Nonlinear Phillips Curves, Complex Dynamics and Monetary Policy in a Keynesian Macro Model

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Abstract: In the framework of a Keynesian monetary macro model we study implications of kinked Phillips curves and alternative monetary policy rules. As alternative monetary policy rules we consider monetary growth targeting and interest rate targeting (the Taylor rule). Our monetary macro model exhibits: asset market clearing, disequilibrium in product and labor markets, sluggish price and quantity adjustments, two Phillips-Curves for wage and price dynamics, and a combination of medium-run adaptive and short-run forward looking expectations. Simulations of the model with our estimated parameters reveal global instability of its steady state. We show that monetary policy can stabilize the dynamics to some extent and that, in addition, an institutionally given kink in the money wage Phillips-Curve (downwardly rigid wages) represents a powerful mechanism for getting bounded, more or less irregular fluctuations in the place of purely explosive ones. The resulting fluctuations can be reduced in their size by choosing the parameters of monetary policy within a certain corridor, the exact position of which may however be very uncertain.

Keywords: Feedback channels, instability, monetary policy, wage floors, fluctuating growth, complex dynamics.

1 Introduction

In the framework of a Keynesian monetary macro model we study the implications of manipulating monetary aggregates or the interest rate as two alternative monetary policy rules. Whereas the former targets the inflation rate indirectly, through the control of the money supply, the latter, also called the Taylor rule, implies more direct inflation targeting. Our monetary macromodel exhibits: asset market clearing, disequilibrium in the product and labor markets, sluggish price and quantity adjustments, two structural Phillips Curves for the wage and price dynamics in the place of a single reduced-from PC and expectations formation which represents a combination of medium-run adaptive and short-run forward looking behavior. The ideas on which this model is built come from a long tradition of Keynes, Kaldor, Metzler, Malinvaud, Mundell, Rose, Tobin and Sargent and others. Here it may suffice to provide a rough description of the structure of the model. Its characteristic laws of motion are however introduced, motivated and studied in their interaction in detail in the next section.

We consider a closed three sector economy, households (workers and asset holders), firms and the government. There exist five distinct markets; labor, goods, money, bonds and equities (which are perfect substitutes of bonds).

To briefly summarize our model we use the following table. In the table real and nominal magnitudes are represented. The index d refers to demand and the same symbol with no index represents supply, while superscript index e is used to denote expectations. We use $\hat{x}$ to denote the rate of growth of a variable x. The symbols in the table in particular denote L, labor, $C = C_w + C_c$, consumption (of workers and capitalists), I, investment, Y, income, M, money, G, government expenditure, δK, depreciation, B, Bonds and E, equity. The table shows the interaction of the sectors and the markets where the rows represent the sectors and the columns the markets.

	Labor market	Goods market	Money market	Bond market	Equity market
Households	L	$C = C_w + C_c$	M^d	B^d	E^d
Firms	L^d	$Y, Y^d, I + \delta K$	–	–	E
Government	–	G	M	B	–
Prices	w	p	1	1	p_e
Expectations	–	$Y^e, \pi^e = \hat{p}^e$	–	–	–

This table represents the basic structure of the closed economy considered in this paper. The connections between the markets and sectors shown, the behavioral relationships and the dynamic adjustment processes that fill this structure have been established in Flaschel, Franke and Semmler (1997) and in Chiarella and Flaschel (2000). They represent significant extensions of Sargent (1987, Ch. 1-5) in various ways. We here extend this framework furthermore by a discussion of the role of monetary policy rules and continue investigations of the stability implications of kinked money-wage Phillips Curves, already asserted to exist in fact by Keynes (1936).

The structure of the model is complete in the sense that it includes all major markets and sectors of a closed economy and all financing conditions and budget restrictions of households, firms and the government, as in Sargent (1987). In contrast to Sargent (1987) we distinguish however between workers and asset holders in the household sector in a Kaldorian fashion. The major difference is however the extent of disequilibrium allowed for and the dynamical processes that follow from these disequilibria. Concerning the extent of these disequilibrium adjustment processes, we want to note already here, that firms have desired capacity utilization rates and desired ratios of inventory to expected sales. Temporary deviations from those benchmarks are caused by unexpected changes in aggregate goods demand. We stress that a distinguishing feature of Keynesian models, in contrast in particular to equilibrium macromodels of the Sargent (1987) type, is that under- or over-utilized capital as well as an under- or over-utilized labor force are important driving factors of the economic dynamics.

Section 2 provides and explains the theoretical core model, that is introduced and motivated from the perspective of its typical adjustment mechanisms solely. This core model is extended, estimated and numerically investigated in section 3. By employing first a linear Phillips Curve we show that the 6D dynamics implied by the model exhibit a unique interior steady state which is locally stable when a strong Keynes-effect is coupled with sluggishly moving prices and quantities – with the exception of the dynamic multiplier which, by contrast, must be sufficiently fast. This steady state, however, loses its stability by way of a Hopf-bifurcation when adjustment parameters are sufficiently increased. Our feedback-guided stability analysis is extended in the second part of the paper to a 7D dynamical system, with theoretically similar properties as the 6D dynamics, by considering alternative dynamic monetary policy rules: a money supply and an interest rate policy rule. These extended systems as well as the original 6D dynamics are studied from the empirical and the numerical point of view and found to give rise to interesting fluctuations in economic activity and inflation.

In section 3 of the paper, we add, on the one hand, a money supply rule and, on the other hand, an interest rate policy rule to the model (whereas money had earlier been assumed to be growing at a constant rate). We extend our stability assertions to these two cases. Estimated parameters of the two model variants are reported, estimated partly through single equation and partly through subsystem estimations for U.S. time series data 1960.1-1995.1. With the estimated parameters system simulations for the two monetary policy rules of the paper are presented and the stability as well as impulse-response properties of the two rules explored.

One important finding of the paper is that, given our parameter estimates, the four feedback channels of the model, the Keynes-, Mundell-, Metzler- and the Rose-feedback chains, are such that the Rose adverse-type real wage adjustments dominate the stabilizing Keynes-effect and the stabilizing dynamic multiplier – here coupled with a weak inventory accelerator. The destabilizing Mundell-effect is also found to be weak. The steady state of the dynamics is therefore found to be explosive. This instability can be overcome in the considered case by making monetary policy react more strongly to the deviation of actual inflation from its target value (establishing cyclical convergence). The instability may also be overcome quite generally, by introducing a nonlinear money wage Phillips Curve, based on an institutionally determined kink in this PC (thereby establishing persistent fluctuations). This kink has recently been discussed and estimated in a number of studies, see Hoogenveen and Kuipers (2000) for an excellent example. This latter modification indeed bounds the dynamics for a larger range of parameter values (compared to the stability implications of monetary policy rules) such that limit cycles or more complex types of attractors are generated. Such cyclical and complex behaviour occurs in particular when the adjustment speed of the expected inflationary climate π^e is increased, since this parameter is found to greatly affect the local stability/instability of the equilibrium. The role of monetary policy in such an environment is also briefly discussed, but must be left by and large for future research into the model of this paper.

2 Keynesian Macrodynamics

This section provides the building blocks of our Keynesian AS-AD macrodynamics. We do this from the perspective of its fundamental adjustment mechanisms and the feedback structures that are implied. We therefore motivate the structure of the model without presenting the many details which underlie its extensive form representation in Chiarella and Flaschel (2000, Ch.6). The stability properties of the interaction of those feedback structures are then studied analytically and numerically, by means of estimated parameters in particular.

2.1 3D Rose wage-price dynamics

The full dynamics, basically presented in ratio or intensive form directly, is best introduced and motivated by starting from a very basic, yet unfamiliar, wage-price module. In our first specification we follow Rose (1967, 1990) and assume two Phillips Curves or PC's in the place of only one, providing wage and price dynamics separately, both based on a measure of demand pressure $V-\bar{V}, U_c-\bar{U}_c$, in the market for labor and for goods, respectively. We here denote by V the rate of employment on the labor market and by $\bar{V}$ the NAIRU-level of this rate, and similarly by U_c the rate of capacity utilization of the capital stock and $\bar{U}_c$ the normal rate of capacity utilization of firms. These demand pressure influences on wage and price dynamics, or on the formation of wage and price inflation, $\hat{w}, \hat{p}$, are here both augmented by a weighted average of cost-pressure terms based on forward looking myopic perfect foresight and a backward looking measure of the prevailing inflationary climate, symbolized by π^e. Cost pressure perceived by workers is thus a weighted average of the currently evolving price inflation $\hat{p}$ and some longer-run concept of price inflation, π^e, based on past observations. Similarly, cost pressure perceived by firms is given a weighted average of the currently evolving (perfectly foreseen) wage inflation $\hat{w}$ rate and again this measure of the inflationary climate in which the economy is operating. Taken together we thus arrive at the following two Phillips Curves for wage and price inflation, which in this core version of the model are formulated in a fairly symmetric way.

<u>Structural form of the wage-price dynamics:</u>

$$\hat{w} \quad = \quad \beta_w(V-\bar{V})+\kappa_w\hat{p}+(1-\kappa_w)\pi^e,$$

$$\hat{p} \;=\; \beta_p(U_c - \bar{U}_c) + \kappa_p \hat{w} + (1-\kappa_p)\pi^e.$$

In the empirical application of the model we have to take account labor productivity growth $n_x = \hat{x}$ in addition,[1] which from the theoretical perspective augments the cost pressure terms in the wage PC by the addition of n_x, while it reduces the wage cost pressure term $\hat{w}$ in the price PC by the same amount, see our calculations below. In the empirical estimate of the model we find that this full indexation of wage and price inflation with respect to productivity growth does not apply to the investigated historical situation where only roughly 50 percent of productivity growth seemed to have entered the wage-price dynamics.[2]

Inflationary expectations over the medium run, π^e, i.e., the inflationary climate in which current inflation is operating, may be adaptively following the actual rate of inflation (exponential weighting scheme), may be based on a rolling sample (hump-shaped weighting schemes), or on other possibilities for updating such an expression. We shall in fact make use of the conventional adaptive expectations mechanism in the presentation of the full model below. Besides demand pressure we thus use (as cost pressure expressions) in the two PC's weighted averages of this climate and the (foreseen) relevant cost pressure term for wage setting and price setting. In this way we get two PC's with very analogous building blocks, which despite their traditional outlook will have interesting and novel implications. In the later part of the paper we will introduce a non-linearity in the money wage Phillips Curve in addition.

