Interacting two-country business fluctuations: 'Euroland and the USA'

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Abstract

In this paper we investigate the closed-economy Keynes-Wicksell-Goodwin model of Chiarella and Flaschel (2000) for the case of two interacting open economies. We introduce these coupled two-country KWG dynamics on the extensive form level by means of a subdivision into nine modules describing the behavioral equations, the laws of motion and the identities or budget equations of the model. We then derive their intensive form representation and the 10 laws of motions of the model on the basis of certain simplifying assumptions. Thereafter we present the uniquely determined steady state solution of the dynamics and discuss in a mathematically informal way its stability properties, concerning asymptotic stability and loss of stability by way of super- or subcritical Hopf-bifurcations. In a final section we explore numerically a variety of situations of interacting real and financial cycles, where the steady state is locally repelling, but where the overall dynamics are bounded in an economically meaningful domain by means of a kinked money wage Phillips curve, exhibiting downward rigidity of the money-wage, combined with upward flexibility of the usual type. The paper will be extended in a next step towards the proper inclusion of Keynesian goods market disequilibrium and the quantity dynamics this implies.

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JEL Classification: E12, E32.

1 Introduction

In this paper we reformulate and extend the analysis of small open economies of Asada, Chiarella, Flaschel and Franke (2003, ch.s 8,9) towards some initial theoretical considerations and some numerical explorations of the case of two interacting large open economies like Euroland and the USA. However we shall here reconsider primarily simplified, compared to the 14D two-country KMG dynamics of Asada, Chiarella, Flaschel and Franke (2003, ch.10), only 10D open KWG growth and inflation dynamics.[1] The results achieved in this paper still represent work in progress and thus surely need extension in order to truly judge the potential of the proposed model type for a discussion of the international transmission of the business cycle through positive or negative phase synchronization and other important topics of the literature on coupled oscillators of economic as well as of other origin.[2]

Analytical propositions are indeed obtained much more easily in the KWG case than in the case of two interacting KMG economies, since in the two-country case we can indeed then economize on four laws of motion (describing the quantity adjustments in the two open KMG economies) which reduces the dimension of the considered dynamics from 14D to 10D. The economically more convincing KMG approach with its less than full capacity growth considerations is considerably more difficult to analyze analytically and will therefore here remain excluded from consideration. From the economic perspective we thus concentrate on the generation and transmission of international inflation by means of the KWG case and do not yet really consider Keynesian quantity driven business cycle dynamics and their transmission throughout the world economy.

This paper first investigates the interaction of two monetary growth models of the KWG inflation dynamics type, which when assumed as closed would each generate an intrinsically nonlinear dynamics of dimension 4 of the kind that has been investigated in detail in Chiarella and Flaschel (2000, ch.3) for the closed economy case.[3] When there is trade in goods and financial assets between them, as in the Dornbusch (1976) model of overshooting exchange rate dynamics, KWG type models are coupled by way of the 2D dynamics of expected and actual exchange rate depreciation that lead in sum to nonlinear dynamics of dimension 10. We derive local stability conditions for these dynamics and show that there exist a variety of situations where Hopf-bifurcations will occur, giving rise to the local birth or death of stable or unstable limit cycles. Furthermore extrinsic nonlinearities are then introduced to limit the trajectories of the dynamics from a global point of view in the numerical analysis of this paper. In this way limit cycles and more complex types of attractors are generated that can exhibit the co-movements typical of national business

[1]The two labels chosen distinguish the so-called Keynes-Metzler-Goodwin (KMG) approach from the simpler Keynes-Wicksell-Goodwin (KWG) approach where there is no quantity adjustment in the market for goods and where therefore primarily an IS-driven inflation dynamics is the focus of interest. These two model types where established in Chiarella and Flaschel (1996a,b) and reconsidered in a larger context in Chiarella and Flaschel (2000) as well as Chiarella, Flaschel, Groh and Semmler (2000).

[2]The theory of coupled oscillators represents a topic with many interesting features. Due to space constraints the proper application of this theory to the questions treated here remains a subject for future research; see however Haxholdt (1995) and Brenner, Weidlich and Witt (2002) on these matters.

[3]See also Chiarella and Flaschel (1996a) with respect the first presentation of the KWG model type, based on the literature on the Keynes-Wicksell monetary growth dynamics of the 1960s and 1970s.

cycles (so-called 'phase locking'), but also the counter-movements typical of such cycles, in both cases primarily with respect to inflation dynamics.

It has been shown in Chiarella and Flaschel (2000) that the supply side oriented KWG approach may be considered as a reasonable simplification of the demand side oriented KMG approach (where prices and quantities both adjust according to demand conditions on the market for goods) if attention is restricted to topics such as income distribution and inflation, since in such situations it provides a pragmatic short-cut for the feedbacks that go from IS-disequilibrium to its impact on wage- and price-inflation. The advantage of the KWG approach is that it reduces the number of the laws of motion needed to describe a monetary growth model of the Keynesian variety, by restricting the adjustment processes considered to the dynamics of the real wage, to savings- or investment driven capital stock growth, the law of motion for real balances (representing the inflationary forces) and the one for inflationary expectations. The consideration of only these state variables simplifies the stability analysis of the closed economy case considerably. In a similar fashion it allows us to establish situations of local asymptotic stability for the case of two interacting KWG economies by first starting from a weak coupling of the two considered economies. Thereafter, a host of situations can be provided where the economies lose their asymptotic stability (by way of Hopf-bifurcations, since it can in particular be shown that the system's determinant has a positive sign throughout). Of course, global stability properties have to be studied then from the numerical point of view since the dynamical system is of too high a dimension to allow for global analytical results.

In section 2 the coupled two-country KWG dynamics is introduced and discussed on the extensive form level, by means of a subdivision into nine modules describing the behavioral equations, the laws of motion and the identities or budget equations of the model. Section 3 then derives their intensive form representation on the basis of certain simplifying assumptions. In section 4 we present the uniquely determined steady state solution of the dynamics and discuss on this basis in a mathematically informal way its stability properties, concerning asymptotic stability and the loss of this stability by way of super- or subcritical Hopf-bifurcations. Rigorous stability proofs that follow the here applied methodology (of starting from an appropriate 3D dynamical subsystem and enlarging it in a feedback guided and systematic way to its full dimension by making certain adjustment speeds – formerly set equal to zero – slightly positive) are provided in Asada, Chiarella, Flaschel and Franke (2003, ch.10). Section 5 explores numerically a variety of situations of interacting real and financial cycles of the KWG type, where the steady state is locally repelling, but where the overall dynamics are bounded in an economically meaningful domain by means of a kinked money wage Phillips curve, with downward rigidity of the money-wage, but with its upward flexibility of the usual type. Section 6 concludes.

2 Two interacting KWG economies

In this section we introduce for KWG approach to open economies the case of two large open economies that are interacting which each other through trade in goods as well as financial assets and the resulting net interest flows. The KWG approach of this paper to the formulation of two-country monetary macrodynamics is not yet a complete description of such a two-country world. This holds in particular, since the allocation and accumulation of domestic and foreign bonds is not completely specified. We make some convenient technical assumptions that will ensure that the accumulation of internationally traded bonds does not feedback into the core 10D-dynamics of the model and may thus be neglected for the time being. Note furthermore that the following presentations of the equations of the model involve many accounting identities that are here simply presented to ease and supplement the understanding of the model. They are however of no importance for the dynamical equations that result from this model (four for each country and two for their interconnection) that will be analyzed in this paper from a theoretical as well as from a numerical point of view. Note finally that we use linear equations to model behavioral relationships as often as this is possible in order to have a model with only intrinsic nonlinearities as a starting point of our investigations. Extrinsic nonlinearities based for example on intertemporal constraints, changing adjustment behavior and the like will be introduced in future extensions of the model type considered here. One such extrinsic nonlinearity is discussed in section 5.

In the model presented below we have chosen the units of measurement such that domestic expressions are in terms of the domestic good or the domestic currency, and foreign country expressions in terms of the commodity produced by the foreign country (or – if nominal – in the foreign currency) as far as this has been possible. For the sake of concreteness, we shall refer to the domestic and foreign economy, as 'Euroland' and the 'USA' with their currencies '€' and '\$' respectively. An asterisk indicates a foreign country variable whilst a subscript '2' on a variable indicates that the variable is sourced from the other country. For notational simplicity we use π in the place of π^e in this paper to denote the rate of inflation expected to apply over the medium-run. Since both countries are modelled analogously we will focus on the domestic economy in the following presentation of the components of the model. The description and justification of the equations presented in the various modules of the model will be brief, since many of the structural equations of this two-country KWG dynamics are already well-documented and explained in Chiarella and Flaschel (2000, ch.4).

1. Definitions (remunerations, wealth, real exchange rate):

$$\omega = w/p, \quad \rho = (Y - \delta K - \omega L^d)/K, \tag{1}$$

$$W = (M + B_1 + eB_2 + p_e E)/p, \quad p_b = p_{b^*} = 1, \tag{2}$$

$$\omega^* = w^*/p^*, \quad \rho^* = (Y^* - \delta^* K^* - \omega^* L^{d*})/K^*, \tag{3}$$

$$W^* = (M^* + B_1^*/e + B_2^* + p_e^* E^*)/p^*, \quad p_b = p_{b^*} = 1, \tag{4}$$

$$\eta = p/(ep^*), \quad [\text{Goods}^*/\text{Goods}]. \tag{5}$$

The equations in the first module of the model provide definitions of important macroeconomic magnitudes, namely the real wage ω, the actual rate of profit ρ and real wealth W. The latter consists of real money balances, equities, bonds issued by the domestic government (B_1) and bonds issued by the foreign government (B_2). These bonds have a constant price, normalized to one, and a variable interest rate (r and r^*, respectively). Since adding the possibility of holding foreign equities as well does not affect the main features of this model in its present formulation, we restrict ourselves to bonds as the only foreign asset domestic residents can hold. The real exchange rate is defined by $p/(ep^*)$ and thus in the present paper describes the exchange ratio between foreign and domestic goods.

2. Households (workers ($s_w, \tau_w = 0$) and asset-holders:

$$
\begin{aligned}
W &= (M^d + B_1^d + eB_2^d + p_e E^d)/p, && (6)\\
M^d &= h_1 pY + h_2 pW(1-\tau_c)(r_o - r), && (7)\\
Y_c^D &= (1-\tau_c)(\rho K + rB_1/p) + e(1-\tau_c^*)r^* B_2/p, && (8)\\
C_1 &= \gamma_w \omega L^d + \gamma_c(\eta)(1-s_c)Y_c^D, \quad \gamma_w, \gamma_c(\eta) \in [0,1], && (9)\\
C_2 &= \eta((1-\gamma_w)\omega L^d + (1-\gamma_c(\eta))(1-s_c)Y_c^D), && (10)\\
S_p &= \omega L^d + Y_c^D - C = s_c Y_c^D = (\dot{M}^d + \dot{B}_1^d + e\dot{B}_2^d + p_e \dot{E}^d)/p, && (11)\\
C &= C_1 + C_2/\eta, && (12)\\
\hat{L} &= n = \text{const.}, && (13)
\end{aligned}
$$

$$
\begin{aligned}
W^* &= (M^{d*} + B_1^{d*}/e + B_2^{d*} + p_e^* E^{d*})/p^*, && (14)\\
M^{d*} &= h_1^* p^* Y^* + h_2^* p^* W^*(1-\tau_c^*)(r_o^* - r^*), && (15)\\
Y_c^{D*} &= (1-\tau_c^*)(\rho^* K^* + r^* B_2^*)/p^*) + (1-\tau_c) r B_1^*/(ep^*), && (16)\\
C_2^* &= \gamma_w^* \omega^* L^{d*} + \gamma_c^*(\eta)(1-s_c^*)Y_c^{D*}, \quad \gamma_w^*, \gamma_c^*(\eta) \in [0,1], && (17)\\
C_1^* &= ((1-\gamma_w^*)\omega^* L^{d*} + (1-\gamma_c^*(\eta))(1-s_c^*)Y_c^{D*})/\eta, && (18)\\
S_p^* &= \omega^* L^{d*} + Y_c^{D*} - C^* = s_c^* Y_c^{D*}\\
&= (\dot{M}^{d*} + \dot{B}_2^{d*} + \dot{B}_1^{d*}/e + p_e^* \dot{E}^{d*})/p^*, && (19)\\
C^* &= C_1^*/\eta + C_2^*, && (20)\\
\hat{L}^* &= n^* = \text{const.} && (21)
\end{aligned}
$$

We assume two groups of households in our model that differ with respect to their savings behavior – workers who do not save (for reasons of simplicity) and asset holders who have a constant average propensity to save, s_c, out of their disposable income. Furthermore, both groups spend a fraction ($1-\gamma_w$ and $1-\gamma_c$, respectively) of their consumption expenditures on imports (C_2). We assume that the fraction γ_c is a negative function of the real exchange rate η. This indicates that asset holders shift their consumption expenditures in favor of the commodity that becomes relatively cheaper. The disposable income Y_c^D of asset holders consists of profits, interest payments from domestic bonds and interest payments from foreign bonds – all net of taxes (which are paid in the country from where this interest income originates). Note that we have assumed – again for reasons of simplicity – that the tax rate on wage income is zero. Furthermore, the asset holders decide how to split up their wealth between the different assets (a superscript d indicates demand). Here we assume that domestic bonds and domestic equities are perfect substitutes, which provides an equation for the price of equities. The stock demand for real money balances depends on output Y (reflecting the transaction motive), on wealth and the nominal interest rate. Equation (11) indicates that the asset holders have to hold their intended savings in the four assets that are available to them domestically. Finally, we assume that the labor force L grows at a constant exogenous rate n.

