# Subtransmission Substation Placement Using Lagrange Method 

PAYMAN NAZARIAN<br>Computer Department<br>Zanjan Islamic Azad University<br>Zanjan Regional Electric Company IRAN


#### Abstract

Expansion of power system networks has made difficult the planning but using simple and fast computational methods simplifies it. This paper explains a method for finding the optimum placement of subtransmission substation by Lagrange method which minimizes the number of power flow studies and decreases the simulation time in planning process.


Key-Words: - Substation placement - Lagrange method - Nonlinear programming - Power flow

## 1 Introduction

The usual way for placement of subtransmission substation in power system networks is performing power flow study for some selected points.

ENPP software uses a more advanced method for simplifying the study by defining floating points [1]. A floating point is a set of acceptable geographical points for substation. Increasing the number of this points multiplies the study time but if we desist from earth cost in comparison with investment cost included high voltage line(s) and substation construction , we can define a nonlinear programming problem as (1):

```
Minimize Investment Cost
S.t.
Power Flow Equations
Voltage Limits
Variables (Geographical Points)
```

Points distances should be selected so that desired error is satisfied. Also increasing the number of points multiplies the study time and it can be reduced by using Lagrange optimization method that in this case nonlinear programming problem is divided to some similar problem with fewer points.

## 2 Problem Formulation

Suppose a rectangular limits for substation placement with length $x$ and width $y$. In stage one, length x is devided to $\mathrm{m}_{1}$ parts and width y to $\mathrm{n}_{1}$ which is produced $\left(m_{1}+1\right)\left(n_{1}+1\right)$ points on the hole; then (1) is solved and one of the points is selected as optimum solution by ENPP software. In the next
stage the small rectangles is divided to $\mathrm{m}_{2} * \mathrm{n}_{2}$ smaller ones around the optimum point as shown in Fig. 1 and this process is similarly performed to the next stages.


Figure 1 - Division of an area in two stages
It is assumed the stage k is the final stage and $\mathrm{e}_{\mathrm{k}}$ is the desired error d . Also in stage j the maximum error and the number of points are illustrated with $\mathrm{e}_{\mathrm{j}}$ and $c_{j}$ respectively. For each rectangle the maximum error is the diagonal of it; thus we have:
$e_{j}=2^{j-1} \sqrt{\frac{\left(\frac{x}{\prod_{i=1}^{j} m_{i}}\right)^{2}+\left(\frac{y}{\prod_{i=1}^{j} n_{i}}\right)^{2}}{}}$
$c_{j}=\left(m_{j}+1\right)\left(n_{j}+1\right)$
$e_{k}=d ; \quad \mathrm{k}$ is the final stage
Also we define M and N as (5):

$$
\begin{equation*}
M=\prod_{i=1}^{k} m_{i} \quad, \quad N=\prod_{i=1}^{k} n_{i} \tag{5}
\end{equation*}
$$

From (2), (4) and (5) we have:
$x^{2} N^{2}+y^{2} M^{2}-d^{2} M^{2} N^{2} / 4^{k-1}=0$
From (3) the total number of points in k stages is obtained:
$C=\sum_{i=1}^{k} c_{i}=\sum_{i=1}^{k}\left(m_{i}+1\right)\left(n_{i}+1\right)$
If we minimize the C then the total simulation time for solving (1) in k stages decreases and this is possible by Lagrange optimization method.

## 3 Using Lagrange Method

From goal function (7) and equality constraint (6) the Lagrange function L is obtained as below:
$L=C+p\left(x^{2} \mathrm{~N}^{2}+\mathrm{y}^{2} \mathrm{M}^{2}-d^{2} M^{2} N^{2} / 4^{k-1}\right)$
For Simplifying the problem assume the number of parts be equal in length and width:
$m_{j}=n_{j}=f_{j} ; j=1,2, \ldots, k$
Also we define $F$ and $r$ as below:
$F=\prod_{i=1}^{k} f_{i}$
$r=\sqrt{x^{2}+y^{2}}$
Then we obtain :

$$
\begin{equation*}
L=C+p\left(r^{2}-d^{2} F^{2} / 4^{k-1}\right) \tag{12}
\end{equation*}
$$

For minimizing C by Lagrange method, we must partially derivate from (8) as follows:

$$
\left\{\begin{array}{l}
\frac{\partial L}{\partial f_{j}}=0 \quad ; \quad j=1,2, \ldots, k  \tag{13}\\
\frac{\partial L}{\partial p}=0 \\
\frac{\partial L}{\partial k}=0
\end{array}\right.
$$