Note that for our current version, the inflationary climate variable does not matter for the evolution of the real wage $\omega = w/p$ – or, due to our addition of productivity growth – the wage share $u = \omega/x$, the law of motion of which is given by:[3]

$$\hat{u} \;=\; \hat{\omega} - n_x = \kappa[(1-\kappa_p)\beta_w(V - \bar{V}) - (1-\kappa_w)\beta_p(U_c - \bar{U}_c)].$$

This follows easily from the obviously equivalent representation of the above two PC's:

$$\begin{aligned} \hat{w} - \pi^e - n_x &= \beta_w(V - \bar{V}) + \kappa_w(\hat{p} - \pi^e), \\ \hat{p} - \pi^e &= \beta_p(U_c - \bar{U}_c) + \kappa_p(\hat{w} - \pi^e), \end{aligned}$$

by solving for the variables $\hat{w} - \pi^e - n_x$ and $\hat{p} - \pi^e$. It also implies the two across-markets or reduced form PC's given by:

$$\begin{aligned} \hat{p} &= \kappa[\beta_p(U_c - \bar{U}_c) + \kappa_p \beta_w(V - \bar{V})] + \pi^e, \\ \hat{w} &= \kappa[\beta_w(V - \bar{V}) + \kappa_w \beta_p(U_c - \bar{U}_c)] + \pi^e + n_x, \end{aligned}$$

which represent a considerable improvement over the conventional view of a single-market price PC with only one measure of demand pressure, the one in the labor market!

[1] We denote by $x = Y/L^d$ labor productivity and by $y^p = Y^p/K$ the potential output – capital ratio (capital productivity) and assume that x is growing at a constant rate while y^p is constant, i.e., we assume as production function a fixed proportions technology with Harrod-neutral technical change.

[2] This is in line with estimates of the wage equation in the macroeconometric model of, for example, the German Bundesbank, see the α_2 estimates in Deutsche Bundesbank (2000, p.52).

[3] Note that $\kappa = 1/(1 - \kappa_w \kappa_p)$.

This traditional expectations-augmented PC formally resembles the above reduced form $\hat{p}$-equation if Okun's Law holds in the sense of a strict positive correlation between $U_c - \bar{U}_c$ and $V - \bar{V}$, our measures of demand pressures on the market for goods and for labor. Yet, the coefficient in front of the traditional PC would even in this situation be a mixture of all of the $\beta's$ and $\kappa's$ of the two originally given PC's and thus represent a composition of goods and labor market characteristics. The currently prominent New Keynesian Phillips Curve, see for example Gali (2000), is based on the reduced-form representation for $\hat{p}$ shown above, but generally with $\beta_p = 0.\kappa_p = 1, \kappa_w = 0$ and π^e a one-period ahead forecast of the rate of price inflation. Under perfect foresight this basically implies in a continuous time set-up the following type of price Phillips Curve:

$$d\hat{p}/dt = \beta_w(V - \bar{V}),$$

which provides an interesting alternative to our reduced form price PC, yet one where the medium-run climate expression for price inflation plays no role. Reducing in this way inflation dynamics to short-term expressions solely, in our view, provides one of the reasons why the New Keynesian PC behaves strangely from an empirical perspective.

Taken together our above structural approach to wage and price PC's gives rise to three independent laws of motion:

$$\begin{aligned}
\hat{u} &= \kappa[(1-\kappa_p)\beta_w(V-\bar{V}) - (1-\kappa_w)\beta_p(U_c - \bar{U}_c)],\\
\hat{m} &= \bar{\mu} - \hat{K} - \hat{p}, \quad (m = M/(pK), \quad \bar{\mu} = const. \quad \text{still}),\\
\dot{\pi}^e &= \beta_{\pi^e}(\hat{p} - \pi^e) = \beta_{\pi^e}\kappa[\beta_p(U_c - \bar{U}_c) + \kappa_p\beta_w(V - \bar{V})]
\end{aligned}$$

These are the first three differential equations of the full 6D Keynesian dynamics summarized in subsection 4. The essential elements in these three laws of motion are the three adjustment speeds β_w, β_p and β_{π^e} for wages, prices and the inflationary climate which strongly influence the stability properties of our Keynesian macrodynamics. Note that the law of motion for the capital stock K has not yet been provided, but will be introduced when the full 6D dynamics are presented.

2.2 2D Metzlerian quantity dynamics and growth

Next, we consider the quantity dynamics of the Keynesian macromodel, where we consider goods market adjustment dynamics and capital stock growth. The resulting 3D dynamics provides the quantity side of our Keynesian macrodynamical model:[4]

$$\begin{aligned}
Y^d &= C + I + \delta K + G,\\
\dot{Y}^e &= \beta_{y^e}(Y^d - Y^e) + (n + n_x)Y^e,\\
N^d &= \beta_{n^d}Y^e,\\
\mathcal{I} &= \beta_n(N^d - N) + (n + n_x)N^d,\\
Y &= Y^e + \mathcal{I},\\
\dot{N} &= Y - Y^d,\\
\hat{K} &= I/K.
\end{aligned}$$

[4]These quantity dynamics have recently been studied in isolation, with a nonlinearity in the inventory adjustment process, in Franke and Lux (1993) and with capital stock growth in Franke (1996).

These equations, though many, represent a still simple, yet consistently formulated output and inventory adjustment process. They define aggregate demand Y^d as the sum of consumption, investment and government demand and state that expected sales Y^e follow aggregate demand in an adaptive fashion. Desired inventories N^d are then assumed to be determined as a constant fraction of expected sales, while intended inventory adjustment $\mathcal{I}$ is based on the inventory adjustment process $\beta_n(N^d - N)$, with N the actual inventory holdings and β_n the speed with which the gap between desired and actual inventory holdings is closed, augmented by a term that accounts for trend growth (n the natural rate of growth of the labor force). Actual production Y must then of course be defined by the sum of expected sales and intended inventory changes, while actual inventory changes $\dot{N}$ are finally given by definition by the discrepancy between actual production and actual sales. Again, the crucial parameters in these adjustment equations are the adjustment speeds, β_{y^e}, β_n, of sales expectations and of intended inventory changes. It is obvious from the above presentation of the Metzlerian inventory adjustment process that this process will add two further laws of motion to those of the wage-price dynamics, see the first two equations in the presentation of the full dynamics (7) – (12) below.

We here already add the growth dynamics of the model which in the case of a Keynesian regime is based on the net investment demand of firms as indicated in the last equation of the above quantity dynamics. We briefly state in addition that aggregate demand is based, on the one hand, on differentiated saving habits as far as the two groups of households of the model, workers and asset holders and their consumption functions are concerned. On the other hand, the other part of aggregate demand, investment, is determined by the excess of the expected profit rate over the real rate of interest, on excess capacity and natural growth (including productivity growth). Moreover, there are given fiscal policy parameters for government behavior in the intensive form of the model. We thereby in particular get that aggregate demand depends on income distribution and the wage share u, positively if consumption dominates investment and negatively if the opposite holds true. We add finally that the nominal interest rate is determined by a conventional LM curve or the Taylor interest rate policy rule, to be introduced below.

We already observe here that the short-run quantity dynamics are difficult to estimate, see the next section for some first results in this matter. This is partly due to the need to distinguish between output, demand and sales expectations on the one hand and between desired and actual inventory changes on the other hand. Furthermore, as was remarked by Åke Anderssön after the presentation of this paper on the conference in honor of Professor Puu in Odense, national product also includes services besides goods as a very significant item, which of course in general are not subject to an inventory adjustment mechanism as described above. But also with respect to goods more modern cost-minimizing inventory adjustments must sooner or later be taken into account. Yet, at present a procedure that is a consistent extension of the familiar dynamic multiplier process is all that we need to make the model an internally coherent one.

2.3 The Keynesian or DAS-DAD version of AS-AD growth dynamics

Let us finally make explicit the sixth law of motion, i.e., the one for economic growth, before we collect all laws of motion in the box presented below. As already stated, in a Keynesian context, capital stock growth is given by net investment per unit of capital and thus based on the assumption of an investment function of firms. This function is now postulated to read:

$$i = I/K = i_1(\rho^e - (r - \pi^e)) + i_2(U_c - \bar{U}_c) + n + n_x, \quad U_c = y/y^p, \tag{1}$$

with the expected rate of profit defined by

$$\rho^e = y^e - \delta - uy, \quad y^e = Y^e/K, y = Y/K, \quad u = \omega/x \text{ the wage share}, \tag{2}$$

and the nominal rate of interest given by the reduced form LM-equation

$$r = r_o + \frac{h_1 y - m}{h_2}, \quad m = M/(pK) \text{ real balances per unit of capital.} \tag{3}$$

We use $y = Y/K$ to denote the actual output-capital ratio which – due to the assumed Metzlerian quantity adjustment process – is determined by:

$$y = (1 + (n + n_x)\beta_{n^d})y^e + \beta_n(\beta_{n^d}y^e - \nu), \ y^e = Y^e/K, \nu = N/K. \tag{4}$$

Taken together the investment equation thus entails that net investment depends on excess profitability with respect to the expected real rate of interest, on capacity utilization in its deviation from desired capital utilization and on a trend term which here has been set equal to the natural rate (including the rate of labor productivity growth) for reasons of simplicity.[5]

The sixth state variable of our model is l, the full employment labor intensity, which in the context of Harrod-neutral technical change, $x = Y/L^d, \hat{x} = n_x, \quad y^p = Y^p/K = const.$, is best represented by $l = xL/K$, where L denotes labor supply (which grows at the given natural rate of growth $n = \hat{L}$). Due to the assumed trend growth term in the investment equation shown above we get for the evolution of this state variable

$$\hat{l} = -i_1(\rho^e - (r - \pi^e)) - i_2(U_c - \bar{U}_c)$$

We add as final (algebraic) equation of the model the equation for aggregate demand per unit of capital:

$$y^d = (1 - s_w)uy + (1 - s_c)\rho^e + \gamma + i + \delta = (1 - s_c)y^e + (s_c - s_w)uy + \gamma + i + s_c\delta \tag{5}$$

and the defining equations for the rate of employment, the rate of capacity utilization and κ :

$$V = y/l \quad (= L^d/L = xL^d/xL), \quad U_c = y/y^p, \quad \kappa = 1/(1 - \kappa_w\kappa_p). \tag{6}$$

[5] We note in passing that our Dynamic-AS Dynamic-AD model is at least in one respect still unbalanced, since we make use of a mixture of short- and medium-run expressions in the risk premium term in the investment function. Correcting this basically would introduce a further law of motion – for the investment climate – into the dynamics considered below without too much change in the model's implications, see for example Asada and Flaschel (2002) in this regard.