3. Firms (production-units and investors):

$$
\begin{aligned}
Y &= yK, \quad L^d = Y/x, \quad y, x = \text{const.}, \quad V = L^d/L, && (22)\\
I &= i(\rho - (r - \pi))K + nK, && (23)\\
\Delta Y &= Y - \delta K - C_1 - C_1^* - I - G, && (24)\\
p_e \dot{E}/p &= I + \Delta Y = I^a \quad (S_f = 0), && (25)\\
\hat{K} &= I/K + (1 - \beta_k)\Delta Y/K, \quad \beta_k \in [0, 1], && (26)\\
\dot{N} &= \delta_2 K + \beta_k \Delta Y, && (27)
\end{aligned}
$$

$$
\begin{aligned}
Y^* &= y^* K^*, \quad L^{d*} = Y^*/x^*, \quad y^*, x^* = \text{const.}, \quad V^* = L^{d*}/L^*, && (28)\\
I^* &= i^*(\rho^* - (r^* - \pi^*))K^* + n^* K^*, && (29)\\
\Delta Y^* &= Y^* - \delta^* K^* - C_2 - C_2^* - G^*, && (30)\\
p_e^* \dot{E}^*/p^* &= I^* + \Delta Y^* = I^{a*}, \quad (S_f^* = 0), && (31)\\
\hat{K}^* &= I^*/K^* + (1 - \beta_k^*)\Delta Y^*/K^*, \quad \beta_k^* \in [0, 1], && (32)\\
\dot{N}^* &= \delta_2^* K^* + \beta_k^* \Delta Y^*. && (33)
\end{aligned}
$$

Module 3 describes the behavior of firms. Output is produced with the help of the two factors, labor and capital, using a technology with fixed input coefficients. Capital is always fully utilized whereas demand for labor, L^d, may differ from the total work force L. The investment per unit of capital depends on the difference between the profit rate and the real interest rate and n as a trend component. Equation (24) defines the excess supply, ΔY, on the domestic goods market. Since we have assumed that firms' factor payments, (in the form of wages and profits), always amount to Y, they have to finance ΔY as well as their intended investment by issuing equities. This is indicated in (25) where actual investment, I^a, is defined as the sum of intended and involuntary investment. By equation (26) this involuntary investment, ΔY, can either result in unintended capital accumulation or in unintended changes in inventories. For $\beta_k = 0$, all excess supply of goods leads to involuntarily capital accumulation. This implies that if output falls short of aggregate demand ($\Delta Y < 0$) investment plans are cancelled by the respective amount. We see from equation (27) that for this value of β_k, there is no need to explicitly consider inventories. Hence in this case, δ_2 – the ratio of intended inventory holdings to the capital stock – can be set equal to zero. For $\beta_k = 1$, in contrast, intended investment will be the only force affecting the capital stock since all unsold production results in a change in the stock of inventories. This stock increases [decreases] if actual output exceeds [falls short of] aggregate demand. Moreover, a positive δ_2 now indicates the assumption that firms try to hold the stock of inventories proportional to output. Because of our assumption concerning the production technology this implies a constant ratio of inventories and capital stock in the steady state.[4] Besides these polar cases, on which we will concentrate in the ensuing analysis, intermediate ones ($\beta_k \in (0,1)$), where part of the unsold production leads to involuntary capital accumulation and part to changes in inventories, are also possible and indeed more plausible. Due to our assumptions on firm behavior it follows finally that the savings of firms are always identically zero.

Module 4 describes the government. In equation (34) it levies a tax with a constant tax rate τ_c on profits and on interest payments from domestic bonds, i.e. only the asset holders pay taxes. Note that the interest payments going to foreigners who hold domestic bonds are also taxed. Equation (35) characterizes government expenditures in the simplest way possible as far as steady state analysis is concerned, namely as being a constant fraction of the capital stock K. Equation (36) is simply the definition of government savings; fiscal receipts net of interest payments minus government spending. Equation (37) expresses the assumption that the central bank of the home country keeps the domestic money supply on a growth path with an exogenous rate μ. Consistent with this assumption, the government budget constraint then states in (38) that the time rate of change of the supply of government bonds (that in fact reaches the public) is determined by two items; the negative of government savings (the government deficit that must be financed) minus that part of the new money supply that is injected into the economy via open market operations (which reduces the supply of new government) and not via the foreign exchange market.[5]

4. Government (fiscal and monetary authority):

$$
\begin{aligned}
T &= \tau_c(\rho K + rB/p), \quad B = B_1 + B_1^*, && (34)\\
G &= gK, \ g = \text{const.}, && (35)\\
S_g &= T - rB/p - G, && (36)\\
\hat{M} &= \dot{M}/M = \mu, && (37)\\
\dot{B} &= pG + rB - pT - \dot{M}, && (38)
\end{aligned}
$$

$$
\begin{aligned}
T^* &= \tau_c^*(\rho^* K^* + r^* B^*/p^*), \quad B^* = B_2^* + B_2, && (39)\\
G^* &= g^* K^*, \ g^* = \text{const.}, && (40)\\
S_g^* &= T^* - r^* B^*/p^* - G^*, && (41)\\
\hat{M}^* &= \dot{M}^*/M^* = \mu^*, && (42)\\
\dot{B}^* &= p^* G^* + r^* B^* - p^* T^* - \dot{M}^*. && (43)
\end{aligned}
$$

With regard to the asset markets we assume continuous market clearing at the end of each 'trading day' (ex post). Equation (44) indicates the respective stock equilibria for the three domestic assets. Note that the demand for domestic bonds stems from domestic as well as from foreign asset owners. The equation (45) directly follows from the assumption

[4] However equation (27) excludes the possibility that inventories N grow even in a non-growing economy (K = const.) Let $\hat{\delta}_2$ denote the desired ratio of inventories and output, $\hat{\delta}_2 Y = \hat{\delta}_2 yK = N$. Differentiating with respect to time and noting that the steady state is characterized by $\dot{K} = nK$, the following definition of δ_2 seems appropriate: $\delta_2 = \hat{\delta}_2 ynK$. In a stationary economy, $n = 0$, the equilibrium is then characterized by a constant stock of inventories.

[5] We do not yet consider foreign market operations by the central banks.

that domestic bonds and equities are perfect substitutes. Hence, the rate of interest net of taxes, $(1-\tau_c)r$, has to be equal to the actual rate of return on equities. This rate can be calculated as follows. In each period, all expected profits, ρpK, are paid out to equity holders. Taking the tax and perfectly foreseen untaxed capital gains into account, the rate of return on equities, therefore, amounts to $(1-\tau_c)\rho pK/p_eE + \hat{p}_e$. Equation (46) then characterizes the respective flow equilibria. We assume that the government and the firms face no demand problems when issuing new bonds or equities, respectively. Note that the division of new bonds between domestic and foreign asset holders is ambiguous.[6] Once their flow demands fulfill the condition $\dot{B} = \dot{B}_1^d + \dot{B}_1^{d*}$, however, these demands are realized ($\dot{B}_1 = \dot{B}_1^d$ and $\dot{B}_1^* = \dot{B}_1^{d*}$).

5. Equilibrium conditions and consistency requirements (asset-markets):

$$\begin{aligned}
M &= M^d = h_1pY + h_2pW(1-\tau_c)(r_o - r), \\
B &= B_1^d + B_1^{d*}, \quad E = E^d, && (44)\\
(1-\tau_c)r &= \frac{(1-\tau_c)\rho pK}{p_eE} + \hat{p}_e, && (45)\\
\dot{M} &= \dot{M}^d, \ \dot{B} = \dot{B}_1^d + \dot{B}_1^{d*}, \ \dot{E} = \dot{E}^d, && (46)
\end{aligned}$$

$$\begin{aligned}
M^* &= M^{d*} = h_1^*p^*Y^* + h_2^*p^*W^*(1-\tau_c^*)(r_o^* - r^*), \\
B^* &= B_2^d + B_2^{d*}, \ E^* = E^{d*}, && (47)\\
p_e^*E^* &= (1-\tau_c^*)\rho^*p^*K^*/((1-\tau_c^*)r^* - \pi^*), && (48)\\
\dot{M}^* &= \dot{M}^{d*}, \ \dot{B}^* = \dot{B}_2^d + \dot{B}_2^{d*}, \ \dot{E}^* = \dot{E}^{d*}. && (49)
\end{aligned}$$

In module 6, the first two equations describe the disequilibrium situations on the market for the domestic and the foreign good, respectively. Then, as the first line in (52) shows, aggregate savings which consists of private and public savings is equal to the sum of actual investment and net private capital exports ($(e\dot{B}_2 - \dot{B}_1^*)/p$). The whole expression is equal – due to our assumptions on income, consumption and the allocation of savings – to actual investment plus, net exports of goods and, plus the excess of foreign interest payments to domestic residents holding foreign bonds over domestic interest payments to foreigners holding home-country bonds (both net of taxes).[7] This is a direct implication of the fact that the surpluses in all accounts of the balance of payments have to sum up to zero. See

[6] For $\dot{M} = \dot{M}^d, \dot{E} = \dot{E}^d, \dot{M}^* = \dot{M}^{d*}, \dot{E}^* = \dot{E}^{d*}$, equations (11), (19), (46) and (49) lead to the following set of four equations in the four unknowns $\dot{B}_1^d, \dot{B}_2^d, \dot{B}_1^{d*}$ and $\dot{B}_2^{d*}$:

$$\begin{bmatrix} 1 & e & 0 & 0 \\ 0 & 0 & 1/e & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{B}_1^d \\ \dot{B}_2^d \\ \dot{B}_1^{d*} \\ \dot{B}_2^{d*} \end{bmatrix} = \begin{bmatrix} pS_g - \dot{M} - p_e\dot{E} \\ p^*S_g^* - \dot{M}^* - p_e^*\dot{E}^* \\ \dot{B} \\ \dot{B}^* \end{bmatrix}.$$

As can easily be verified, the rank of the 4×4 matrix on the left hand side is three, yielding one degree of freedom. Hence, once the division of new domestic bonds between domestic residents and foreigners is chosen the division of foreign bonds is determined as well.

[7] Whereas the first line in (52) follows directly from inserting the use of private and government savings from (11) and (36), the derivation of the second line is slightly more complicated. Using the definitions of

also below where we explain the balance of payments in greater detail. Naturally, as shown in (54), for the world as a whole, aggregate savings equal aggregate actual investment.

