),

\[
G(d) = \sum_{m=0}^{N_{\text{FFT}}-1} z(d-m) \cdot z(d-m - N_{\text{FFT}})\text{T}
\]

which calculates the correlation over a sliding window of the received signal. \( G(d) \) expresses the sum of correlation between a pair of samples in the two sequences of \( N_g \) samples length spaced by \( N_{\text{FFT}} \) samples. The \( d \) denotes the index of the most recent input samples, \( z(d) \) the received complex signal sample, \( N_{\text{FFT}} \) the number of FFT points used for the OFDM modulation, \( N_g \) is the length of the guard interval, and \( T \) denotes the conjugate of the complex value. Plugging this equation into (12) at \( d = d_{\text{max}} \), yields

\[
G(d_{\text{max}}) = \sum_{m=0}^{N_{\text{FFT}}-1} z(d_{\text{max}}-m) \cdot z(d_{\text{max}}-m - N_{\text{FFT}})\text{T}
\]

\[
= \sum_{m=0}^{N_{\text{FFT}}-1} |z(d_{\text{max}}-m)|^2 \cdot e^{i 2 \pi m N_g T_s T}
\]
Figure 5 shows a schematic plot of the computation of $G(d)$. Due to the frequency offset, these samples are received as (neglecting channel effects)

$$ y_n = s_n e^{i(2\pi f_n T_s + \phi)} $$

where $f_n$ is the transmitter carrier frequency and $T_s$ is the sampling period.

This implies that the more the samples in the sum, the better the quality of the estimate. Based on the preceding discussion, it is clear that we require a two-step frequency estimation process with a coarse frequency estimate performed from the short training symbols and fine frequency synchronization from the long training symbols.

### 3.1 Channel Estimation with Frequency-Domain Processing

The received Time-Domain (TD) OFDM signal $y(n)$ in Figure 1 is a function of the transmitted signal, the channel TF, and additive white Gaussian noise $w(n)$. It can be expressed as

$$ y(n) = x(n) \overset{\ast}{\otimes} h(n) + w(n), \quad 0 \leq n \leq N-1, $$

where “$\ast$” denotes convolution. The received FD signal $Y(k)$ is the Fourier Transform of $y(n)$, which can be expressed as

$$ Y(k) = X(k) \cdot H(k) + I(k) + W(k) $$

where $W(k)$ is the FT of $w(n)$, $H(k)$ is a recognized function of the transmitted signal, and $I(k)$ is the ICI component in the received signal at the $k^{th}$ SC, which is independent of the transmitted signals $\{X(k)\}$ and $\{H(k)\}$ given by [7]:

$$ H(k) = \sum_{i=0}^{N-1} h_i e^{i2\pi f_{ni} T} \sin(\pi f_{Dj} T) e^{-j\pi f_{nj}} $$

where $f_{nj}$ is the $i^{th}$ path Doppler frequency shift which causes ICI of the received signals, and $\tau_i$ is the $i^{th}$ path delay time normalized by sampling time. $I(k)$ in (20) is the ICI component in the received signal at the $k^{th}$ SC, depending upon the signal values $X(k)$ modulated on all of the other SCs, which is given by

$$ I(k) = \frac{1}{N} \sum_{j=0}^{N-1} h_i X(K) \frac{1 - e^{i2\pi f_{nj} T}}{1 - e^{i2\pi f_{nj} T / N}} e^{-j\pi f_{nj}} $$

where $h_i$ is the complex impulse response of the $i^{th}$ path, $f_{nj}$ is the $i^{th}$ path Doppler frequency shift.

The FD channel model is used to describe the frequency variability. Thereby, we confine ourselves to the case of the exponentially decaying delay power spectrum, where a direct relation can be given between the channel parameters $\{P_0 – average power, K – Rician factor, and \tau_{rms} - RDS\}$ and the channel correlation function

$$ \phi_H(Df) = E[H^*(f, t)H(f + Df, t)] = \frac{P_0}{K + 1} \left\{ \frac{1}{1 + j2\pi f \tau_{rms} K_i} \right\} $$

In this equation, $K_i = (K + 1) / \sqrt{2K + 1}$, $Df$ is the frequency lag, and $*$ denotes the complex conjugate.
The Normalized Received Power (NRP) (average power) is defined as $P_0 = E\{|h_k|^2\}$. To model the time variability, the so-called Jakes Doppler spectrum can be used [4], augmented by an line of sight (LOS) component $\rho e^{j(2\pi f+\phi_0)}$ at a given Doppler frequency $f_\rho$. Such a Doppler spectrum corresponds to a space-time correlation function $\tilde{\phi}_n(\Delta t) = E\{H^*(f,t)H(f,t+\Delta t)\} = \frac{P_0}{K+1}(K e^{j2\pi f\Delta t} + J_0(2\pi f_\rho \Delta t))$ where $J_0(\cdot)$ denotes the zeroth-order Bessel function of the first kind, $\Delta t$ is the time lag, and $f_m$ is the maximum Doppler frequency. $(f_m = \nu_m / \lambda = \nu_m f_c / c$, where $\nu_m$ is the mobile’s velocity, $\lambda$ is the wavelength, $f_c$ is carrier frequency, and $c$ is the speed of light.)