Due to our assumption of Kaldorian saving habits $s_w < s_c$ (with $s_w = 0$, i.e., classical saving habits as special case) we have that aggregate demand depends positively on the wage share u through consumption and negatively on the wage share through the investment component in aggregate demand. There is wage taxation and property income taxation which are assumed to be constant per unit of capital, the latter net of interest as in Sargent (1987). These fiscal policy parameters as well as government expenditures per unit of capital, assumed to be constant as well, are collected in the parameter γ of the aggregate demand function shown above.

We are now in the position to present the full macrodynamical model, here for brevity immediately in intensive or state variable form. The dynamic model is based on five markets: labor, goods, money, bonds and equities and three sectors: households (workers and asset holders, with Kaldorian differentiated saving habits), firms and the fiscal and monetary authority. We stress again that all budget equations are fully specified on the extensive form level, so that all stock-flow interactions are present, though not yet fully interacting here.[6]

The resulting integrated six laws of motion of the dynamics to be investigated include the state variables: sales expectations $y^e = Y^e/K$ and inventories $\nu = N/K$ per unit of capital, real balances per unit of capital $m = M/(pK)$ and the inflationary climate π^e, the wage share $u = \omega/x$ and labor intensity $l = L/K$. They read:

$$\dot{y}^e = \beta_{y^e}(y^d - y^e) + \hat{l}y^e, \tag{7}$$
the law of motion for sales expectations
$$\dot{\nu} = y - y^d + (\hat{l} - (n + n_x))\nu, \tag{8}$$
the law of motion for inventories
$$\hat{m} = \bar{\mu} - \pi^e - (n + n_x) + \hat{l} - \kappa[\beta_p(U_c - \bar{U}_c) + \kappa_p\beta_w(V - \bar{V})], \tag{9}$$
the growth law of real balances
$$\dot{\pi}^e = \beta_{\pi^e}\kappa[\beta_p(U_c - \bar{U}_c) + \kappa_p\beta_w(V - \bar{V})], \tag{10}$$
the evolution of the inflationary climate
$$\hat{u} = \kappa[(1 - \kappa_p)\beta_w(V - \bar{V}) - (1 - \kappa_w)\beta_p(U_c - \bar{U}_c), \tag{11}$$
the growth law of the wage share
$$\hat{l} = -i_1(\rho^e - (r - \pi^e)) - i_2(U_c - \bar{U}_c), \tag{12}$$
the growth law for labor intensity

These equations can be easily understood from what has been stated about wage-price, quantity and investment dynamics if note is taken of the fact that everything is now expressed – with the exception of the wage share – in per unit of capital form. Inserting

[6] See Chiarella and Flaschel (2000) for the details of this Keynesian working model, including the specification of all budget and behavioral equations on the extensive form level, and Chiarella, Flaschel, Groh and Semmler (2000) for various extensions of this model type.

the algebraic equations (1) – (6) into these laws of motion one obtains a nonlinear autonomous 6D system of differential equations that we will investigate with respect to the stability properties of its unique interior steady state in the remainder of the paper.

2.4 Feedback-guided local stability analysis

As the model is formulated we can distinguish four important feedback chains which we now describe in isolation from each other. Of course these interact with each other in the full 6D dynamics and various feedback mechanisms can become dominant contingent on the model parameters chosen.

1. The Keynes effect: We assume $IS - LM$ equilibrium in order to explain this well-known effect in simple terms. According to $IS - LM$ equilibrium, the nominal rate of interest r depends positively on the price level p. Aggregate demand and thus output and the rate of capacity utilization therefore depend negatively on the price level implying a negative dependence of the inflation rate on the level of prices through this channel. A high sensitivity of the nominal rate of interest with respect to the price level (a low parameter h_2, the opposite of the liquidity trap) thus should exercise a strong stabilizing influence on the dynamics of the price level and on the economy as a whole, which is further strengthened if price and wage flexibility increase.[7]

2. The Mundell effect: We assume again $IS - LM$ equilibrium in order to explain this less well-known (indeed often neglected) effect. Since net investment depends (as is usually assumed) positively on the expected rate of inflation π^e, via the expected real rate of interest, aggregate demand and thus output and the rate of capacity utilization depend positively on this expected inflation rate. This implies a positive dependence of $\hat{p} - \pi^e$ on π^e and thus a positive feedback from the expected rate of inflation on its time rate of change. Faster adjustment speeds of inflationary expectations will therefore destabilize the economy through this channel.

The two discussed effects are working with further delays if Metzlerian quantity adjustment processes are allowed for.

3. The Metzler effect: In the Metzlerian quantity adjustment process, output y depends positively on expected sales y^e and this the stronger, the higher the speed of adjustment β_n of planned inventories. The time rate of change of expected sales therefore depends positively on the level of expected sales when the parameter β_n is chosen sufficiently large. Flexible adjustment of inventories coupled with a high speed of adjustment of sales expectations thus lead to a loss of economic stability. There will, of course, exist other situations where an increase in the latter speed of adjustment may increase the stability of the dynamics.

4. The Rose effect: In order to explain this effect we assume for the time being again $IS - LM$ equilibrium. We know from our formulation of aggregate goods demand that output and in the same way the rate of employment and the rate of capacity utilization may depend positively or negatively on real wages, due to their opposite effects on

[7] The same argument applies to wealth effects which, however, are not yet included here.

consumption and investment shown in eq. (5). According to the law of motion for real wages (7) we thus get a positive or negative feedback channel of real wages on their rate of change, depending on the relative adjustment speed of nominal wages and prices. Either price or wage flexibility will therefore always be destabilizing, depending on investment and saving propensities, i_1, s_c, s_w, with respect to the expected rate of profit and the wage share. The destabilizing Rose effect (of whatever type) will be weak if both wage and price adjustment speeds β_w, β_p are low.

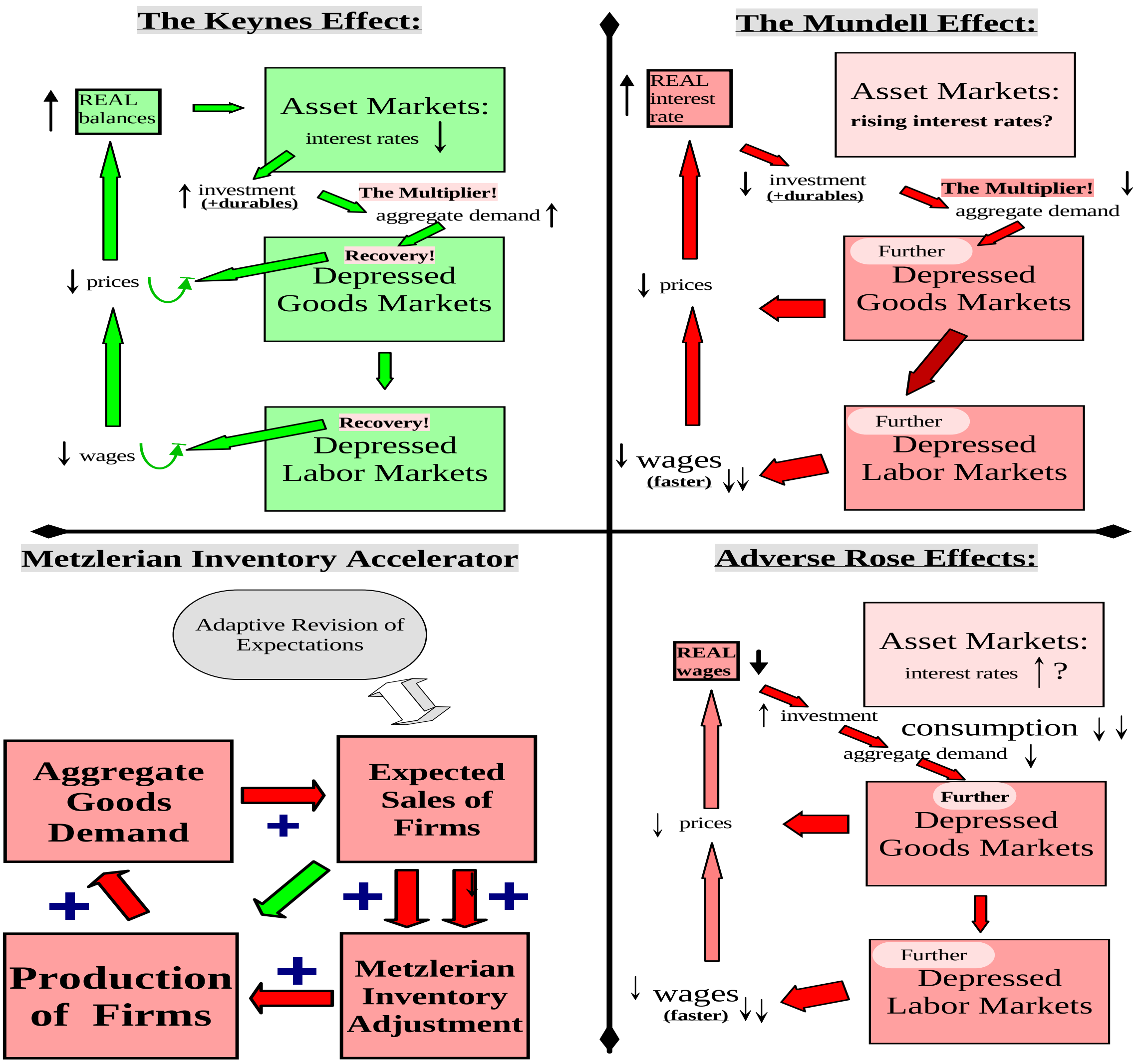

Figure 1: The feedback channels of the model.