6. Disequilibrium situation (goods-markets):

$$
\begin{aligned}
Y &\neq C_1 + C_1^* + I + \delta K + G \;\; [\Delta Y \neq 0], && (50)\\
Y^* &\neq C_2^* + C_2 + I^* + \delta^* K^* + G^* \;\; [\Delta Y^* \neq 0], && (51)\\
S &= S_p + S_g = I^a + (e\dot{B}_2 - \dot{B}_1^*)/p && (52)\\
&= I^a + \{C_1^* - (ep^*/p)C_2\} + \{e(1-\tau_c^*)r^*B_2/p - (1-\tau_c)rB_1^*/p\}, && (53)\\
S^* &= S_p^* + S_g^* = I^{a*} + (\dot{B}_1^*/e - \dot{B}_2)/p^* &&\\
&= I^{a*} + \{C_2 - (p/ep^*)C_1^*\} + \{(1-\tau_c)rB_1^*/(ep^*) - (1-\tau_c^*)r^*B_2/p^*\}, &&\\
S^w &= S + (ep^*/p)S^* = I^a + (ep^*/p)I^{a*} = I^{aw}. && (54)
\end{aligned}
$$

Module 7 contains the adjustment of wages, prices, and inflationary expectations. Wage and price inflation are modelled analogously. In both cases, there is a combination of demand pressure and cost pressure factors. Wage inflation depends on the deviation of the actual rate of employment from the NAIRU rate of employment. Furthermore, it is influenced by the actual rate of change in the workers' price index, $\hat{p}_w$, and the expected future rate of change, π_w. Underlying this formulation is the assumption that not only current but also medium run workers' price inflation is important in the wage bargaining process. From (58), the current rate of workers' price inflation amounts to the weighted sum of domestic price inflation and foreign price inflation (converted in domestic currency), where the weights are the proportions of the respective goods in workers consumption expenditures. The construction of π_w is completely analogous, using only expected magnitudes. Note that the use of $\hat{p}$ and π in lieu of $\hat{p}_w$ and π_w in equation (55) would imply an exchange rate illusion on the part of workers. Price inflation, on the other hand, depends on the actual excess supply on the goods market as a demand pressure factor and on wage inflation as a cost push force. Furthermore, the expected price trend π influences today's price inflation in a similar way as today's wage inflation. Equation (67) describes the formation of inflationary expectations concerning the medium-run. It consists of a backward looking first term (adaptive expectations with weight α_π) and a forward looking second term (with weight $1-\alpha_\pi$) that refers to a theoretical of price forecasting method (the p-star concept of the FED for example).

private and government savings,

$$S = S_p + S_g = \omega L^d + Y_c^D - C + T - rB/p - G.$$

Inserting expressions for Y_c^D, C and T according to (8), (12) and (34) yields

$$S = \omega L^d + \rho K - C_1 - (ep^*/p)C_2 - G - rB_1/p + e(1-\tau_c^*)r^*B_2/p.$$

Making use of the definition of ρ in (1) and noting that from (24) and (25) that $Y - \delta K - G - C_1 = \Delta Y + I + C_1^* = I^a + C_1^*$, one finally obtains the desired expression:

$$S = I^a + \{C_1^* - (ep^*/p)C_2\} + \{e(1-\tau_c^*)r^*B_2/p - (1-\tau_c)rB_1^*/p\}.$$

7. Wage-Price-Sector (adjustment equations):

$$\begin{aligned}
\hat{w} &= \beta_w(V-\bar{V})+\kappa_w\hat{p}_w+(1-\kappa_w)\pi_w, && (55)\\
\hat{p} &= -\beta_p(\Delta Y/K)+\kappa_p\hat{w}+(1-\kappa_p)\pi, && (56)\\
\dot{\pi} &= \beta_\pi(\alpha_\pi(\hat{p}-\pi)+(1-\alpha_\pi)(\hat{p}^+-\pi)), && (57)\\
\hat{p}_w &= \gamma_w\hat{p}+(1-\gamma_w)(\hat{e}+\hat{p}^*),\ \ p_w=p^{\gamma_w}(ep^*)^{1-\gamma_w}, && (58)\\
\pi_w &= \gamma_w\pi+(1-\gamma_w)(\epsilon+\pi^*), && (59)
\end{aligned}$$

$$\begin{aligned}
\hat{w}^* &= \beta_w^*(V^*-\bar{V}^*)+\kappa_w^*\hat{p}_w^*+(1-\kappa_w^*)\pi_w^*, && (60)\\
\hat{p}^* &= -\beta_p^*(\Delta Y^*/K^*)+\kappa_p^*\hat{w}^*+(1-\kappa_p^*)\pi^*, && (61)\\
\dot{\pi}^* &= \beta_\pi^*(\alpha_\pi^*(\hat{p}^*-\pi^*)+(1-\alpha_\pi^*)((\hat{p}^+)^*-\pi^*), && (62)\\
\hat{p}_w^* &= \gamma_w^*\hat{p}^*+(1-\gamma_w^*)(\hat{p}-\hat{e}),\ \ p_w^*=(p^*)^{\gamma_w^*}(p/e)^{1-\gamma_w^*}, && (63)\\
\pi_w^* &= \gamma_w^*\pi^*+(1-\gamma_w^*)(\pi-\epsilon). && (64)
\end{aligned}$$

Note again with respect to the above that expected inflation variables are now no longer carrying a superscript e in order to simplify to some extent the notation of the many expressions for inflation rates now involved.

Module 8 describes the dynamics of (the rate of change of) the exchange rate and the formation of expectations about this rate of change. Here we assume as a first approach to this dynamic interaction that the interest rate differential (augmented by depreciation expectations) in the international market for bonds determines via corresponding international capital flows the way and the extent by which the growth rate of the exchange rate deviates from its steady state value[8] in conjunction with the imbalance that exists in the trade account (per unit of capital) at each moment in time. Dornbusch type models of the open economy here often assume perfect capital mobility (i.e. $\beta=\infty$) and perfect substitutability of the assets traded internationally. These assumptions are the root cause of the prevalence of the UIP-condition (Uncovered Interest Parity) as the theory that determines the exchange rate dynamics. Our formulation extends this approach and allows for (some) imperfection with respect to capital mobility and exchange rate flexibility. Furthermore, the mechanism by which exchange rate expectations are formed is – as the mechanism that determined inflationary expectations – again a weighted average of 'backward' and 'forward' looking expectations. On the one hand, we use adaptive expectations, as the simplest expression for a chartist type of behavior and, theory based expectations,[9] using for example the relative form of the PPP, on the other hand, as a simple description of a fundamentalist sort of behavior. We assume here that domestic and foreign asset holders form the same expectations regarding the exchange rate.

[8] We thus allow here for imperfect capital mobility – in contrast to the approach assumed for domestically traded bonds and equities.

[9] For simplicity, only asymptotically rational expectations are assumed here.

8. <u>Exchange rate dynamics:</u>

$$\hat{e} = \beta_e(\beta((1-\tau_c^*)r^* + \epsilon - (1-\tau_c)r) - NX/K) + \hat{e}_o, \tag{65}$$

$$\hat{e}_o = \hat{p}_o - \hat{p}_o^*, \tag{66}$$

$$\dot{\epsilon} = \beta_\epsilon(\alpha_\epsilon(\hat{e} - \epsilon) + (1-\alpha_\epsilon)(\hat{e}^+ - \epsilon)). \tag{67}$$

Module 9 deals with the balance of payments and its components. The first three equations concern the trade balance and denote exports and imports of goods and also net exports (see also module 2). Note that domestic imports are foreign exports and vice versa. Then, equation (71) indicates net interest payments or exports (NIX) from abroad; foreign interest payments to domestic asset holders minus domestic interest payments to foreigners (assumed to be transferred through the foreign exchange market). In the balance of payments statistics NIP is part of exports of services (and thus also concerns the current account) and in national income accounting it is subsumed under net factor income from abroad. Another international transaction is the change in the stock of foreign bonds domestic residents hold. NCX denotes net capital exports, i.e. the deficit in the private capital account; the excess of additional foreign bonds held by domestic asset owners over additional domestic bonds held by foreigners. Note that, taking the exchange rate into account, NIX^* and NCX^* are simply mirror images of the respective domestic magnitudes. In (73), Z denotes the overall surplus in the balance of payments. It consists of the surplus in the current account (first and second bracket), the surplus in the private capital account (third bracket). As stated in (73), Z is identically equal to zero on the basis of what has been assumed so far. This is the well-known accounting identity, i.e., the magnitudes considered are ex post or equilibrium magnitudes. The same is true for the various terms in (52) – (54) described above, from which it immediately follows that $Z = 0$ is indeed fulfilled in this model type.

9. <u>Balance of Payments:</u>

$$Ex = ((1-\gamma_w^*)\omega^* L^{d*} + (1-\gamma_c^*(\eta))(1-s_c^*)Y_c^{D*})/\eta = C_1^* = Im^*/\eta, \tag{68}$$

$$Im = (1-\gamma_w)\omega L^d + (1-\gamma_c(\eta))(1-s_c)Y_c^D = C_2/\eta = Ex^*/\eta, \tag{69}$$

$$NX/p = Ex - Im = -NX^*/\eta, \tag{70}$$

$$NIX = e(1-\tau_c^*)r^* B_2 - (1-\tau_c)rB_1^* = -eNIX^*, \tag{71}$$

$$NCX = e\dot{B}_2^d - \dot{B}_1^{*d} = -eNCX^*, \tag{72}$$

$$\begin{aligned} Z &= pNX + NIX - NCX \\ &= \{pC_1^* - ep^*C_2\} + \{e(1-\tau_c^*)r^*B_2 - (1-\tau_c)rB_1^*\} \\ &- \{e\dot{B}_2^d - \dot{B}_1^{*d}\} \\ &= 0, \end{aligned} \tag{73}$$

$$\begin{aligned} Z^* &= p^*NX^* + NIX^* - NCX^* \\ &= \{p^*C_2 - pC_1^*/e\} + \{(1-\tau_c)rB_1^*/e - (1-\tau_c^*)r^*B_2\} \\ &\quad -\{\dot{B}_1^{*d}/e - \dot{B}_2^d\} \\ &= -Z/e = 0. \end{aligned} \tag{74}$$

3 The core 10D KWG growth dynamics

We now derive the intensive form representation of the two-country KWG growth dynamics on the basis of certain assumptions that simplify its structure without sacrificing too much in generality. In this way we obtain a structure that can be easily decomposed and later on re-integrated in order to allow for various stability investigations and also numerical comparisons between the closed economy case and the case of two interacting economies. The assumptions for the somewhat restricted variant of the two-country KWG model that will be investigated in the remainder of this paper (where the accumulation of assets other than money and real capital is still left in the background) are the following:[10]

- W, in the money demand function, is replaced by K as a narrow definition of domestic wealth (this removes feedbacks from bond and equity accumulation from part of the model).
- $t_c = (T_c - rB_1/p - er_o^* B_2/p)/K =$ const., where $T_c = \tau_c(\rho K + rB_1/p) + \tau_c^* er^* B_2/p$ represents the sum of all taxes paid by domestic asset holders worldwide. This rule of tax collection is used in the place of the earlier profit tax collection rule and removes another feedback route of the accumulation of domestic and foreign bonds from the model. The question, of course, is how important such feedbacks routes are for the dynamics of the model in general.

For reasons of simplicity we also employ the following assumptions:

- $\gamma_w \equiv 1$: Wage earners consume domestic goods solely (but $\gamma_c(\eta), \gamma_c' < 0$). This simplifies the consideration of the wage/price dynamics in a way that makes it identical to that of a closed economy.
- $\rho_o^{e*} = \rho_o^e$: The domestic steady state rate of profit is identical to that of the foreign economy. This allows the interest rate parity condition to coincide with the relative form of the PPP in the steady state or (equivalently) allows the removal of any trend from the real exchange rate in the steady state.
- $n = n^*$ in order to have a uniform real rate of growth in the world economy in the steady state for reasons of analytical simplicity.
- $\hat{p}^+ = \hat{p}_o = \mu - n$: The simplest rule for the formation of forward looking expectations of the rate of inflation by means of the quantity theory of money.
- $\hat{e}^+ = \hat{e}_o = \hat{p}_o - \hat{p}_o^* = \mu - \mu^*$: The simplest rule for the formation of forward looking expectations of the rate of change of the exchange rate by means of the relative form of purchasing power parity theory

We furthermore assume that the export and import of commodities is modelled in its mathematical details in the following simple way:

[10]The assumptions we make for the foreign economy are the same as the ones here for the domestic economy and are therefore not made explicit in this section.

According to module 9 of the above presentation of our general model and due to the assumptions just made we have for $c_1^* = Ex/K = C_1^*/K$ and $c_2/\eta = Im/K = C_2/(\eta K)$ the expressions

$$\begin{aligned} c_1^* &= (1-\gamma_c^*(\eta))(1-s_c^*)(\rho^*-t_c^*)(l/l^*)/\eta, && (75)\\ c_2/\eta &= (1-\gamma_c(\eta))(1-s_c)(\rho-t_c). && (76)\end{aligned}$$

These show that imports as well as exports (the first in terms of the domestic commodity and the second in terms of the foreign good) are both a linear function of the real exchange rate if, for the functions that determine the division of consumption between domestic and foreign goods in both countries, it is furthermore assumed that

$$\begin{aligned} \gamma_c(\eta) &= \gamma_c + \gamma(\eta_o - \eta), \quad \gamma > 0, && (77)\\ \gamma_c^*(\eta) &= \gamma_c^* - \gamma^*(\eta_o - \eta), \quad \gamma^* > 0, && (78)\end{aligned}$$

are linear as well. This is justified in the present paper because we want to express the model in as linear a form as possible in order to allow only for intrinsic (unavoidable) nonlinearities at the start of our considerations. Nonlinearities that rest on certain restrictions concerning the postulated behavior of agents when the economy is far off its steady state or on nonlinearities in the assumed speed of adjustment to disequilibrium far off the steady state should then be introduced step-by-step at a later stage of the analysis. With respect to the above $\gamma_c(\eta)$-function the linear relationships in (77) and (78) would then have to be replaced by, say, *tanh* functions in order to guarantee $\gamma_c(\eta)$, $\gamma_c^*(\eta)$ remain between 0 and 1 at large values of $|\eta_0 - \eta|$. The assumptions just made imply that the trade account in determined according to the way depicted in figure 1.