From (13) we have:
$f_{j}\left(f_{j}+1\right)=\frac{p d^{2} F^{2}}{4^{k-1}} ; \quad j=1,2, \ldots, k$
From (16) we see that the number of divisions is equal in different stages; thus we define $f$ as follows:

$$
\begin{equation*}
f_{j}=f \quad ; \quad j=1,2, \ldots, k \tag{17}
\end{equation*}
$$

Combining (17) and (7) and Substituting into (12) results:
$L=k(f+1)^{2}+p\left[r^{2}-4 d^{2}\left(f^{2} / 4\right)^{k}\right]$

From (13) (15) and (17) $f$ is calculated and rounded to the nearest integers towards right:

$$
\begin{equation*}
e^{\frac{f+1}{2 f}}=\frac{f}{2} \Rightarrow f=3.77 \approx 4 \tag{19}
\end{equation*}
$$

Also from (14) the number of stages is obtained as follows:
$k=\frac{\ln \frac{r}{2 d}}{\ln \frac{f}{2}}$

## 4 Case Study

Assume the rectangular area for a subtransmission substation placement with 99 unit length and 59 unit width and a simple single line diagram for this problem as Fig. 2 and the voltage limits are 0.95 and 1.05 per unit.


Figure 2 - Single line diagram of problem
It is assumed the location of slack and load is fix, but the substation can vary in the rectangular area. Also two virtual lines connect the substation to the slack and the load which its length of each line is related to geographical coordinates of the substation that varies between pre-selected points

For 1 unit precision it is necessary 6000 power flow studies in one stage at nonlinear programming framework as (1) for finding optimum location. Using ENPP software we can define a rectangular area from $(0,0)$ to $(99,59)$ with 1 unit spacing that results the problem solving as below:
$\left\{\begin{array}{l}\text { Substation optimum coordinates }:(29,2) \\ \text { Minimum voltage }=0.95008 \text { p.u. } \\ \text { Minimum Investment } \text { Cost }=567.09\end{array}\right.$
If the Lagrange method is used, in addition to decreasing the study time, precision also is increased. From (19) the number of divisions $f$ is selected 4 and from (20) the number of stages $k$
rounds to 6, the nearest integer towards right and then ENPP runs six times separately by user that the results and coordinates of used areas are in Table 1 and Table 2 respectively.

Table 1 -Results of substation placement using Lagrange method

| Stage | Optimum <br> Point <br> Coordinates | Minimum <br> Voltage | Minimum <br> Investment <br> Cost |
| :---: | :---: | :---: | :---: |
| 1 | $(24.75,0)$ | 0.95164 | 578.47 |
| 2 | $(24.75,0)$ | 0.95164 | 578.47 |
| 3 | $(30.94,0)$ | 0.95107 | 572.98 |
| 4 | $(27.84,1.84)$ | 0.95029 | 568.67 |
| 5 | $(29.39,1.84)$ | 0.95013 | 567.34 |
| 6 | $(30.16,1.84)$ | 0.95005 | 566.74 |

Table 2 - Area coordinates ( corner to corner )

| Stage | X1 | Y1 | X2 | Y2 |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | -14.75 | 49.5 | 14.75 |
| 2 | 12.375 | -7.375 | 37.125 | 7.375 |
| 3 | 24.75 | -3.6875 | 37.125 | 3.6875 |
| 4 | 24.75 | 0 | 30.9375 | 3.6875 |
| 5 | 27.84375 | 0.921875 | 30.9375 | 2.765625 |
| 6 | 29.390625 | 1.3828125 | 30.9375 | 2.3046875 |

## 5 Comparison of the methods

Usual method uses 6000 points but Lagrange optimization method uses 150 points and this means the placement is simulated 40 times faster than before. Also the maximum placement error is decreased from 1 to 0.9 unit and the minimum voltage of sixth stage is closer to the lower voltage limit than study in one stage and therefore the minimum investment cost in Lagrange method is less than usual way.

## 6 Conclusion

Using this method in power system planning softwares such as ENPP, enables the engineer for fast finding the optimum placement of substation and examining the different alternatives for connecting the equipment to the electrical network and thus many studies will be possible in less time.

## References:

[1] P. Nazarian, Electrical Planning Study Using ENPP software (in Persian), Zanjan Regional Electric Company, 2003.