The effects just discussed are summarized in figure 1. Note with respect to the Keynes-effect that the corresponding figure only indicates that wage and price decline is eventually stopped. However, since wages and prices will have fallen below their equilibrium values in the course of this process they must in fact rise again in certain periods in order to converge back to their equilibrium values. There is therefore some overshooting involved in the working of this partial feedback chain. Note that figure 1 represents the four feedback chains for the case of wage and price deflation. The case of wage and price inflation is of course obtained by simple reversal of the arrows shown in figure 1.

These more or less traditional feedback channels, the nominal-interest rate Keynes-effect, the inflationary expectations Mundell-effect, the Metzler inventory accelerator and the real-wage Rose effect, are here combined and determine in their interaction the stability of the interior steady state position of the model. If inventories are adjusted too fast instability may arise despite the presence of a stable dynamic multiplier process, due to the fact that production is then too responsive to expected demand changes via the planned inventories channel. The Mundell effect is potentially destabilizing, since inflation feeds expected inflation which in turn lowers the real rate of interest and further increases economic activity and thus the rate of inflation. The Rose effect can be destabilizing in two ways, if aggregate demand depends positively on the real wage and the wage share in the case where wage flexibility exceeds, broadly speaking, price flexibility or in the opposite case of depressing effects of real wage increases if price flexibility exceeds wage flexibility.[8] The only unambiguously stabilizing effect is the Keynes-effect whereby increasing prices and wages decrease real liquidity and thus raise nominal rates of interest which not only stops further wage-price increases, but in fact brings wages and prices back to their 'full' employment levels.

Based on the insights gained from these partial feedback chains we are now in a position to formulate proposition 1 on local stability, instability and limit cycle behavior. We stress that such feedback-guided β-stability analysis can be applied in many other situations as well and nicely confirms, for our integrated Keynesian dynamics, what has since long been known (in principle) for its constituent parts.

Proposition 1:
Assume that $0 \leq s_w < s_c \leq 1$ holds. The following statements hold with respect to the 6D dynamical system (7)-(12) (if fiscal policy parameters are chosen in a plausible way):[9]

1. *There exists a unique interior steady state of the model basically of supply side type.*
2. *The determinant of the 6D Jacobian of the dynamics at this steady state is always positive.*
3. *Assume that the parameters $\beta_w, \beta_p, \beta_{\pi^e}, \beta_n$ are chosen sufficiently small and the parameter β_{y^e} sufficiently large and assume that the Keynes-effect works with sufficient strength (h_2 small). Then: The steady state of the 6D dynamical system is locally asymptotically stable.*
4. *On the other hand, if for example β_{π^e} is sufficiently large, the equilibrium is locally repelling and the system undergoes a Hopf-bifurcation at an intermediate value of this parameter, whereby in general stable or unstable limit cycles are generated close to the bifurcation value.*

[8] A more detailed explanation of such adverse Rose effects has to pay attention to the κ-weights in the cost-pressure terms as well.

[9] See Köper (2000) for a detailed proof of this proposition.

Though intrinsically nonlinear, the above 6D Keynesian growth dynamics are generally, however, too weakly nonlinear in order to guarantee the boundedness of trajectories when the adjustment speeds in the above proposition are made sufficiently large. Extrinsic or behavioral nonlinearities thus have to be added later on in order to ensure boundedness for the trajectories of the dynamics.

Sketch of proof (β-stability methodology): Based on our partial knowledge of the working of the 4 feedback channels of the considered 6D dynamics, we choose an independent 3D subsystem of the 6D Keynesian dynamics in the following way, by setting the parameters $\beta_n, \beta_{\pi^e}, \beta_w$ all equal to zero:[10]

$$\begin{aligned}
\dot{y}^e &= \beta_{y^e}(y^d - y^e) + (n + n_x - i)y^e \\
&\quad \text{the stable dynamic multiplier} \\
\widehat{m} &= \bar{\mu} - \kappa\beta_p(U_c - \bar{U}_c) - \pi^e_o - i \\
&\quad \text{the stabilizing Keynes effect} \\
\hat{u} &= -\kappa(\kappa_w - 1)\beta_p(U_c - \bar{U}_c) \\
&\quad \text{sluggish price adjustment}
\end{aligned}$$

In this 3D system, the Keynes-effect (h_2 small) and the dynamic multiplier (β_{y^e} large) dominate the outcome and imply the Routh-Hurwitz conditions for local asymptotic stability if they operate with sufficient strength and if β_p is sufficiently small (which avoids stability problems arising from any type of Rose effect).

We then add step-by-step the further laws of motion by assuming that those adjustment speeds initially assumed to be zero are made slightly positive:

$$\begin{aligned}
&4D: \ \beta_w > 0: \ \hat{l} = -i_1(\rho^e - (r - \pi^e)) - i_2(U_c - \bar{U}_c), \\
&\qquad \text{labor intensity feeds back into the 3D dynamics via V=y/l}, \\
&5D: \ \beta_n > 0: \ \dot{\nu} = y - y^d + ..., \\
&\qquad \text{inventory accumulation feeds back into the 4D dynamics via y }, \\
&6D: \ \beta_{\pi^e} > 0: \ \dot{\pi}^e = \beta_{\pi^e}[c_1\beta_p(U_c - \bar{U}_c) + c_2\beta_w(V - \bar{V})], \\
&\qquad \text{inflationary climate starts moving and influencing the 5D dynamics .}
\end{aligned}$$

Since the determinants of the Jacobian at the steady state of the sequentially enlarged dynamics always have the correct sign, as demanded by the Routh-Hurwitz conditions, we know that the eigenvalue that is departing from zero by making a certain adjustment speed slightly positive must always become negative. In this way, a system with at most one pair of complex eigenvalues (with negative real parts) and at least four real and negative ones is established, which proves the local asymptotic stability asserted in the above proposition, here still even with monotonic convergence generally. Since the determinant of the full Jacobian is always nonzero, loss of stability can only occur

[10] See again Köper (2000) for the full presentation of such a stability investigation by means of varying adjustment speeds β_j where $j = n, \pi^e, w$.

by way of (in general non-degenerate) Hopf-bifurcations, at which eigenvalues cross the imaginary axis with positive speed. ■

This by and large closes the theoretical section of the paper which however is extended in the next section through the introduction of two types of monetary policy rules, for which similar propositions can be formulated, and a nonlinear money wage PC (which helps to bound the dynamics in the case of locally explosive steady states). Apart from such propositions, which can also be proven for a variety of extensions of the considered dynamics, we have, however, to rely on numerical methods in order to gain further insight into the kind of dynamics that is generated by our Keynesian AS-AD dynamics.

3 Extending the Keynesian growth dynamics

In this section we introduce two types of monetary policy rules that augment the dimension of the considered dynamics by one to 7D. The extended dynamics preserves the results obtained in the first part of the paper in a natural way, if policy parameters are chosen sufficiently low. These monetary policy rules need however not be stabilizing for choices of the parameters obtained from empirical estimates.

We then present estimates of the parameters of these extended dynamics, taken from Flaschel, Gong and Semmler (2001), together with some simulation runs for which they still work as intended, at least to some extent. The steady state of the dynamics is – with the estimated parameters – in both cases slightly unstable, since an adverse Rose effect dominates the outcome in the observed situation. This can be and has been remedied in Flaschel, Gong and Semmler (2001) by increasing the parameter for the inflation target of the two monetary policy rules which provides one possibility for overcoming the local instability of the private sector. However, we shall see that this possibility for stabilizing an unstable economy is of limited power, since it may exist for a very restricted region of the employed policy parameters solely. It is therefore not a priori clear whether an increase, for example, of the parameter that is meant to control the evolution of inflation, will improve the situation or make it worse.

There is, however, an important institutional feature of modern market economies, a nonlinearity in the money-wage Phillips Curve that may imply that local instability cannot give rise to global instability if this feature is added to the considered dynamics. The importance of this feature, in stylized form a kink in the money wage PC, expressing downward rigidity of nominal wages, had already been discussed in Keynes (1936) and has recently been estimated for various countries by Hoogenveen and Kuipers (2000), who even obtain the result that not only are wages downwardly rigid, but also the rate of wage inflation, which they found to rarely fall below 2 percent. We will use these findings here only in the stylized form that assumes that the money wage level can rise according to our linear wage PC, but will never fall. Wage deflation will thus now be excluded from consideration. This will imply that even for high wage adjustment speeds in the inflationary area we will get trajectories in the modified dynamics that remain bounded in economically meaningful domains, giving rise to limit cycle behavior of more complex types of attractors and thus to persistent fluctuation of more or less regular type in these domains.

These fluctuations as well as the instabilities of the case of a linear money wage PC can be tamed to some extent by anti-inflationary money supply or interest rate policy rules. Yet, in order to achieve this one has to choose parameters in certain corridors which in principle are unknown to the policy makers. Thus, an active, in contrast to a passive monetary policy will not necessarily increase stability.[11] It may therefore happen that a policy maker suggests that the money supply or interest rate dynamics should react more strongly to deviations of the inflation rate from the inflation target of the central bank, but that the result of this suggested tighter policy increases instability, i.e., creates larger fluctuations than were observed before. This will be demonstrated below by means of numerical simulations for explosive dynamics as well as for the locally explosive, but globally bounded dynamics generated by the kinked wage PC.