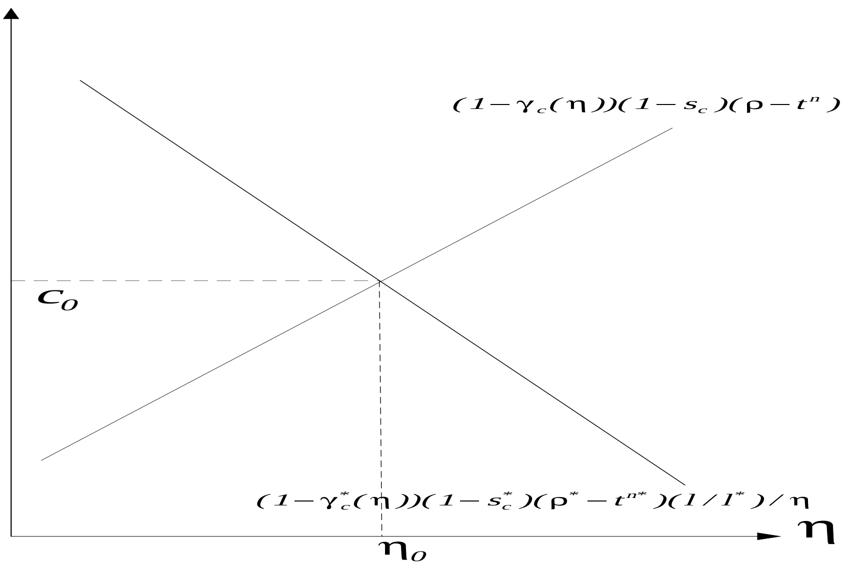


Figure 1: Determination of the balanced trade account ($NX = Ex - Im = 0$).

To simplify even further our treatment of the trade that occurs between the two countries we finally assume that the parameter η_o is given by

$$\eta_o = \frac{l_o(1-\gamma_c^*)(1-s_c^*)(\rho_o^*-t_c^*)}{l_o^*(1-\gamma_c)(1-s_c)(\rho_o-t_c)}. \tag{79}$$

The choice of this particular parameter value for η_o guarantees (as we shall see in the following) that the steady value of η will be η_o and that the trade account (per unit of capital) $nx = NX/K = (Ex - Im)/K = c_1^* - c_2/\eta$ will be balanced in the steady state.

A special case that is often employed in the literature on overshooting exchange rates is recovered by making the following sequence of additional assumptions:

- $\beta_e = \infty, \beta = \infty$: so that $(1-\tau_c)r = (1-\tau_c^*)r_o^* + \epsilon$: Uncovered interest parity (UIP), based on perfect capital mobility, is assumed to hold.
- $\beta_\epsilon = \infty, \alpha_\epsilon = 1$: so that $\epsilon = \hat{e}$: Myopic perfect foresight (MPF) with respect to the exchange rate is assumed to hold.

These assumptions are generally assumed in the literature for a treatment of the Dornbusch model of overshooting exchange rates. There are however also treatments of this model type that make use of adaptive expectations ($\alpha_\epsilon = 1$) in order to investigate from this point of view the MPF-limit ($\alpha_\epsilon = 1, \beta_\epsilon = \infty$) and its properties, see Chiarella (1990a,b, 1992). Furthermore, the case $\alpha_\epsilon = 0$ in which $\dot{\epsilon} = \beta_\epsilon(\hat{e}_o - \epsilon)$, can be considered as a variant of Dornbusch's original choice of a regressive expectations mechanism: $\epsilon = \beta_\epsilon(\ln(e_o/e))$, that by differentiation implies the rule $\dot{\epsilon} = \beta_\epsilon(\hat{e}_o - \hat{e})$.

As far as the mathematical investigation of the general two-country KWG model of the preceding section is concerned we will confine ourselves here to the case t_c =const where lump sum taxes are varied in such a way that the ratio of real total taxes paid by domestic asset holders (net of deflated interest payments they have received) to the capital stock remains constant over time. This assumption will allow us to disregard the GBR and the evolution of worldwide government debt in the following analysis of the model.[11] In making use of this simplifying device we employ similar assumptions to those of Sargent (1987, ch.V) and Rødseth (2000, ch.6).

Let us now show how this model (which ignores the GBR, the government budget restraint) can be rewritten as an nonlinear autonomous dynamical system in the ten state variables $\omega = w/p$, $l = L/K, m = M/(pK)$, π, $\omega^* = w^*/p^*$, $l^* = L^*/K^*$, $m^* = M^*/(p^*K^*)$, π^*, $\eta = p/(ep^*)$ and ϵ:

The domestic economy:

$$
\begin{aligned}
\widehat{\omega} &= \kappa[(1-\kappa_p)\beta_w X^w + (\kappa_w - 1)\beta_p X^p], && (80)\\
\widehat{l} &= -i(\cdot) + (1-\beta_k)X^p, && (81)\\
\widehat{m} &= \mu - \pi - n - \kappa[\beta_p X^p + \kappa_p \beta_w X^w] + \widehat{l}, && (82)\\
\dot{\pi} &= \beta_\pi[\alpha_\pi \kappa(\beta_p X^p + \kappa_p \beta_w X^w) + (1-\alpha_\pi)(\mu - n - \pi)]. && (83)
\end{aligned}
$$

[11] Where the parameter τ_c has thus to be removed from the model's equations, since taxes are now lump-sum.

Financial and trade links between the two economies:

$$\begin{aligned}
\hat{\eta} &= (\hat{p} - \pi) + \pi - [(\hat{p}^* - \pi^*) + \pi^*] \\
&- \beta_e(\beta(r^* + \epsilon - r) - a\cdot)) - \hat{e}_o, && (84)\\
\dot{\epsilon} &= \beta_\epsilon[\alpha_\epsilon((\hat{p} - \pi) + \pi - [(\hat{p}^* - \pi^*) - \pi^*] - \hat{\eta} - \epsilon) \\
&+ (1 - \alpha_\epsilon)(\mu - \mu^* - \epsilon)]. && (85)
\end{aligned}$$

The foreign economy:

$$\begin{aligned}
\widehat{\omega}^* &= \kappa^*[(1 - \kappa_p^*)\beta_w^* X^{w*} + (\kappa_w^* - 1)\beta_p^* X^{p*}], && (86)\\
\widehat{l^*} &= -i^*(\cdot) + (1 - \beta_k^*)X^{p*}, && (87)\\
\widehat{m}^* &= \mu^* - \pi^* - n^* - \kappa^*[\beta_p^* X^{p*} + \kappa_p^* \beta_w^* X^{w*}] + \widehat{l^*}, && (88)\\
\dot{\pi}^* &= \beta_\pi^*[\alpha_\pi^* \kappa^*(\beta_p^* X^{p*} + \kappa_p^* \beta_w^* X^{w*}) + (1 - \alpha_\pi^*)(\mu^* - n^* - \pi^*)]. && (89)
\end{aligned}$$

Here we employ the abbreviations:

$$\begin{aligned}
\rho &= y - \delta - \omega l^d = \rho(\omega),\ y = Y/K,\ l^d = L^d/K = y/x = \text{const.}, \\
X^w &= l^d/l - \bar{V} = y/(xl) - \bar{V},\ l = L/K, \\
X^p &= -\Delta Y/K = c_1 + c_1^* + i(\cdot) + n + \delta + g - y \\
&= \omega l^d + (1 - s_c)(\rho - t_c) + i(\cdot) + n + \delta + g + nx(\cdot) - y, \\
i(\cdot) &= i(\rho - r + \pi), \\
r &= r_o + (h_1 y - m)/h_2 = r(m), \\
c_1 &= C_1/K = \omega l^d + \gamma_c(\eta)(1 - s_c)(\rho - t_c), \\
c_1^* &= C_1^*/K = (l/l^*)(1 - \gamma_c^*(\eta))(1 - s_c^*)(\rho^* - t_c^*)/\eta, \\
nx(\cdot) &= (1 - \gamma_c^*(\eta))(1 - s_c^*)(\rho^* - t_c^*)(l/l^*)/\eta - (1 - \gamma_c(\eta))(1 - s_c)(\rho - t_c), \\
\hat{p} - \pi &= \kappa[\beta_p X^p + \kappa_p \beta_w X^w], \\
\hat{p}^* - \pi^* &= \kappa^*[\beta_p^* X^{p*} + \kappa_p^* \beta_w^* X^{w*}],
\end{aligned}$$

and similarly for the other country.[12] Note again that we are using for the determination of the division of households' consumption into domestic and foreign commodities the simple

[12]Where we in particular have:

$$\begin{aligned}
c_2^* &= C_2^*/K^* = \omega^* l^{d*} + \gamma_c^*(\eta)(1 - s_c^*)(\rho^* - t_c^*), \\
c_2 &= C_2/K^* = (l^*/l)(1 - \gamma_c(\eta))(1 - s_c)(\rho - t_c)\eta.
\end{aligned}$$

Note that the steady state values of the domestic and the foreign economy are dependent on the above choice of the steady state real exchange rate.[13] Note also that the domestic and the foreign rate of profit must be equal to each other in this formulation of a two-country model of international trade.

linear functions

$$\gamma_c(\eta) = \gamma_c + \gamma(\eta_o - \eta), \quad \gamma > 0, \tag{90}$$
$$\gamma_c^*(\eta) = \gamma_c^* - \gamma^*(\eta_o - \eta), \quad \gamma^* > 0, \tag{91}$$

in order to keep the model as close as possible to a linear form. Note finally that we always have $nx_\eta < 0$ due to our assumptions on consumption behavior, i.e., there is no need here for the consideration of so-called Marshall-Lerner conditions to ensure a normal reaction of net exports with respect to exchange rate changes.

We disregard the boundary solutions $\omega, l, m = 0, etc.$ – caused by the growth rate formulation of their laws of motion – in the following determination of the steady state solutions of the above dynamics. These values of the variables $\omega, l, m, etc.$ are economically meaningless and never appear as attractors in the numerical investigations to be performed later. Furthermore, the achieved theoretical results will all be constrained to a neighborhood of the unique interior steady state considered below. Of course, a general and global analysis of the system must take into account the stability properties of such boundary points of rest of the dynamics.

4 Steady state and $\beta-$stability analysis

In this section we present in a mathematically informal way a variety of subsystem stability investigations that eventually allow us to derive the stability of the fully integrated 10D dynamics in a systematic fashion by way of our $\beta-$stability approach to macroeconomic dynamics. We thereby again show the merits of a feedback guided stability analysis, here however from the purely local perspective.[14] Let us first however consider the uniquely determined interior steady state solution of the 10D dynamics of the preceding section.

Theorem 1

There is a unique steady–state solution *or point of rest of the simplified dynamics (80) – (89) fulfilling* $\omega_0, l_0, m_0 \neq 0$. *This steady–state is given by:*[15]

$$l_0 = l^d/\bar{V} = y/(x\bar{V}), \tag{92}$$
$$m_0 = h_1 y, \tag{93}$$
$$\pi_0 = \mu - n, \tag{94}$$
$$\rho_0 = t_c + \frac{n + g - t_c}{s_c}, \tag{95}$$
$$r_o = \rho_o + \pi_o, \tag{96}$$
$$\omega_0 = (y - \delta - \rho_0)/l^d, \tag{97}$$

for the domestic economy and correspondingly $\omega^*, l^*, m^*, \pi^*, r^*, \rho^*$, *for the foreign economy, and*

$$\eta_o = \frac{l_o(1-\gamma_c^*)(1-s_c^*)(\rho_o^* - t_c^*)}{l^*(1-\gamma_c)(1-s_c)(\rho_o - t_c)}, \tag{98}$$
$$\epsilon_o = \mu - n - (\mu^* - n) = \mu - \mu^* = \hat{e}_o, \tag{99}$$

[14]Rigorous stability proofs for the propositions of this section are provided in Asada, Chiarella, Flaschel and Franke (2003, ch.10).

[15]Note again that y, l^d are given magnitudes in the KWG dynamics.

We assume that the parameters of the model are chosen such that the steady state values for $\omega, l, m, \rho, r, \eta$ are all positive. Note in particular that $\pi_0 = \mu_0 - n$ should not be so negative that $r_0 > 0$ will not hold true. All following investigations will be confined to local stability considerations around such steady–state solutions.