### 3.2 Coherent Detection with Perfect Channel Estimation

The $k$th symbol received is defined in (19) as $y_k = x_k h_k + n_k$. Perfect channel estimation means that the receiver has exact knowledge of the attenuation factor $h_k$, denoting by $\hat{h}_k = h_k$. Considering the transmitted symbols $x_{k,j}$ as a constant yields $\bar{X}_i = E\{h_k\} = \rho e^{j\theta}$, and

$\bar{Y}_i = E\{x_k h_k + n_k\} = x_k E\{h_k\} + E\{n_k\} = x_k \rho e^{j\theta}$

(25)

$\Psi_{xx} = \frac{1}{2} E\{|h_k|^2\} - |\bar{X}_i|^2 = \frac{1}{2} [P_0 - \rho^2]$  

$\Psi_{yy} = \frac{1}{2} E\{|x_k h_k + n_k|^2\} - |\bar{Y}_i|^2 = \frac{1}{2} [x_k^2 (P_0 - \rho^2) + \sigma^2_n]$  

$\Psi_{yx} = \frac{1}{2} E\{|h_k (x_k h_k + n_k)\} - \bar{X}_i \bar{Y}_i\} = \frac{1}{2} x_k^2 [P_0 - \rho^2]$

where $\rho e^{j\theta}$ is the LOS-component, with arbitrary phase $\theta$, and with an amplitude defined by

$\rho^2 = \frac{P_0 K}{K+1}$.

### 3.3 Differential Detection

With differential detection, the decision for the received symbol $y_k$ is made based on the adjacent symbol $y_{k-1} = x_k h_{k-1} + n_{k-1}$. For phase modulation schemes, this can be seen as a detection based on the channel estimate $\hat{h}_k = y_{k-1} / x_{k-1} = h_{k-1} + n_{k-1}$, where $E\{|n_{k-1}|^2\} = \sigma^2_n$. Note that $\sigma^2_n = \sigma^2_N$, if the magnitude of $x_{k-1}$ is one. Thus, the additional noise term $n_k$, the correlation between $h_k$ and $h_{k-1}$, and the Doppler shift of the LOS-component are expressed in

$\bar{X}_1 = E\{h_{k-1} + n'_k\} = \rho e^{j(\theta - 2\pi f_\rho T)}$

(24)

$\Psi_{xx} = \frac{1}{2} [NRP + \sigma^2_N - \rho^2]$  

(27)

For evaluating differential detection in the frequency direction, let $T = 0$. Using the channel correlation functions given in last section, the correlation $\Psi_{xy}$ between the attenuation factors at two adjacent symbols becomes

$\Psi_{xy} = \frac{1}{2} [x'_{k+1} (E\{h_{k-1} h'_{k-1}\} - \rho^2 e^{j2\pi f_\rho T}]$

(28)

### 4. Simulation

Some observations can be made from the mathematical expressions derived above:

1) For coherent detection, the statistical parameters, and thus the performance results, only depend on $\{P_0, K, \tau_{rms}\}$, i.e., $P_0$, $\rho$, and $\sigma^2_N$.

2) The same holds in the limits $F \rightarrow 0$ or $T \rightarrow 0$ for differential detection.

3) The performance of differential detection degrades for $F > 0$ (or $T > 0$) because of a systematic estimation error in $\hat{h}_k = h_{k-1} + n'_k$, because $h_{k-1} \neq h_k$. The parameter produces $\tau_{rms}$, and $f_m T$ define the degradation according to (24).

Performance results (average BER and PER) for the above comments (1) and (2) and QPSK modulation are shown in Fig. 6 as a function of the average SNR per bit and as a function of $K$. The advantage of a high $K$-factor is seen. Taking the channel variability into account, irreducible error floors arise (see Figure 6). Both versions of different detection have been evaluated for Rayleigh fading channels, QPSK modulation, and for the following parameters. The channel’s RDS $\tau_{rms}$ is assumed to be three samples, which corresponds to a maximum delay spread of about 30 samples, assuming an exponentially decaying channel delay profile. For
128 FFT points, this value corresponds to about one quarter of the FFT time. It is seen that the irreducible error floor associated with such realistic parameters \(\tau_{rms}F = 3/128\) lies around \(10^{-2}\). In Figure 7, the performance of differential QPSK (in frequency direction) is shown as a function of \(E_b/N_0\) and \(K\), where \(\tau_{rms}\) is a parameter. Since the maximum excess delay of the channel, which should not exceed the GI, is a function of \(\tau_{rms}\) and \(K\), all of these parameters are interrelated. The maximum excess delay of the channel can be written \(\tau_{max} = 10\tau_{rms}K\). This leads to the normalized excess delay, defined as \(T_m = \tau_{max}/T_{FFT} = 10\tau_{rms}K/F\). In Figure 7, the performance of differential QPSK is shown for \(T_m = \{0, 0.11, 0.27\}\).

5. Conclusion

The derivation of the OFDM system model has confirmed that data symbols can be transmitted independently over multipath fading radio channels. It has to be assumed that, however, that the channel’s maximum excess delay is shorter than the GI and that the system has been synchronized sufficiently. If the timing- or frequency-synchronization error becomes too large, the orthogonality of the SCs is partly lost, and the SNR of the system is degraded; that is,ICI and ISI arise. ICI can also result very fast channel variations (Doppler spreads) or from carrier phase jitters.

The system models presents can be utilized in analytical studies of various aspects of the OFDM techniques in the performance evaluation. The performance analysis of an uncoded OFDM scheme are derived based on the classic formulas given by Proakis [5] Appendix B.

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