3.1 Introducing monetary policy rules

Let us first extend the employed model by means of one of the following two monetary policy rules:

1. Money Supply Rule:
   $\dot{\mu} = \beta_{m_1}(\bar{\mu} - \mu) + \beta_{m_2}(\bar{\pi} - \hat{p}) + \beta_{m_3}(\bar{U}_c - U_c), \quad \bar{\mu} = \bar{\pi} + (n + n_x).$
2. Taylor Interest Rate Policy Rule:
   $\dot{r} = -\beta_{r_1}(r - r_o) + \beta_{r_2}(\hat{p} - \bar{\pi}) + \beta_{r_3}(U_c - \bar{U}_c), \quad r_o$ the steady state value of r.

The first rule, a money supply growth rule, states that the growth rate of the money supply is changed on the basis of two targets and one restriction. The first aim of monetary policy is to steer the currently evolving rate of inflation to the target rate $\bar{\pi}$ by lowering the rate of growth of money supply if inflation is too high in view of this target (and vice versa). In correspondence to this anti-inflationary type of behavior we assume that the monetary authority wants to steer the economy to the growth rate of money supply given by $\bar{\mu} = n + n_x + \bar{\pi}$, the steady state rate of nominal growth. Fighting too high or too low inflation (or even deflation) and moving the economy towards its steady state are thus the aims of this type of monetary policy which are pursued in a stronger way if the business cycle is in an expansion ($U_c - \bar{U}_c > 0$), and in a weaker way in the opposite case. Money supply is therefore no longer growing at a constant rate, but responding to its deviation from normal growth, the (perfectly anticipated) inflation gap and the capacity utilization gap of firms. The 6D dynamics of the preceding section with the above used two linear PC's thereby becomes 7D, with the law of motion for μ added to the other laws of motion (and μ in the place of $\bar{\mu}$ in the law of motion for real balances m).

[11]For stability results concerning active and passive monetary policy see Benhabib et al. (2001).

Proposition 2:
The following statements hold with respect to this 7D dynamical system with a money supply policy rule if $\bar{\pi} = \bar{\mu} - (n + n_x)$ holds and if fiscal policy parameters are chosen in an economically meaningful range:

1. *The unique interior steady state of the 7D model is the same as for the 6D dynamics ($\mu_o = \bar{\mu}$).*
2. *The determinant of the 7D Jacobian of the dynamics is always negative.*
3. *Assume in addition to the stability assumptions of the 6D case that the parameters β_{m_2}, β_{m_3} are chosen sufficiently small. Then: The steady state of the 7D dynamical system is locally asymptotically stable.*
4. *On the other hand, if for example β_{m_2} is sufficiently large, the equilibrium can be locally repelling and the system undergoes a Hopf-bifurcation at an intermediate value of this parameter.*

This proposition basically says that the assumed monetary policy rule does not endanger local asymptotic stability if operated sufficiently weakly with respect to the inflation target and the state of the business cycle. The range of policy parameter values that allow for this conclusion may however be a very specific or limited one, in particular in the situation where the private sector is not locally asymptotically stable.

Sketch of Proof: By means of the law of motion for l we can reduce the law of motion for m to $\mu - \hat{p}$ without change in the sign of the determinant of the 7D Jacobian of the dynamics at the steady state. Similarly, we can remove the demand pressure term $V - \bar{V}$ from the law of motion for π^e which leaves the term $U_c - \bar{U}_c$ in this law, again without change in the sign of the considered determinant of the thereby reduced dynamics. These two simplified laws of motion can then in turn be used to remove the $\hat{p}$ and $U_c - \bar{U}_c$ expressions from

$$\dot{\mu} = \beta_{m_1}(\bar{\mu} - \mu) + \beta_{m_2}(\bar{\pi} - \hat{p}) + \beta_{m_3}(\bar{U}_c - U_c),$$

which in fact increases the negative influence of μ on its rate of change by the addition of further terms of this type. Again the sign of the considered determinant does not change under the considered manipulation of the laws of motion of the dynamics. Its 7th row, corresponding to the added monetary policy rule, does however now exhibit a negative entry only in its seventh element (and is zero otherwise). There follows that the 7D determinant has the opposite sign from the 6D determinant considered in the first part of the paper. This basically suffices for the proof of the above proposition if note is taken of the fact that its last assertion can only be proved by way of numerical examples (to be considered below). ∎

We stress that the eigenvalues of the 6D subdynamics are moved only slightly in the considered situation and remain negative with respect to their real parts. These real parts need not, however, become more negative in the considered situation, i.e., the considered policy rule need not improve the stability of the 6D subdynamics. In order to show this one has to investigate the full set of Routh-Hurwitz conditions which is an impossible task at this level of dimensionality.

Turning to the second policy rule, the Taylor type interest rate policy rule, we have instead that nominal interest is raised (lowered) if inflation exceeds (is lower than) the inflationary target and that this policy is exercised in a stronger way in the case of booms (excess demand in the goods market) than in the case of recessions or depressions (excess supply in the goods market). In addition, there is now interest rate smoothing with respect to the steady state nominal rate of interest in the place of money growth smoothing. Note that both these smoothing processes are built on targets that are consistent with the steady state behavior of the dynamics. In the case of a Taylor rule we have again augmented the original 6D dynamics by one dimension to a 7D dynamical system. Yet, in this case, real balances per unit of capital, m, no longer feed back into the rest of the system which means that this law of motion is now an appended one and can be suppressed in the following stability analysis of the interacting state variables.[12]

Proposition 3:
The following statements hold with respect to the 6D dynamical system obtained by adding the Taylor rule to the original 6D dynamics and ignoring the law of motion of real balances:[13]

1. *The unique interior steady state of this modified 6D model is the same as the one for the original 6D dynamics (ignoring real balances m).*
2. *The determinant of the 6D Jacobian of the interacting dynamics is always positive.*
3. *Assume in addition to the stability assumptions of the original 6D case that the parameters β_{r_2}, β_{r_3} are chosen sufficiently small. Then: The steady state of this reformulated 6D dynamical system is locally asymptotically stable.*
4. *On the other hand, if for example β_{r_2} is sufficiently large, the equilibrium can be locally repelling and the system undergoes a Hopf-bifurcation at an intermediate value of this parameter.*

This proposition basically again says that the assumed monetary policy rule does not endanger local asymptotic stability if operated sufficiently weakly with respect to the inflation target and the state of the business cycle. The range of policy parameter values that allow for this conclusion may however be a very specific or limited one, in particular in the situation where the private sector is not locally asymptotically stable.

Sketch of Proof: The proof is of the same type as the one for the original 6D dynamics investigated in subsection 2.3, if the $\hat{m}$ dynamics in the 3D subsystem considered there

[12]The evolution of real balances m should however be confined to a compact set in the positive domain which is not difficult to show if the other state variables are confined to such a set.

[13]The evolution of real balances is now dependent on historical conditions (subject to zero root hysteresis) since the determinant of the 7D dynamics at the steady state is zero.

is replaced by the $\dot{r}$ dynamics of the now employed Taylor rule. ■

In the case of the 6D dynamics considered in the preceding section we have high interest rate sensitivity if the stipulated LM-curve is nearly vertical (the classical case) and found that this is stabilizing (a strong Keynes-effect). Interest sensitivity by way of increasing the second or third parameter in the Taylor rule is not necessarily doing the same job, since we can get instability from stability by increasing these parameters (as is shown numerically below).

3.2 The dynamics with estimated parameters

The two model variants considered in the preceding subsection have been estimated in Flaschel, Gong and Semmler (2001) by means of single equation or appropriate subsystem estimates. For this purpose, the dynamics had to be translated into discrete time. This is simply done by replacing all differential quotients by difference quotients on the left hand side of the equations. This means that growth rates $\hat{x}$ are then represented by $\Delta \ln x$ and time derivatives by Δx on the left hand side of the laws of motion (7) – (12) and also in the two employed policy rules 1 and 2. In all other respects the dynamics are the same as the ones studied in section 2 and in this section.[14]

However, the hybrid measure $\rho^e - (r - \pi^e)$ of excess profitability used so far was substituted in the empirical application of the model by a more balanced one, namely a moving average of the excess profitability $\rho^e - (r - \hat{p} + \xi)$, here in addition augmented by a fixed positive risk premium ξ. On this basis, the two employed policy rules are quantified in the following estimates of the parameters of the model (and found to be too weak).

Estimated parameters of the two model variants are reported in table 1, estimated partly through single equation and partly through subsystem estimations for U.S. time series data 1960.1-1995.1. The details on these estimates and the t-statistics are provided and discussed in detail in Flaschel, Gong and Semmler (2001).[15]

[14]The discrete time version of the considered dynamics is presented in full detail in Flaschel, Gong and Semmler (2001) on the extensive as well as on the intensive form level.

[15]Making use of the term $i_1\epsilon^m$ in the investment function in the place of the original term, where ϵ^m, the investment climate, is determined by:

$$\dot{\epsilon}^m = \beta_{\epsilon^m}(\epsilon - \epsilon^m), \quad \epsilon = \rho^e + \hat{p} - (r + \xi),$$

provides an extended dynamics with delayed adjustment of investment to currently expected excess profitability. This extension of the dynamics allows for similar stability propositions as the ones of this paper and is used in the form of a moving average in the estimation procedure.