Proof: By setting to zero the right hand sides of (81)-(83) and (87)-(89), we have $\pi_o = \mu - n$, $\hat{p}_o = \pi_o$ as well as $\pi_o^* = \mu^* - n$, $\hat{p}_o^* = \pi_o^*$. From (84), (85), also set equal to zero, we then get $\epsilon_o = \mu - \mu^* (= \hat{p}_o - \hat{p}_o^* = \hat{e}_o)$ and thus $r_o^* + \epsilon_o - r_o$ due to our assumption that $\rho_o = \rho_o^*$ and because of $r_o = \rho_o + \pi_o, r_o^* = \rho_o^* + \pi_o^*$. From (84) we then get $a(\cdot) = 0$ which implies $\eta = \eta_o$, since a is nx negatively sloped function of η solely (all other variables in nx are fixed at their steady state values by assumption). We thus have $c_1^* = c_2/\eta_o$ in the steady state and therefore a description of goods-market disequilibrium as if both economies were closed, i.e., for example: $X^p = \omega_o l^d + (1 - s_c)(\rho_o - t_c) + i(\cdot) + n + \delta + g - y$. Equations (80)-(83) and (86)-(89) can therefore now be considered in isolation from each other, as in the case of closed economies. We shall concentrate on equations (80)-(83) in the following analysis.

From the equations (80) and (82) we get the following equation system for the variables X^p, X^w:

$$\begin{aligned} 0 &= (1 - \kappa_p)\beta_w X^w + (\kappa_w - 1)\beta_p X^p, \\ 0 &= \beta_p X^p + \kappa_p \beta_w X^w. \end{aligned}$$

It is easily shown for $\kappa_w \kappa_p < 1$ that this linear equation system can be uniquely solved for X^w, X^p which must both be zero then. This implies the first of our steady state equations (92) as well as $i(\cdot) = 0$, i.e., $r = \rho_o - \pi_o$. Equation (93) then immediately follows and (94) has already been shown above. The equation for ρ_0 is obtained from $X^p = 0$ by solving this equation for $\rho_0 \quad (= y - \delta - \omega_o l^d)$. The calculation of ω_0 is then straightforward. This concludes the proof of existence and uniqueness for the interior steady state solution. □

We now investigate stability properties of a convenient slightly more special case of the above 10D dynamical system which can be written as an nonlinear autonomous dynamical system in the ten state variables $\omega = w/p$, $l = L/K$, p, π, $\omega^* = w^*/p^*$, $l^* = L^*/K^*$, p^*, π^*, e and ϵ. As this list shows we now intend to neglect all trends in the nominal magnitudes, by assuming $\mu - n = \mu^* - n^* = 0$ (no steady state inflation at home and abroad and also no steady depreciation or appreciation). Furthermore, since we have $nx_o = 0$ in steady state we (continue to) assume that $\rho_o = \rho_o^*$ holds in the steady state. This allows for interest rate parity $r_o = r_o^*$ in the steady state (where $\hat{e}_o = \epsilon_o = 0$ holds and where interest rates coincide with the profit rates of firms). Finally, we consider only the case where capital stock growth is driven by investment demand, i.e., we assume $\beta_k = 1$ in the following. We then have the following steady state values of the nominal magnitudes (in addition to what has been listed in theorem 1):

$$\boxed{p_o = \frac{m(0) l_o}{h_1 y}, \quad m(0) = \frac{M(0)}{L(0)}, \quad p_o^* = \frac{m^*(0) l_o^*}{h_1^* y^*}, \quad m^*(0) = \frac{M^*(0)}{L^*(0)}, \quad e_o = \frac{\eta_o p_o^*}{p_o}}$$

and of course $w_o = \omega_o p_o$ and $w_o^* = \omega_o^* p_o^*$ for the level of money wages. The laws of motion of the two economies and their interaction in the situation now being considered simply read, in the case $\beta_k = 1$:

The domestic economy:

$$\begin{aligned}
\widehat{\omega} &= \kappa[(1-\kappa_p)\beta_w X^w + (\kappa_w - 1)\beta_p X^p], && (100)\\
\widehat{l} &= -i(\rho + \pi - r), && (101)\\
\widehat{p} &= \kappa[\beta_p X^p + \kappa_p \beta_w X^w] + \pi, && (102)\\
\dot{\pi} &= \beta_\pi[\alpha_\pi(\hat{p} - \pi) + (1-\alpha_\pi)(-\pi)]. && (103)
\end{aligned}$$

Financial and trade links between the two economies:

$$\begin{aligned}
\hat{e} &= \beta_e(\beta(r^* + \epsilon - r) - a(\cdot)), && (104)\\
\dot{\epsilon} &= \beta_\epsilon[\alpha_\epsilon(\hat{e} - \epsilon) + (1-\alpha_\epsilon)(-\epsilon)]. && (105)
\end{aligned}$$

The foreign economy:

$$\begin{aligned}
\widehat{\omega}^* &= \kappa^*[(1-\kappa_p^*)\beta_w^* X^{w*} + (\kappa_w^* - 1)\beta_p^* X^{p*}], && (106)\\
\widehat{l}^* &= -i^*(\rho^* + \pi^* - r^*), && (107)\\
\widehat{p}^* &= \kappa^*[\beta_p^* X^{p*} + \kappa_p^* \beta_w^* X^{w*}] + \pi^*, && (108)\\
\dot{\pi}^* &= \beta_\pi^*[\alpha_\pi^*(\hat{p}^* - \pi^*) + (1-\alpha_\pi^*)(-\pi^*)]. && (109)
\end{aligned}$$

Here for the domestic economy we employ the abbreviations:

$$\begin{aligned}
\rho &= y - \delta - \omega y/x, \quad y = \text{const.}\\
X^w &= y/(xl) - \bar{V}, \quad X^p = c_1 + c_1^* + i(\cdot) + n + \delta + g - y,\\
i(\cdot) &= i(\rho + \pi - r), \quad r = r_o + (h_1 y - m)/h_2, \quad m = m(0)l/p,\\
c_1 &= \omega y/x + \gamma_c(\eta)(1-s_c)(\rho - t_c), \quad c_1^* = (l/l^*)(1-\gamma_c^*(\eta))(1-s_c^*)(\rho^* - t_c^*)/\eta,\\
a(\cdot) &= (1-\gamma_c^*(\eta))(1-s_c^*)(\rho^* - t_c^*)l/l^*/\eta - (1-\gamma_c(\eta))(1-s_c)(\rho - t_c), \quad \eta = p/(ep^*),
\end{aligned}$$

and similarly for the foreign economy.[16] Note again that we are using for the determination of the division of households' consumption into domestic and foreign commodities, the simple linear functions:

$$\begin{aligned}
\gamma_c(\eta) &= \gamma_c + \gamma(\eta_o - \eta), \quad \gamma > 0, \eta = p/(ep^*),\\
\gamma_c^*(\eta) &= \gamma_c^* - \gamma^*(\eta_o - \eta), \quad \gamma^* > 0, \eta = p/(ep^*),
\end{aligned}$$

[16] Where we in particular have:

$$c_2^* = \omega^* y^*/x^* + \gamma_c^*(\eta)(1-s_c^*)(\rho^* - t_c^*), \quad c_2 = (l^*/l)(1-\gamma_c(\eta))(1-s_c)(\rho - t_c)\eta.$$

Note also that X^p can be rewritten as $X^p = \omega y/x + (1-s_c)(\rho - t_c) + i(\cdot) + n + \delta + g + nx(\cdot) - y$.

in order to keep the model as close as possible to a linear form for the time being.

We now start our local stability investigations by a series of propositions and their proofs which are both concentrated on the essential issues to be dealt with and thus do not present every detail that is necessary for their final formulation. A detailed proof of the local stability of the steady state of the fully integrated 10D dynamics will be presented in the next section. In the following theorems we neglect all border line cases where parameters other than adjustment speed parameters, like the $\kappa's$, are set equal to zero or one.

Theorem 2

Assume that the parameters β_p, β_p^*, β_e, β_ϵ, are all set equal to zero.[17] *Then:*

1. *The dynamics of the two countries are completely decoupled from each other and the determinants of the Jacobians at the steady states of the two separate 4D dynamics at home and abroad are both zero.*
2. *These dynamics can both be reduced two 3D systems, each with a locally asymptotically stable steady state, if β_π, β_π^* are chosen sufficiently small. Concerning the eigenvalue structure of the dynamics at the steady state, we therefore have in this case six eigenvalues with negative real parts and four that are zero in the considered situation.*

Proof: 1. As the KWG model is formulated it only links the two countries via excess demands X^p and X^{p*}, terms which are suppressed when price adjustment speeds with respect to demand pressure are set equal to zero. The first and the third block of laws of motion are therefore then independent from each other and can be investigated separately. Furthermore, there exist positive numbers a and b such that $-a\hat{\omega}+\hat{p}+b\dot{\pi} \equiv 0$ which implies the statement on the 4D determinants.
2. Integrating the linear dependency just shown gives (for example for country 1) with respect to the price level p : $\quad p = +const \cdot \omega^a \exp(-b\pi)$. This equation feeds into the investment equation via

$$i(\cdot) = i(\rho + \pi - r), \quad r = r_o + (h_1 y - m)/h_2, \quad m = m(0)l/p, \quad p = +const \cdot \omega^a \exp(-b\pi),$$

which thereby reduces the originally 4D dynamics to dimension 3. The Jacobian of the reduced 3D dynamics (for ω, l, π) is characterized by

$$\begin{pmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{pmatrix} = \begin{pmatrix} 0 & - & 0 \\ + & - & - \\ 0 & - & - \end{pmatrix}.$$

The trace is unambiguously negative in this case. For β_π sufficiently small we have that $J_{22}J_{33} - J_{23}J_{32}$ will be dominated by $J_{12}J_{21}$ which gives the local asymptotic stability result, since the Routh-Hurwitz coefficients $a_1 a_2$ will always be larger than $a_3 = -\det J$ in the considered situation, due to the fact that the determinant will be just one expression in the product $a_1 a_2$. □

[17] The first two assumption imply that trade does not influence the price - quantity dynamics in the two countries considered. The other imply that both e and ϵ can be frozen at their steady state values.

As the proof has shown, we have zero root hysteresis present in each country, i.e., the price levels in both countries are not uniquely determined in their long-run position, but depend on the history of the economy and the shocks it has experienced. This is due to the fact that demand pressure in the market for goods does not matter for the dynamics of the price level. It is also due to this fact that neither Mundell-effects nor Keynes-effects are present in the currently considered situation in their typical format (since there is no positive feedback of expected inflation on its time rate of change by way of the third law of motion and no negative effect of the price level onto its rate of change by the law of motion for this price level). Furthermore, a positive dependence of aggregate demand on real wages cannot be destabilizing here via the Rose effect, while a negative dependence is destabilizing, but only if the price level reacts with sufficient strength with respect to demand pressure on the market for goods.

Theorem 3

Assume that the parameters β_p^*, β_e and β_ϵ, remain fixed at zero, but that the parameter β_p is made positive such that the negative real parts considered in theorem 2 remain so. Then:

1. *The dynamics of the home country now depends on what happens in the foreign economy.*
2. *There are now seven eigenvalues of the full dynamical system with negative real parts, while three remain at zero.*

The hysteresis argument can now only be applied to the foreign economy and the price level there, while the price level at home now has a unique long-run position (as it has been determined above). Note here also that we only consider an 8D dynamical system for the moment, since e and ϵ are kept frozen at their steady state values. We thus have an 8D system with vanishing 8D determinant ($a_8 = 0$), but with all other conditions of the Routh Hurwitz theorem being fulfilled (i.e. for the Routh Hurwitz coefficients $a_1, ..., a_7$).