Table 1: Estimated Parameters (US-Data)[16]

1	w-p-dynamics	$\beta_p = 0,\ \kappa_p = 0.34,\ \beta_{\pi^e} = 0.65$
		$\beta_w = 0.1,\ \kappa_w = 0.9\ \bar{V} = 0.9, \bar{U}_c = 0.82$
2	expected sales	$\beta_n = 0.04,\ \beta_{n^d} = 0.47,\ \beta_{y^e} = 1.26$
3	savings	$s_c = 0.62,\ s_w = 0.05$
4	investment	$i_1 = 0.13,\ i_2 = 0.034$
5	money demand	$h_1 = 0.17,\ h_2 = 2.14$
	(money rule)	$\beta_{m_1} = 0.55,\ \beta_{m_2} = 0.05,\ \beta_{m_3} = 0.05$
6	money demand	$h_1 = 0.17,\ h_2 = 2.14$
	(Taylor rule)	$\beta_{r_1} = 0.05,\ \beta_{r_2} = 0.07,\ \beta_{r_3} = 0.015$
7	other parameters	$y^p = 0.25,\ \gamma = 0.083,\ \delta = 0.048$
		$n + n_x = 0.008$
		$\beta_x = 0.47,\ \bar{\mu} = 0.0154,\ r_0 = 0.02$

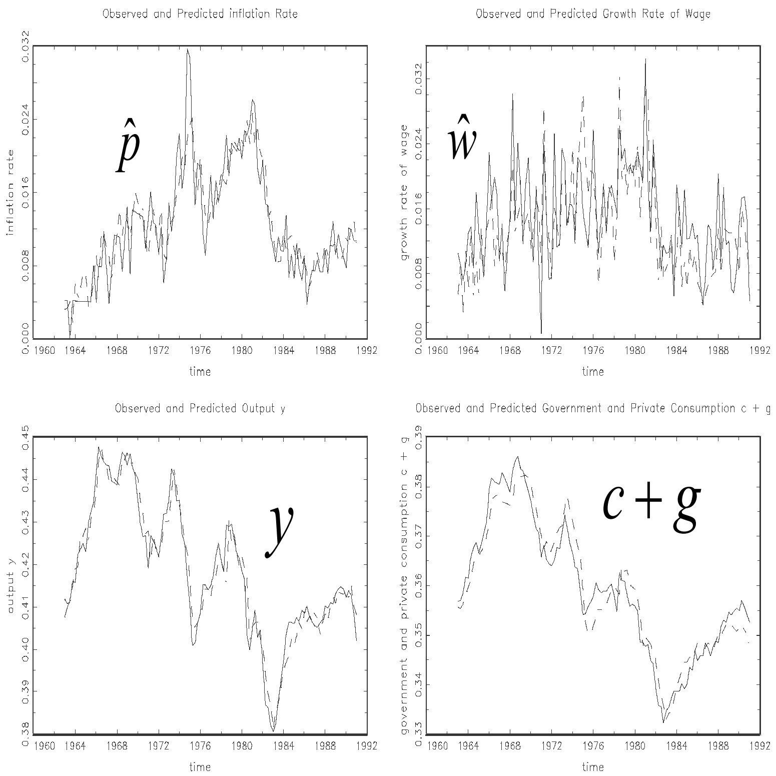

Figure 2: Observed and predicted variables

[16]Flaschel, Gong and Semmler (2002) show similar results to the ones presented here for the case of the German economy, while Flaschel and Krolzig (2002) obtain with respect to the wage and price PC's $\beta_w = 0.193, \beta_p = 0.039, \kappa_w = 0.266, \kappa_p = 0.286$ using different data for the US-economy. In all these cases we have – from the partial perspective – a destabilizing Rose effect. We note again that the estimates for investment and money demand are not yet really convincing and that the inventory dynamics is subject to the problems raised when it was first formulated here.

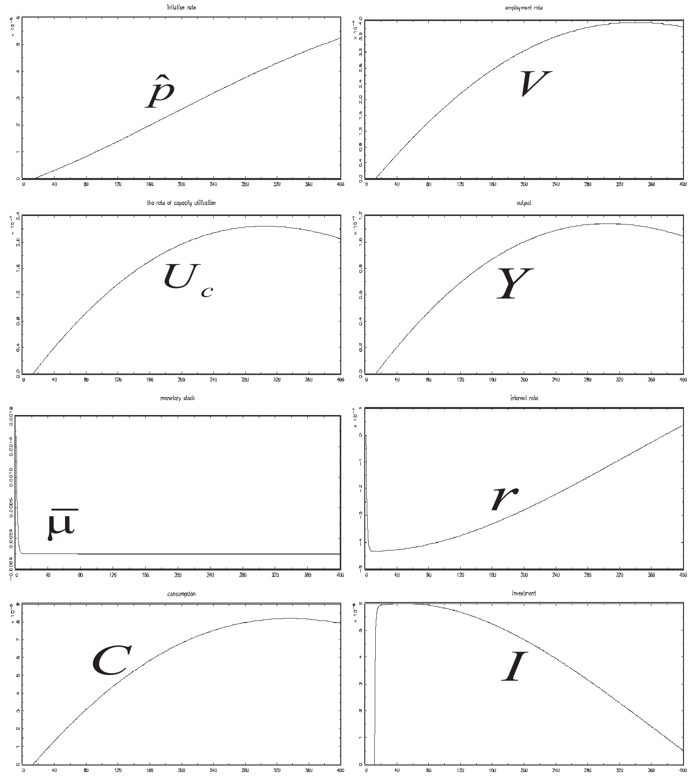

Figure 3: Impulse-responses for the money supply rule

Using the estimated parameters we, for example, obtain the in-sample fit of single variables shown in figure 2. The four examples shown in figure 2 indicate how well the model fits the data, but we should point out that the fit is significantly less convincing in the case of investment and money demand (not shown here), which is not too surprising. As impulse-response simulations resulting from a (positive) money supply shock we obtain in figure 3 reasonable results from a Keynesian perspective , and in the case of the Taylor interest rate policy rule the results look even more convincing. This also holds for the type of simulations shown in figure 2.[17]

The estimated parameter values – taken together – suggest in view of figure 1 that the Keynes-effect, the Mundell-effect and the Metzler accelerator effect are weak, while the Rose effect implies an adverse real wage adjustment, basically based on the result that aggregate demand is found to depend positively on the real wage and that wage flexibility is larger than price flexibility. We thus should expect – and find this in figure 4 – that the steady state of the dynamics is surrounded by (slightly) explosive forces.[18] The economy therefore operates slightly above the Hopf-bifurcation point where local asymptotic stability is lost.[19]

Since interest rate sensitivity, as characterized by the parameter h_2, is high, we should expect that the Taylor rule works better than the money supply rule. This is indeed the

[17]For details see Flaschel, Gong and Semmler (2001).

[18]The above estimates imply $y^d_u = s_c y \approx 0.27u$, $\hat{u}_y = \frac{1-\kappa_p}{1-\kappa_p \kappa_w} \frac{\beta_w}{y^p} \approx 0.18$ which indeed provides a strong positive feedback of the wage share on its rate of growth.

[19]Such a finding also holds for the Bergstrom model of the UK economy, see Barnett and He (1999) for a numerical study of this macroeconometric model.

case, but nevertheless does not suppress the local instability of the dynamics as figure 4 exemplifies, possibly due to the fact that both policy rules only operate weakly. Increasing to some extent the parameters of the two policy rules that concern the influences of the inflation gap and the output gap will in the situation currently under consideration bring about convergence to the steady state of the dynamics, which is what one would expect from the application of such policy rules.

The destabilizing force in the presented estimates thus seems to be an adverse Rose-type real wage effect. Figure 5 again summarizes the various possibilities of real wage adjustments when income distribution and wage-price flexibility matter. We see that the situation that seems to characterize our empirical findings is given by the case 2.b in figure 5 (corresponding to the first row in the preceding feedback chain representations).

3.3 The kinked money wage Phillips Curve

Empirical observations, already made by Phillips (1958), and recently reinforced especially in the empirical study of Hoogenveen and Kuipers (2000), suggest that the Phillips Curve cannot be linear from the global point of view. The local and global instability obtained from the above estimates suggest a reconsideration of our model for the case where the trajectories depart too much from the steady state. In order to investigate the role of such nonlinearity briefly, we follow Hoogenveen and Kuipers (2000) to some degree and consider now a simple border case of their observations, namely a stylized money wage PC that is defined as follows:

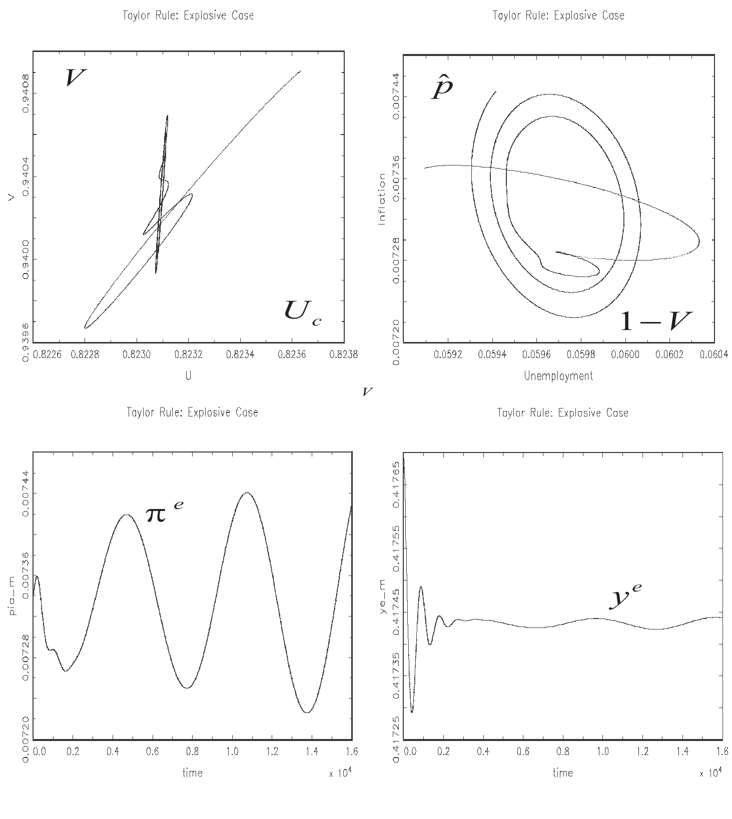

Figure 4: Simulation of the model with Taylor rule (unstable case)

The Four Partial Rose Real Wage Adjustment Mechanisms

$$- - -- \Rightarrow C \uparrow Y^d, Y^e, Y \uparrow w \uparrow w/p \uparrow$$

$$- - -- \Rightarrow C \uparrow Y^d, Y^e, Y \uparrow p \uparrow w/p \downarrow$$

$$w/p \uparrow$$

$$- - -- \Rightarrow I \downarrow Y^d, Y^e, Y \downarrow w \downarrow w/p \downarrow$$

$$- - -- \Rightarrow I \downarrow Y^d, Y^e, Y \downarrow p \downarrow w/p \uparrow$$

Normal Rose Effects:

1a. Real wage increases (decreases) will be reversed in the case where they reduce (increase) economic activity when nominal wages respond stronger than the price level to the decrease (increase) in economic activity

1b. Real wage increases (decreases) will be reversed in the case where they increase (reduce) economic activity when the wage level responds weaker than the price level to the increase (decrease) in economic activity

Adverse Rose Effects:

2a. Real wage increases (decreases) will be further increased in the case where they reduce (increase) economic activity when the wage level responds weaker than the price level to the decrease (increase) in economic activity

2b. Real wage increases (decreases) will be further increased in the case where they increase (reduce) economic activity when the wage level responds stronger than the price level to the increase (decrease) in economic activity

Figure 5: The dominant Rose effect is of form 2b. in the estimated Keynesian dynamics.

$$\widehat{w} = \max\{\beta_w(V - \bar{V}) + \kappa_w(\hat{p} + n_x) + (1 - \kappa_w)(\pi^e + n_x), 0\}.$$

Hoogenveen and Kuipers (2000) find in fact that there exist lower limits even to wage inflation, which in their findings is bounded away from zero to a significant degree. Our above formula assumes a weaker type of nonlinearity, since it only states that the wage level may rise as before under corresponding circumstances, but will never fall. This Phillips Curve thus assumes that money wages behave as in the preceding section if their growth rate is positive, but stay constant if they would be falling in the situations considered so far. There is thus no wage deflation possible now. The kink could be smoothed and restricted wage deflation could even be allowed for, without altering the conclusions of this section significantly. We consider such kinked money wage PC's a much better description of reality than the former strictly linear one.

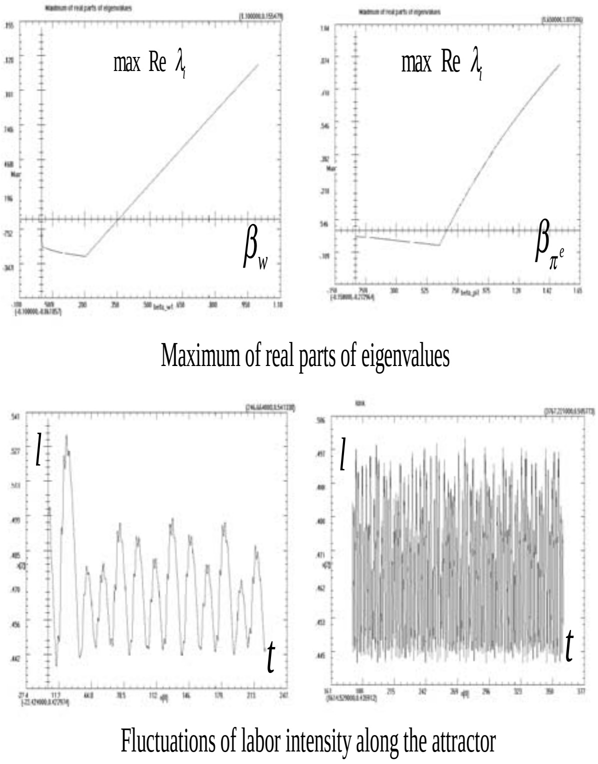

Figure 6a: The dynamics with kinked wage PC.

Increasing the adjustment speed of the inflationary climate to a sufficient degree makes the originally considered dynamics generally unstable and in fact much more explosive and non-viable than is indicated by figure 4. However with the above kink in the money wage PC in operation, the instability of the steady state remains bounded in an economically meaningful domain even for large values of this adjustment speed. This kink provides therefore a powerful institutional means of obtaining economically viable dynamics in the place of purely explosive and collapsing ones. This is exemplified in figures 6a and 6b.[20]

In the top row of figure 6a we have plotted the maximum real part of the eigenvalues of the dynamics at the steady state against the parameters that characterizes wage flexibility and the speed of adjustment of the inflationary climate. As one can see, the steady state loses its stability when one of these parameters becomes sufficiently large (greater than 0.2 and 0.6, due to the working of the Rose and the Mundell effect, respectively). The orbits generated by the dynamics however remain bounded as the bottom time series for labor intensity l exemplifies. They show the movement of labor intensity l along the attractor and indicate irregular economic fluctuations that represent

[20]The numerical investigations of this section were performed with the software package SND, see Chiarella, Flaschel, Khomin and Zhu (2001) for its description. The software package and a range of projects or applications can be downloaded from the homepage of Carl Chiarella at UTS.

complex dynamics from the mathematical point of view. These persistent fluctuations are transformed into damped ones if the parameter h_2 that characterizes the interest rate sensitivity of the money demand function is decreased (from 0.1 to 0.06) whereby the Keynes-effect is made stronger and indeed then stabilize the economy.[21]

Figures 6b shows in more detail that some sort of complex dynamics is generated as the speed of adjustment of the inflationary climate towards the actually observed inflation rates is increased, already indicated by the times series presented in figure 6a. We show in figure 6b the projection of the attractor (and the transient towards it) in the $\omega, l-$plane (for $\beta_{\pi^e} = 1$), the bifurcation diagrams for the state variables ω and l for different lengths of transients and, the calculation of the maximum Liapunov exponent. In the bifurcation diagrams we have plotted vertically all local maxima and minima of labor intensity l and the real wage ω ($n_x = 0$ here) for a certain range of the parameter β_{π^e}. We can see that nearly all amplitudes for the fluctuation of the real wage become possible (in a certain range that increases with the considered speed of adjustment) if this parameter is sufficiently high, with both transient phase and explicitly shown simulation phase chosen sufficiently large. We conclude that integrated Keynesian dynamics can give rise to interesting irregular economic fluctuations (that are complex from the mathematical point of view) in parameter ranges that are not implausible from the economic point of view.

3.4 The role of monetary policy rules reconsidered

We now add again the money supply and interest rate policy rules 1 and 2 of subsection 3.1 and consider first two eigenvalue diagrams for the parameters that refer to the inflationary gap and the state of the business cycle. We start – as in the empirical example – from a situation of unstable dynamics of the private sector and see that all these policy parameters can enforce stability, when increased from zero to certain positive values. Yet, after a certain corridor they lose their stabilizing power again in a pronounced way. This also holds for the parameters β_{m_1} and β_{r_1}, characterizing money supply and interest rate smoothing, which implies that the negative entries in the trace of the Jacobian of the dynamics caused by these items also do not always work in favor

[21]Other crucial parameter values in this simulation of the 6D dynamics are $s_c = 0.8, s_w = 0, i_1 = 0.25, \beta_w = 2.5, \beta_p = 1.5, \kappa_w = \kappa_p = 0.5$. In this situation, the adverse Rose effect dominates, since aggregate demand depends positively on the wage share and since wage flexibility exceeds price flexibility. The very explosive local situation is nevertheless tamed by the kink in the money wage PC, and gives rise to complex dynamics.

of stability.

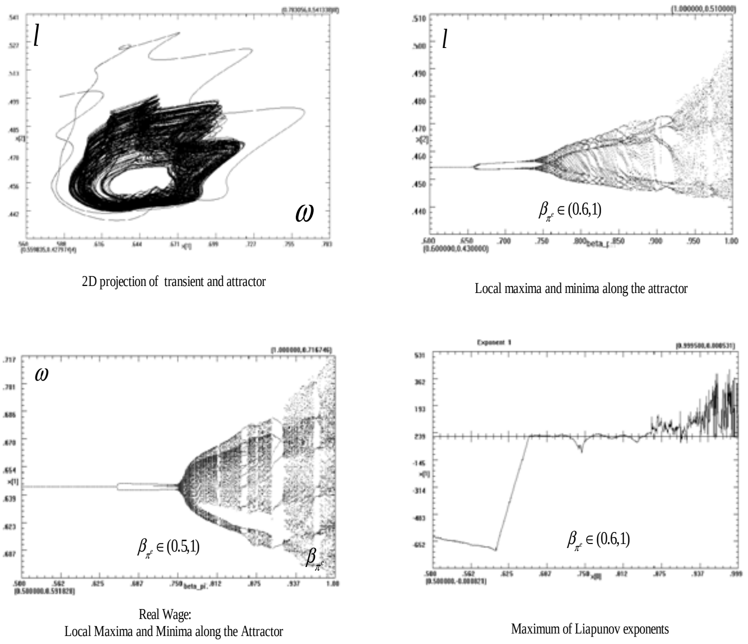

Figure 6b: The dynamics with kinked wage PC.

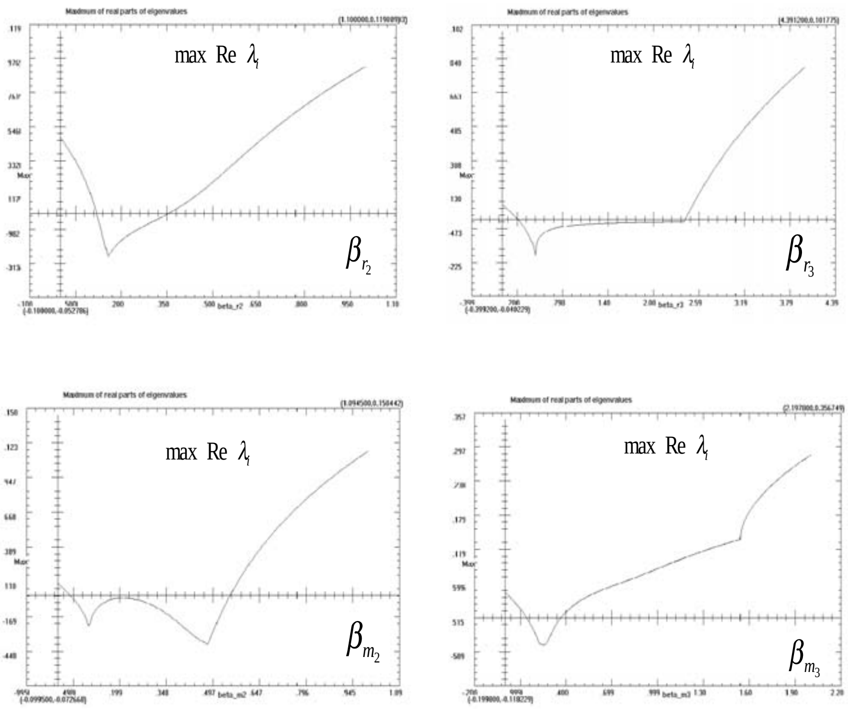

Figure 7: Local stability regions for money supply and interest rate policy rules.