Proof: We reduce the dynamics in the foreign economy to 3D according to the proof strategy of theorem 2. The 8D dynamics is thereby made 7D. The Jacobian to be investigated then is of the form (with the domestic economy shown first):

$$\begin{pmatrix} J_{11} & J_{12} & J_{13} & J_{14} & ? & ? & ? \\ J_{21} & J_{22} & J_{23} & J_{24} & ? & ? & ? \\ J_{31} & J_{32} & J_{33} & J_{34} & ? & ? & ? \\ J_{41} & J_{42} & J_{43} & J_{44} & ? & ? & ? \\ 0 & 0 & 0 & 0 & J_{55} & J_{56} & J_{57} \\ 0 & 0 & 0 & 0 & J_{65} & J_{66} & J_{67} \\ 0 & 0 & 0 & 0 & J_{75} & J_{76} & J_{77} \end{pmatrix}.$$

The entries with question mark do not matter for the calculation of the eigenvalues of this Jacobian. Furthermore, the foreign country exhibits three eigenvalues with negative real parts according to what has been shown in theorem 2. These eigenvalues are independent of what happens in the domestic economy. For the latter economy we have assumed that three of its eigenvalues still have negative real parts when β_p is made positive. It suffices therefore to show that

$$\det \begin{pmatrix} J_{11} & J_{12} & J_{13} & J_{14} \\ J_{21} & J_{22} & J_{23} & J_{24} \\ J_{31} & J_{32} & J_{33} & J_{34} \\ J_{41} & J_{42} & J_{43} & J_{44} \end{pmatrix}$$

is always positive in order to get the result that the eigenvalue that moves away from zero must become negative. The sign of the determinant can – as usual – be obtained by removing linear dependencies from the involved laws of motion as follows:

$$\begin{aligned} \widehat{\omega} &= X^w, \\ \widehat{l} &= -i(\cdot), \\ \widehat{p} &= X^p, \\ \dot{\pi} &= -\pi. \end{aligned}$$

Continuing in this way we get

$$\begin{aligned} \widehat{\omega} &= -l, \\ \widehat{l} &= +\omega + p, \\ \widehat{p} &= +\omega - p, \\ \dot{\pi} &= -\pi. \end{aligned}$$

Note here that we have to employ $m = m(0)l/p$ in the rate of interest expression in the investment function, but that the influence of l does not matter due to what is shown in the first row of the considered 4D matrix J. We thus finally get (with the usual interpretation that the equality sign only indicates that there is no change in the sign of the corresponding determinant):

$$\begin{aligned} \widehat{\omega} &= -l, \\ \widehat{l} &= +\omega, \\ \widehat{p} &= -p, \\ \dot{\pi} &= -\pi. \end{aligned}$$

This last form of dynamic interdependence indeed implies that $\det J$ must be positive in sign.

□

We have so far considered the domestic economy as – so to speak – a satellite of the foreign one (with convergence to a steady state however). We therefore next assume that the adjustment speed β_p^* is also made positive. In this case the two economies become dependent on each other, like in a monetary union, since the exchange rate is still kept fixed and can therefore be set equal to 1. In this 8D case we have full interdependence though only via the excess demand channels and their influence on domestic and foreign price dynamics and thus now investigate the international price level connection. We therefore consider the first and the third block of our laws of motion in full interaction, yet still an inactive Dornbusch type of exchange rate dynamics. In this case the following theorem holds:

Theorem 4

Assume that the parameters β_e and β_ϵ, remain fixed at zero, but that the parameters β_p and β_p^* are now both positive, but chosen sufficiently small (such that the negative real parts of the eigenvalues considered in theorem 3 remain negative). Then:

1. *The determinant of the Jacobian at the steady state of the considered 8D dynamics is always positive (independently of speed of adjustment conditions).*
2. *There are now eight eigenvalues with negative real parts, i.e., the steady state is locally asymptotically stable in the considered situation.*

Proof: We proceed again by removing from the laws of motion of the 8D case (where e, ϵ are still kept fixed at their steady state values) all expressions that are irrelevant for the sign of the determinant of their Jacobian at the steady state. In a first step this leads us again to:

The domestic economy:

$$\boxed{\begin{array}{rcl} \widehat{\omega} & = & X^w, \\ \widehat{l} & = & -i(\cdot), \\ \widehat{p} & = & X^p, \\ \dot{\pi} & = & -\pi. \end{array}}$$

The foreign economy:

$$\boxed{\begin{array}{rcl} \widehat{\omega}^* & = & X^{w*}, \\ \widehat{l}^* & = & -i^*(\cdot), \\ \widehat{p}^* & = & X^{p*}, \\ \dot{\pi}^* & = & -\pi^*. \end{array}}$$

We then simplify in the same way even further (due to $nx_o = 0$):

The domestic economy:

$$\boxed{\begin{array}{rcl} \widehat{\omega} & = & -l, \\ \widehat{l} & = & +\omega + p, \\ \widehat{p} & = & \omega y/x + (1-s_c)\rho + nx(\cdot), \\ \dot{\pi} & = & -\pi. \end{array}}$$

The foreign economy:

$$\begin{aligned}
\widehat{\omega}^* &= -l^*, \\
\widehat{l}^* &= +\omega^* + p^*, \\
\widehat{p}^* &= \omega^* y^*/x^* + (1 - s_c^*)\rho^* - \frac{l_o^*}{l_o}\eta_o nx(\cdot), \\
\dot{\pi}^* &= -\pi^*.
\end{aligned}$$

From this result we finally obtain by continuing the employed method of reduction (since a depends negatively on η and ω^* and positively on ω):

The domestic economy:

$$\begin{aligned}
\widehat{\omega} &= -l, \\
\widehat{l} &= +\omega + p, \\
\widehat{p} &= +\omega + \omega^*, \\
\dot{\pi} &= -\pi.
\end{aligned}$$

The foreign economy:

$$\begin{aligned}
\widehat{\omega}^* &= -l^*, \\
\widehat{l}^* &= +\omega^* + p^*, \\
\widehat{p}^* &= +\omega^* - \omega + p - p^*, \\
\dot{\pi}^* &= -\pi^*.
\end{aligned}$$

We are now in a position to calculate the sign of the determinant under consideration. Note first of all that the laws of motion for π and π^* can be neglected in this calculation, since their two rows and columns in the Jacobian do not change the sign of its determinant. For the remaining entries of J (in the order $\omega, l, p, \omega^*, l^*, p^*$) we have according to what has been shown above:

$$\det\begin{pmatrix} 0 & - & 0 & 0 & 0 & 0 \\ + & 0 & + & 0 & 0 & 0 \\ + & 0 & 0 & + & 0 & 0 \\ 0 & 0 & 0 & 0 & - & 0 \\ 0 & 0 & 0 & + & 0 & + \\ 0 & 0 & + & 0 & 0 & - \end{pmatrix} = +\det\begin{pmatrix} + & + & 0 & 0 & 0 \\ + & 0 & + & 0 & 0 \\ 0 & 0 & 0 & - & 0 \\ 0 & 0 & + & 0 & + \\ 0 & + & 0 & 0 & - \end{pmatrix}$$

$$= +\det\begin{pmatrix} + & + & 0 & 0 \\ + & 0 & + & 0 \\ 0 & 0 & + & + \\ 0 & + & 0 & - \end{pmatrix} = +\det\begin{pmatrix} 0 & + & 0 \\ 0 & + & + \\ + & 0 & - \end{pmatrix} - \det\begin{pmatrix} + & + & 0 \\ 0 & + & + \\ 0 & 0 & - \end{pmatrix}$$

$$= -\det\begin{pmatrix} 0 & + \\ + & - \end{pmatrix} - \det\begin{pmatrix} + & + \\ 0 & - \end{pmatrix} > 0$$

This proves assertion 1 of theorem 4. Assertion 2 than follows immediately from what has been shown for the 7D case and the fact that the positive 8D determinant enforces a negative eigenvalue if the real parts of the eigenvalues of the 7D case are all negative. □

We thus have shown the result that monetary unions of KWG type exhibit cyclical or even monotonic convergence of trajectories to their interior steady state position if wages and in particular prices adjust sufficiently sluggishly in both countries. Though the proofs concern only the local validity of such a statement, numerical simulations suggest that such a result also holds from the global perspective, since the nonlinearities intrinsically present in the employed laws of motion are generally of a type that generate for such a result. The same however generally also applies to situations of divergence which therefore demand the introduction of extrinsic nonlinearities in order to get viable dynamics.

Let us now allow for $\beta_e > 0$, but not yet for adjusting expectations of depreciation or appreciation. In this situation we leave the case of a monetary union and consider now the role of capital mobility and of adjusting nominal exchange rates, again at first with respect to asymptotic stability and with the presence of just intrinsic nonlinearities.

Theorem 5

Assume that the parameter β_ϵ remains fixed at zero, but that the parameters β_e and β are now positive, and chosen sufficiently small (such that the negative real parts of the eigenvalues considered in theorem 4 remain negative). Then:

1. *The determinant of the Jacobian at the steady state of the considered 9D dynamics is always negative (independently of speed of adjustment conditions).*[18]

[18]Note here that the parameter β does not represent a speed of adjustment condition, but characterizes the degree of capital mobility. Setting this parameter to a small value has the convenient effect that the law of motion for the exchange rate is basically dependent on trade and can thus be used to eliminate the net export term $nx(\cdot)$ from the laws of motion for the domestic and the foreign economy as far as the calculation of determinants is concerned. We conjecture however that the obtained result on determinants also holds for large values of β, though row operations are considerably more difficult then.

2. *Assume that β, the degree of capital mobility, is chosen sufficiently small. The considered 9D dynamics then exhibits nine eigenvalues with negative real parts, i.e., their interior steady state is locally asymptotically stable in this situation.*

Proof: In the case $\beta = 0$ we get, because of

$$X^p = \omega y/x + (1 - s_c)(\rho - t_c) + i(\cdot) + n + \delta + g + nx(\cdot) - y,$$

$$X^{p*} = \omega^* y^*/x^* + (1 - s_c^*)(\rho^* - t_c^*) + i^*(\cdot) + n^* + \delta^* + g^* - \frac{l^*}{l} a(\cdot)\eta - y^*,$$

that the a expression can be removed both from the domestic and the foreign economy as far as the calculation of determinants is concerned, since we then simply have $\hat{e} = -\beta_e a$. The system decomposes into two 4D dynamics with positive determinants and $\hat{e} = -e$, again of course solely as far as the calculation of the determinant of the Jacobian at the steady state is concerned. This proves the first assertion, but – due to the method chosen – only for $\beta's$ that are sufficiently small (all other speed of adjustment parameters can be arbitrary). We conjecture that this results holds for all positive β as well.[19] The second assertion of the theorem finally follows immediately, and in the usual way, from the continuity of eigenvalues on the parameters of the considered dynamics. □

Assume finally that the parameter β_ϵ is made positive, in the situation considered in theorem 5. Then:

1. *The determinant of the Jacobian at the steady state of the considered 10D dynamics is always positive.*
2. *Assume that β_ϵ, the speed of adjustment of expectations on exchange rate depreciation, is chosen sufficiently small. The considered 10D dynamics then exhibits ten eigenvalues with negative real parts, i.e., their interior steady state solution is locally asymptotically stable in the considered situation.*

Proof: Obvious from what has been shown so far, since the $\dot{\epsilon}-$law of motion can be reduced to $\dot{\epsilon} = -\beta_\epsilon \epsilon$ by means of the $\hat{e}$ law of motion, as usual, though only as far as the calculation of determinants is concerned. □

Theorem 7

From the locally asymptotically stable situation of theorem 6, the steady state must lose its local stability by way of Hopf-bifurcations if one of the parameters β_π (carrying the destabilizing Mundell effect), β_ϵ (carrying the destabilizing Dornbusch effect) or β_p (carrying the destabilizing Rose effect) is made sufficiently large, the latter however only in the case where the real wage effect in investment demand dominates the real wage effect in consumption demand.

[19]In which case $r^* - r$ can be reduced to $\omega - \omega^*$, but in this form remains as a new item in the fifth row of the considered Jacobian.

Proof: Straightforward, since the trace of the Jacobian J of the dynamics at the steady state can be made positive, by way of $\dot{\pi}'(\pi) > 0$, $\dot{\epsilon}'(\epsilon) > 0$ and $\hat{\omega}'(\omega) > 0$, respectively. □

Fast adjustments of expectations and fast adjustments of prices (in the case of a negative dependence of aggregate demand on the real wage level) are thus dangerous for asymptotic stability and will lead to loss of stability which is always accompanied by business fluctuations, possibly persistent ones if a supercritical Hopf-bifurcation occurs, but generally explosive ones as long as only intrinsic nonlinearities are present in the considered dynamical system. Numerical simulations have then to be used to gain insights into the global dynamics. These indicate that stable limit cycle situations or persistent cycles can be generated by the additional assumption of extrinsic nonlinearities, such as asymmetries in the money wage Phillips curve.

5 Numerical investigation of the KWG dynamics

In this section we provide some numerical illustrations of the dynamic features of the two-country KWG growth model that has so far only been studied from the local perspective around its unique interior steady state.[20] It is not difficult to provide numerical examples of damped oscillations or even monotonic adjustment back to the steady state based on what has been shown for the speed of adjustment parameters in the two preceding sections. Increasing such speed of adjustment parameters will then also provide examples of supercritical Hopf-bifurcations where – after the loss of local stability – stable limit cycles and thus persistent economic fluctuations will be born for a certain parameter range. However there will often simply be purely explosive behavior after such loss of stability, indicating that the intrinsic nonlinearities are generally too weak to bound the dynamics within economically meaningful ranges. The addition of extrinsic or behavioral nonlinearities is thus generally unavoidable in order to arrive at an economically meaningful dynamic behavior.