Figure 7 thus demonstrates that there may be only small domains for the parameters of the monetary policy rules where these rules indeed successfully stabilize the economy.

Policy parameters that are too low or too high allow for or even increase local instability in the considered situation. This means that it is not clear in a given situation whether these policy parameters should be increased (a more active monetary policy) or decreased (a more passive policy) in order to dampen business fluctuations. There are therefore examples – even for relatively small values of these parameter values – where it may be wiser to make anti-inflationary policy weaker instead of making it stronger in view of the observed fluctuations. This is the corridor problem of monetary policy, which thus has to determine on which side of the corridor it is operating.[22]

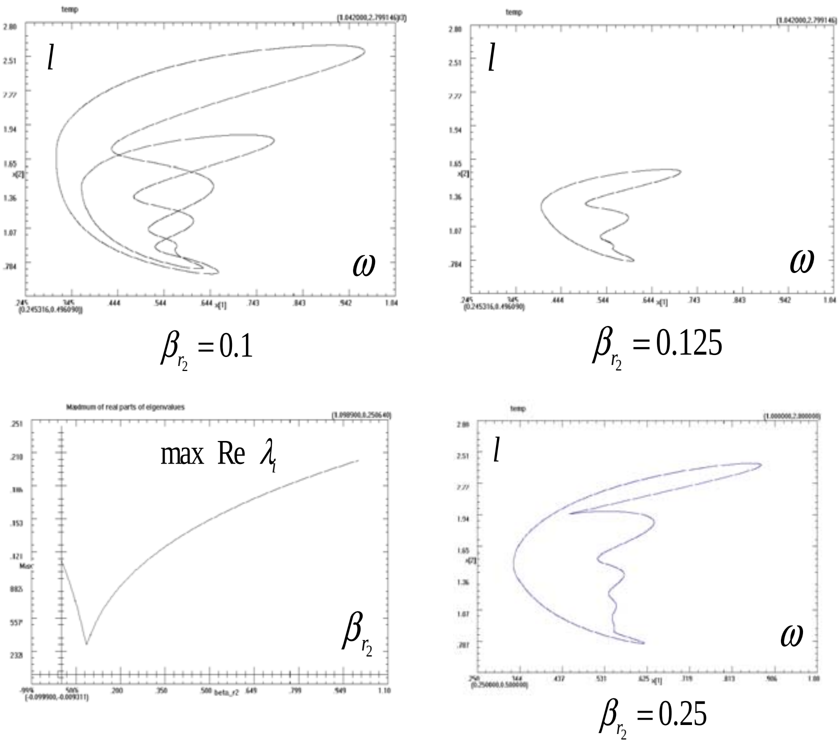

Figure 8: Impacts of interest rate policy rules on persistent fluctuations.

Next we increase the parameter β_w to 0.5 and get thereby a repelling steady state. We assume in addition a kinked money wage PC and consequently get globally bounded dynamics, as exemplified by the three attracting limit cycles shown in figure 8. We here consider the Taylor interest rate policy rule and find in the present situation that no choice of the parameter β_{r_2} will produce convergence, as shown in the eigenvalue diagram bottom left. Yet, the degree of local instability is varying with this parameter and this has global implications. Indeed, a given limit cycle situation, shown top left, will be improved (towards smaller limit cycles) if this parameter is moved towards smaller values of the maximum real parts of eigenvalues (the figure top right), while limit cycles will increase again if the maximum real part of eigenvalues is increasing again (the figure

[22]The base parameter values in the simulations shown are: $\beta_w = 0.1, \beta_p = 2, \kappa_w = \kappa_p = 0.5, s_c = 0.6, s_w = 0.05, i_1 = 0.1, \beta_{\pi^e} = 0.1$. The Rose effect is therefore again destabilizing with respect to wage flexibility and stabilizing with respect to price flexibility.

bottom right). The corridor problem of monetary policy thus also applies when more complex attractors than just point attractors are considered. We note in passing that a floor to wage deflation of -1 percent in the place of no wage deflation will make the limit cycle much larger, while a floor of -10 percent will completely remove the stable limit cycle and will make the dynamics purely explosive and economically non-viable.

With a final numerical example we show[23] – for the case of the money supply growth rule – again a limit cycle situation (for $\beta_{m_2} = 0.5$) which however is turned into convergence if the reaction to the inflation gap is made more moderate ($\beta_{m_2} = 0.4$). This therefore provides an example that a more active monetary policy can in fact be destabilizing.[24] This finding is again related to the eigenvalue calculations shown bottom left, which again – though locally in nature – predict correctly what happens in the situation of the more complex limit cycle dynamics, generated by a kinked money wage PC. The plots on the right hand side of figure 9 finally show that the Rose effect – which here again predicts destabilizing wage and stabilizing price flexibility, due to the chosen parameter values in the aggregate demand function – need not completely dominate the outcome on price flexibility, since instability due to the destabilizing Mundell effect may from some point onwards dominate the Rose effect in this regard.

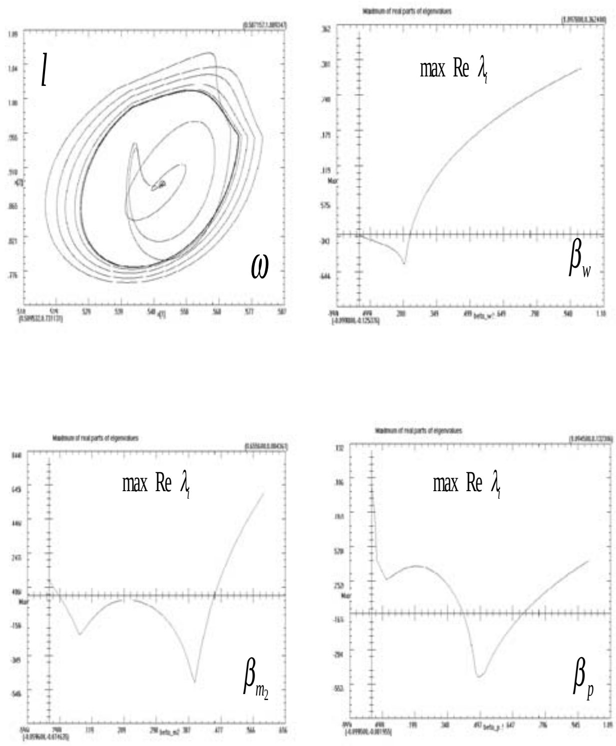

Figure 9: Impacts of money supply policy rules on persistent fluctuations and the role of wage / price flexibilities.

[23]The important base parameter values here are: $\beta_w = 0.2, \beta_p = 0.5, \kappa_w = \kappa_p = 0.5, s_c = 0.6, s_w = 0.08, i_1 = 0.2, \beta_{\pi^e} = 0.2$. The Rose effect is therefore again destabilizing with respect to wage flexibility and stabilizing with respect to price flexibility.

[24]The fact that a more active monetary policy can be destabilizing is also shown in Benhabib et al. (2001), there however in an optimizing macro-model.

3.5 Conclusions

We have shown in this paper that a proper Keynesian version of AS-AD growth dynamics demands at least six laws of motion that integrate prominent feedback chains of Keynesian macrodynamics. We have also shown that the steady state of these dynamics with our standard PC's is asymptotically stable, broadly speaking, for sluggish adjustment speeds and conjecture on the basis of numerical experience that it often becomes purely explosive soon after the necessary occurrence of Hopf-bifurcations when adjustment speeds become larger. Monetary policy can help to avoid this explosiveness to some extent, but only for a limited range of adjustment speeds and policy parameters.

Estimation of the parameters of the dynamics indeed showed that mildly explosive forces around the steady state may be given in reality. To this situation we then superimposed an important behavioral nonlinearity, a kink in the money wage Phillips Curve in the place of the linear wage PC and argued that this kink may radically increase the viability of the dynamics for large ranges of its adjustment speeds, leading to some sort of complex dynamics eventually when adjustment speeds become sufficiently large. Again, policy can help, by reducing economic fluctuations to some extent, but this only for a limited range of its parameters (that may not easily be determined).

Our basic conjecture for future research therefore is that steady states of Keynesian macrodynamics are typically surrounded by centrifugal forces which come to a halt and give way to more or less complex real and nominal fluctuations if in particular an important nonlinearity, the kink in the money wage Phillips Curve, comes into operation. Monetary policy rules – only if their parameters are correctly chosen in a certain corridor – can improve the resulting situation of persistent, often irregular fluctuations, by reducing amplitudes or even bring about a return to the steady state under certain conditions. Monetary policy by itself, however, being more active, does not automatically deliver better stability results. We conclude that the prevailing studies of only damped fluctuations and of only monetary shocks applied to such convergent processes represent a much too limited scenario to really grasp the implications of properly formulated Keynesian labor and goods market (price and quantity) adjustment processes when all markets dynamically interact and the aforementioned four macroeconomic feedback mechanisms are at work. Monetary policy has to learn to live with the kink in the money wage Phillips Curve and the irregular endogenous fluctuations generated by it in interaction with the unstable feedback chains of non-market clearing macrodynamics.

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