In the following we will however make use of another prominent behavioral nonlinearity, already discussed in Keynes (1936), namely a kinked money wage Phillips curve, expressing in stylized form the fact that wages are much more flexible upwards than downwards. This nonlinearity is often already sufficient to limit the dynamics to economically viable domains, though in reality of course coupled with other behavioral nonlinearities, also in operation at some distance from the steady state. Downward nominal wage rigidity however can often already by itself overcome the destabilizing feedback channels of Mundell-type (working through the real interest rate) or Rose-type (working through the real wage rate) and thus succeed in stylizing the economy in a certain area outside the steady state. This in particular holds if wages are assumed to be completely inflexible in the downward direction and if there is zero steady state inflation, where they can even stylize an economy towards damped oscillations that would otherwise – without this inflexibility – break down immediately as for example in the following first simulation exercise of the KWG dynamics.

[20]The simulations that follow were performed using the SND software package described in Chiarella, Flaschel, Khomin and Zhu (2002), which can be downloaded together with the project files for the simulations of this paper from Carl Chiarella's homepage: http//:www.business.uts.edu.au/finance/staff/carl.html.

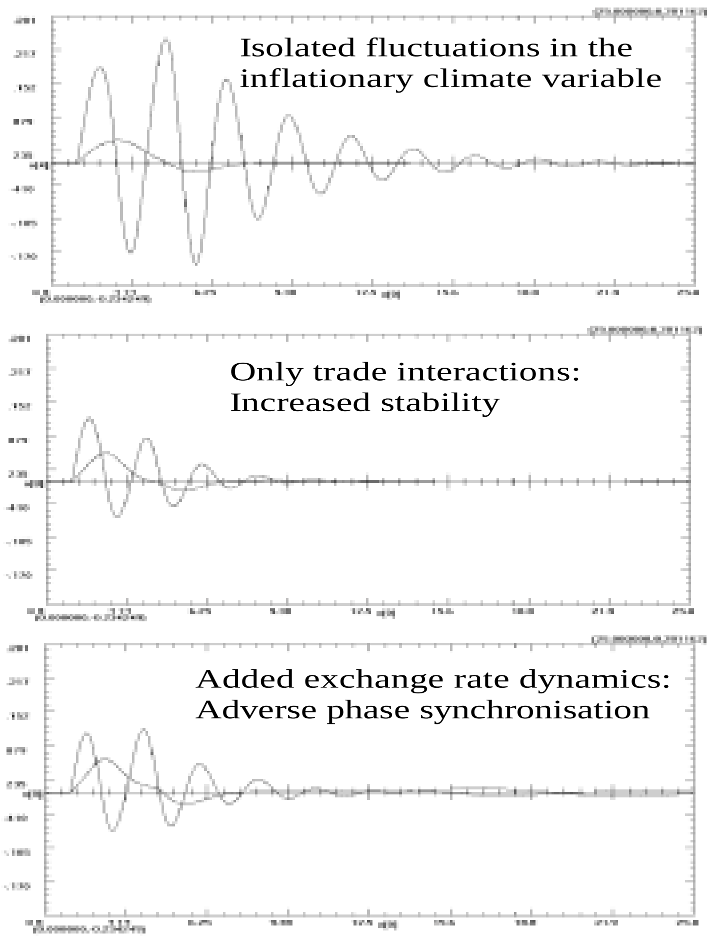


Figure 2: KWG cycles: Isolated, with trade interactions, and finally with financial market interaction: π and π^*.[21]

We show in figure 2, at the top, the time series for the inflationary climate π and π^*, when both countries are still completely decoupled from each other with country 1 exhibiting the larger fluctuations (and shorter phase length) in this state variable. Due to the strict kink in the money wage Phillips curve we have a marked convergence to the steady state in both countries. Note here that, though wage deflation is excluded from the model, nevertheless goods price deflation occurs. Allowing now for trade in goods between the two countries (but not yet for financial links) dampens the cycle in country 1 considerably and makes the one in country two slightly more pronounced, as shown in the middle of figure 2. This change remains true if financial links are added (as shown in the parameter set). Now, however, the dynamics converge to a limit cycle and no longer to the steady state (only crudely shown at the bottom of figure 2 to the right). This limit cycle exhibits nearly

[21]The parameters of this simulation run are as follows (the modifications (1) in trade and (2) in financial links are shown in brackets): $s_c = 0.8; \delta = 0.1; t_c = 0.35; g = 0.35; n = 0.05; \mu = 0.05; h_1 = 0.1; h_2 = 0.2; y^p = 1.0; x = 2.0; \beta_w = 2; \beta_p = 5; \kappa_w = 0.5; \kappa_p = 0.5; \beta_\pi = 3; \alpha_\pi = 0.5; i = 0.5; \beta_k = 1.0; \bar{V} = 0.8; s_c^* = 0.8; \delta^* = 0.1; t_c^* = 0.35; g^* = 0.35; n^* = 0.05; \mu^* = 0.05; h_1^* = 0.1; h_2^* = 0.2; y^{p*} = 1.0; x^* = 2.0; \beta_w^* = 2.0; \beta_p^* = 1.0; \kappa_w^* = 0.5; \kappa_p^* = 0.5; \beta_\pi^* = 3; \alpha_\pi^* = 0.5; i^* = 0.5; \beta_k^* = 1.0; \bar{V}^* = 0.8; \beta_e = 0(\beta_e = 1); \beta = 0(\beta = 2.5); \beta_\epsilon = 0(\beta_\epsilon = 1); \alpha_\epsilon = 0.5; \gamma_c = 0.99(\gamma_c = 0.5); \gamma = 0(\gamma = 1); \gamma_c^* = 0.99(\gamma_c^* = 0.5); \gamma^* = 0(\gamma_2 = 1); m_{shock} = 1.1$

completely adverse phase synchronization at least in the inflationary climate of the two considered countries, since the exchange rate dynamics now dominate the outcome and produce the negative correlation in inflation dynamics shown. There is thus no positive international transmission of inflation dynamics, contrary to what is generally expected, if trade is dominated by exchange rate movements and their (always adverse) effect on one of the two countries.

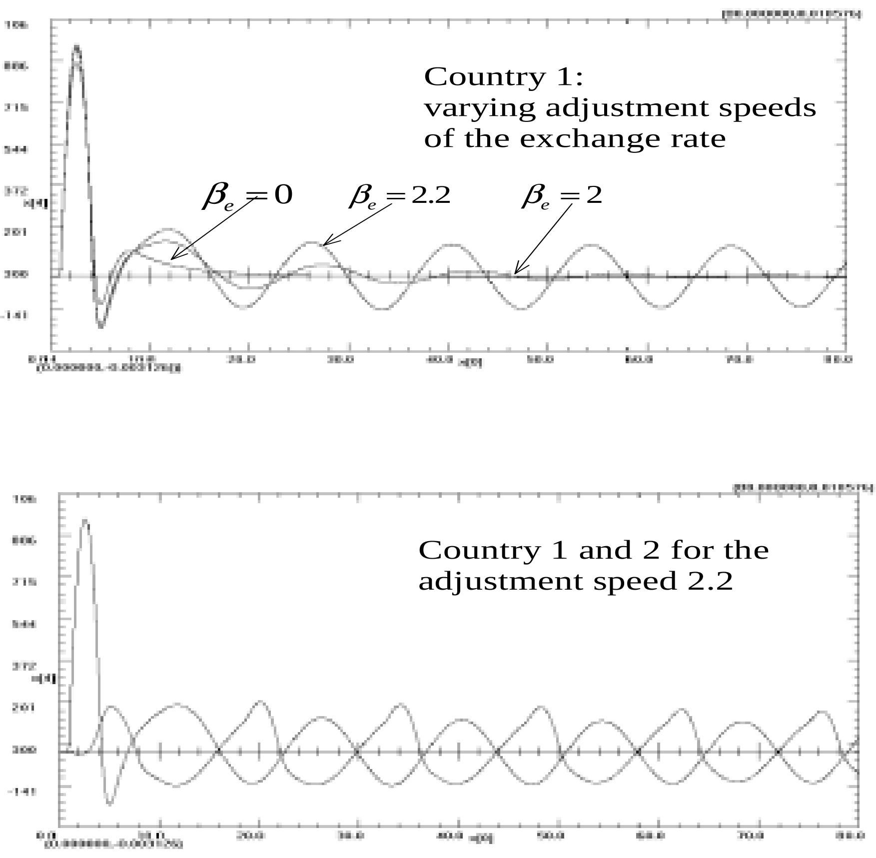


Figure 3: The occurrence of limit cycles and of negative transmissions of inflation.[22]

In figure 3 we show (again for π, π^*) with the time series in the top figure that increasing speed of adjustment of the exchange rate produces increasing volatility, here shown for the inflationary climate variable π. The final outcome shown is convergence to a persistent business cycle (stable limit cycle) in both countries, yet – as the lower time series show – with nearly perfect negative correlation. This figure again demonstrates that business fluctuations need not at all be synchronized with respect to upswings and downswings, though they are clearly synchronized here with respect to phase length. Note that setting

[22]The parameters of this simulation run are as follows (with $\beta_e = 0,\ 2,\ 2.2$ in the top time series): $s_c = 0.8; \delta = 0.1; t_c = 0.35; g = 0.35; n = 0.05; \mu = 0.05; h_1 = 0.1; h_2 = 0.2; y^p = 1.0; x = 2.0; \beta_w = 2; \beta_p = 1; \kappa_w = 0.5; \kappa_p = 0.5; \beta_\pi = 3; \alpha_\pi = 0.5; i = 0.5; \beta_k = 1.0; \bar{V} = 0.8; s_c^* = 0.8; \delta^* = 0.1; t_c^* = 0.35; g^* = 0.35; n^* = 0.05; \mu^* = 0.05; h_1^* = 0.1; h_2^* = 0.2; y^{p*} = 1.0; x^* = 2.0; \beta_w^* = 2.0; \beta_p^* = 1.0; \kappa_w^* = 0.5; \kappa_p^* = 0.5; \beta_\pi^* = 3; \alpha_\pi^* = 0.5; i^* = 0.5; \beta_k^* = 1.0; \bar{V}^* = 0.8; \beta_e = 0; \beta = 1.0; \beta_\epsilon = 1.0; \alpha_\epsilon = 0.5; \gamma_c = 0.5; \gamma = 1.0; \gamma_c^* = 0.5; \gamma^* = 1.0; m_{shock} = 1.02$

$\beta_e = 0$ (no exchange rate dynamics) is already sufficient to decouple the real dynamics from what happens in the foreign exchange market.

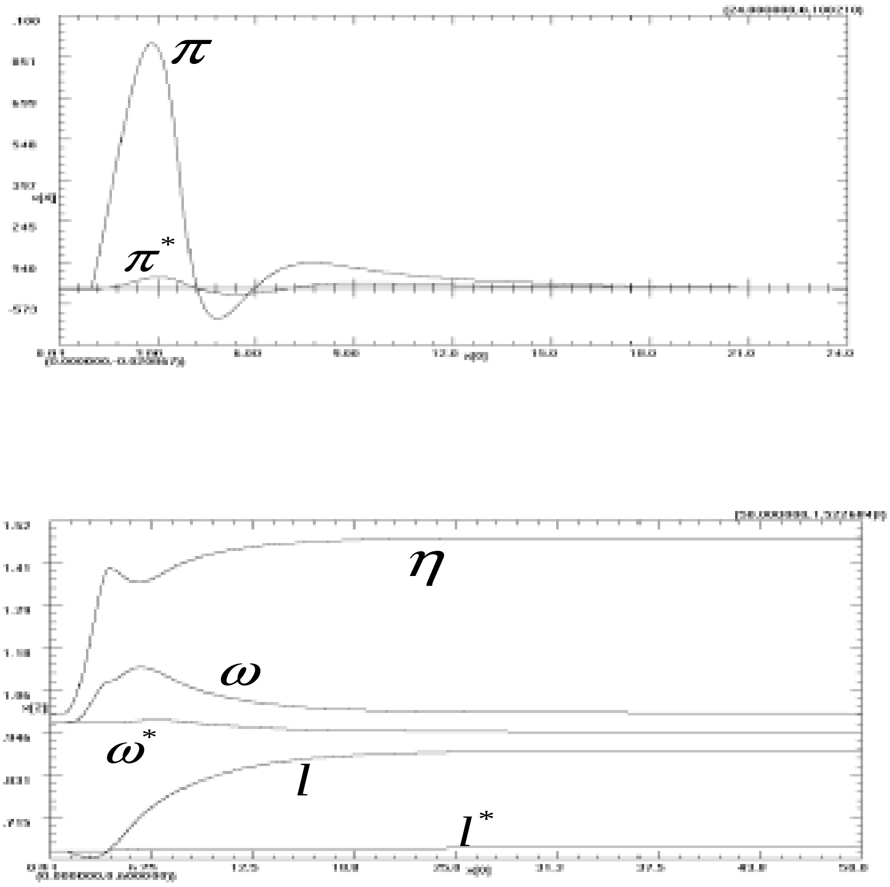


Figure 4: Positively correlated trade and hysteresis in a monetary union.[23]

The top figure in figure 4 shows that business fluctuations (represented here again by the two inflation climate variables) are now fairly synchronized and also fairly damped again (with the home country the one with initially more volatility in inflation, since the expansionary monetary shock is occurring in this country solely, there lowering the interest rate and thus increasing investment and inflation directly). Wage flexibility is very high ($\beta_w = 5$) in the simulation under consideration, but is again tamed in a radical way by the assumption that there is no wage deflation possible (which is more restrictive than just the assumption $\beta_w = 0$). In the lower graph of figure 4 we show in addition that there is now zero root hysteresis involved in the evolution of the nominal as well as the real variables. This is due to the fact that the relevant 9D dynamics, with its suppression of the Dornbusch nominal exchange rates, but still with changing real exchange rate dynamics

[23]The parameters of this simulation run are as follows (with $\beta_e = 0$ and thus a fixed exchange rate throughout): $s_c = 0.8; \delta = 0.1; t_c = 0.35; g = 0.35; n = 0.05; \mu = .05; h_1 = 0.1; h_2 = 0.2; y^p = 1.0; x = 2.0; \beta_w = 5; \beta_p = 1; \kappa_w = 0.5; \kappa_p = 0.5; \beta_\pi = 3; \alpha_\pi = 0.5; i = 0.5; \beta_k = 1.0; \bar{V} = 0.8; s_c^* = 0.8; \delta^* = 0.1; t_c^* = 0.35; g^* = 0.35; n^* = 0.05; \mu^* = .05; h_1^* = 0.1; h_2^* = 0.2; y^{p*} = 1.0; x^* = 2.0; \beta_w^* = 5; \beta_p^* = 1.0; \kappa_w^* = 0.5; \kappa_p^* = 0.5; \beta_\pi^* = 3; \alpha_\pi^* = 0.5; i^* = 0.5; \beta_k^* = 1.0; \bar{V}^* = 0.8; \beta_e = 0; \beta = 0; \beta_\epsilon = 0; \alpha_\epsilon = 0.5; \gamma_c = .7; \gamma = 1; \gamma_c^* = .7; \gamma^* = 1; m_{shock} = 1.1$

due to differing inflation in the two countries considered, now exhibits a law of motion for the real exchange rate η that is 'linearly dependent' on the two laws of motion for the two price levels of the investigated economies. There is thus hysteresis present in the evolution of the real exchange rate which is transmitted also to hysteresis in real wages and full employment labor intensity, as shown in figure 4. We note that hysteresis can here also be partly due to the kink in the Phillips curve, which when based on this fact implies that the steady state employment rate need no longer coincide with the given NAIRU rate $\bar{V}$, if it is characterized by zero inflation rates in the steady state so that the kink becomes operative immediately below the steady state.[24]

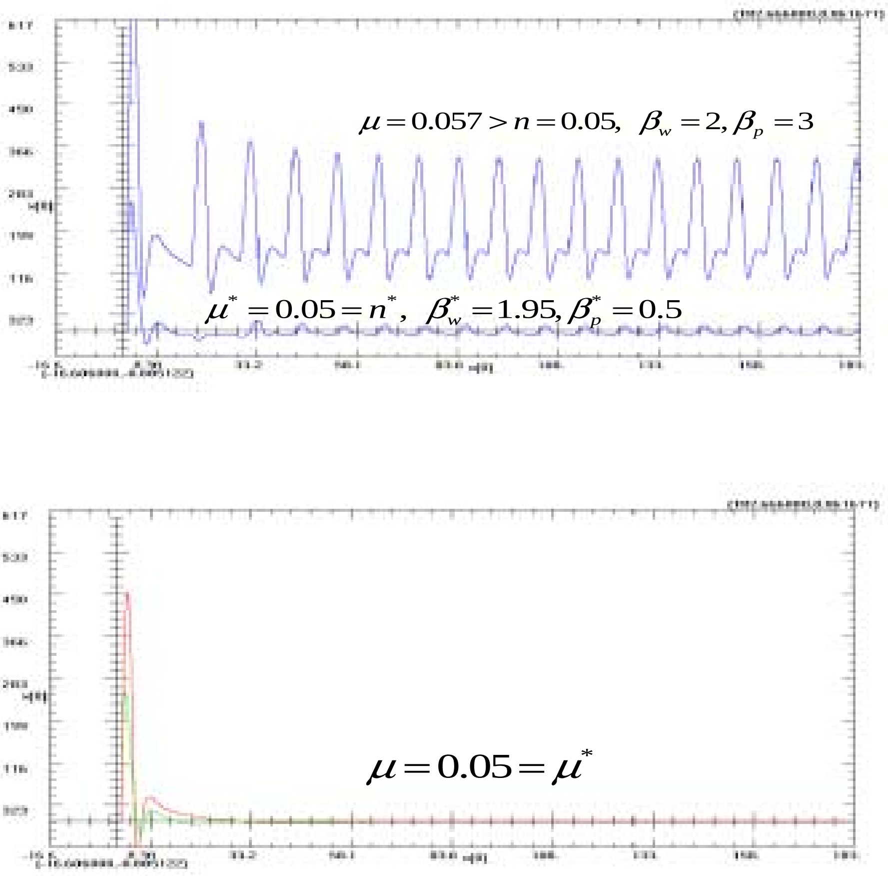


Figure 5: The generation of persistent economic fluctuations in the case of positive steady state inflation in country 1[25]

Figure 5 shows in its lower part, and, in a striking fashion (for π and π^*), that only radically damped oscillations may occur in the case where both countries pursue the policy

[24]Note that the shown fluctuations are obtained by throwing the economy out of the steady state via a ten percent increase in the money supply. This is a large shock and one which shocks the economy the more the further from the unstable steady state is the unstable limit cycle surrounding it.

[25]The parameters of this simulation run are as follows: $s_c = 0.8; \delta = 0.1; t_c = 0.35; g = 0.35; n = 0.05; \mu = 0.057; h_1 = 0.1; h_2 = 0.2; y^p = 1.0; x = 2.0; \beta_w = 2; \beta_p = 3; \kappa_w = 0.5; \kappa_p = 0.5; \beta_\pi = 3; \alpha_\pi = 0.5; i = 0.5; \beta_k = 1.0; \bar{V} = 0.8; s_c^* = 0.8; \delta^* = 0.1; t_c^* = 0.35; g^* = 0.35; n^* = 0.05; \mu^* = 0.05; h_1^* = 0.1; h_2^* = 0.2; y^{p*} = 1.0; x^* = 2.0; \beta_w^* = 1.95; \beta_p^* = .5; \kappa_w^* = 0.5; \kappa_p^* = 0.5; \beta_\pi^* = 3; \alpha_\pi^* = 0.5; i^* = 0.5; \beta_k^* = 1.0; \bar{V}^* = 0.8; \beta_e = 0; \beta = 0; \beta_\epsilon = 0; \alpha_\epsilon = 0.5; \gamma_c = 0.5; \gamma = 1; \gamma_c^* = .5; \gamma^* = 1; m_{shock} = 1.1$

of zero steady state inflation. In the upper part however we show what happens if country one allows for 0.7 percent of inflation in the steady state by increasing its money supply growth rate accordingly. There are now persistent fluctuations not only occurring in the country that allows for such monetary policy, but also induced persistent fluctuations in the other country, here with a significant degree of phase synchronization, since the Dornbusch dynamics is again absent from the considered situation. The inflationary environment in which the kinked money wage Phillips curve is operating does therefore matter very much and may give rise to situations where the economy is no longer viable (which occurs here for $\mu = 0.07$).

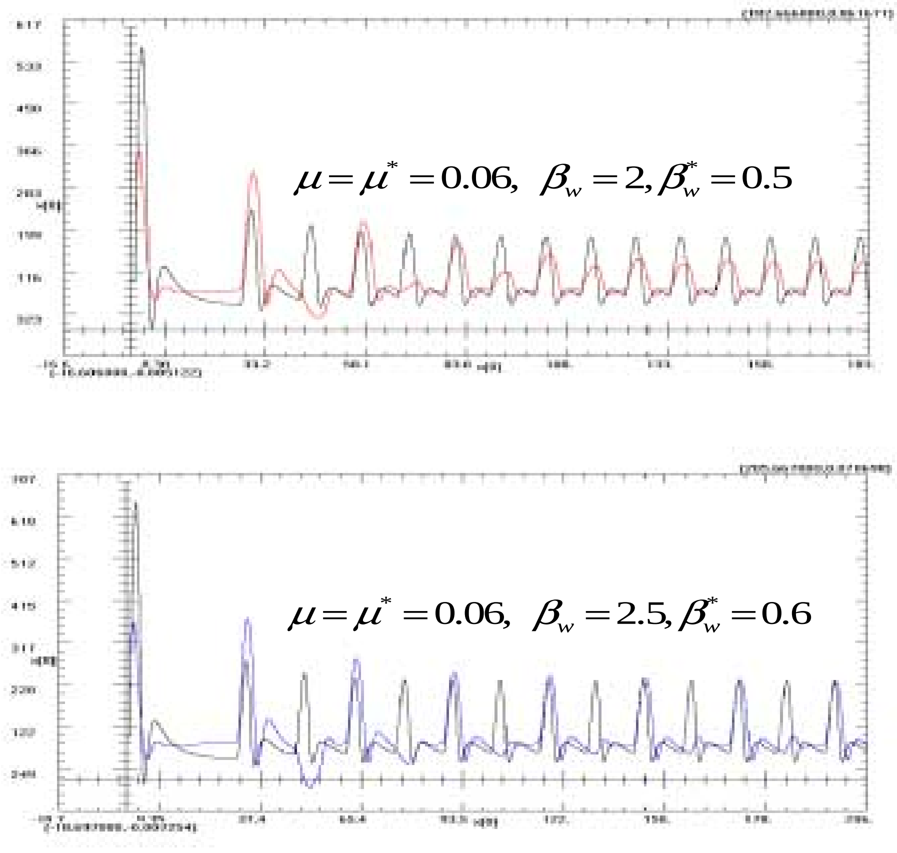
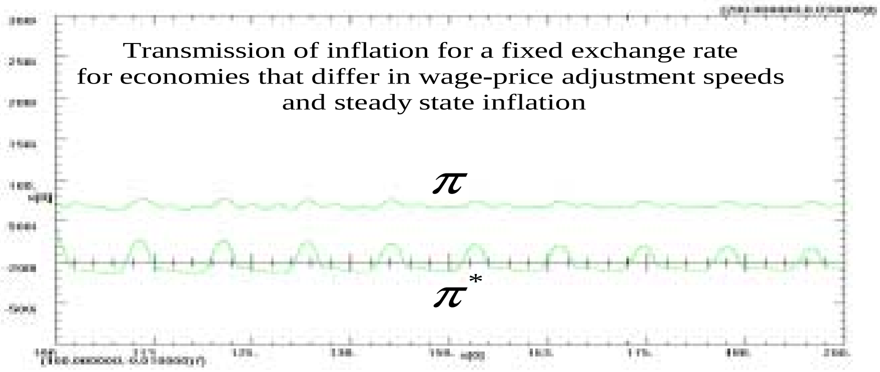


Figure 6: Phase synchronization in a fixed exchange rate system.[26]

The time series in figure 6 (as usual for the inflationary climate variable in both countries) show for varying wage adjustment speeds (and a 1 percent inflation rate in both countries in the steady state) how phases get synchronized in the two countries, here with respect to inflation rates. Due to the higher wage adjustment speed in country 1 we find in the case of independent fluctuations that phase lengths differ considerably in the more volatile

[26]The parameters of this simulation run are as follows: $s_c = 0.8; \delta = 0.1; t_c = 0.35; g = 0.35; n = 0.05; \mu = 0.06; h_1 = 0.1; h_2 = 0.2; y^p = 1.0; x = 2.0; \beta_w = 2.5; \beta_p = 1; \kappa_w = 0.5; \kappa_p = 0.5; \beta_\pi = 3; \alpha_\pi = 0.5; i = 0.5; \beta_k = 1.0; \bar{V} = 0.8; s_c^* = 0.8; \delta^* = 0.1; t_c^* = 0.35; g^* = 0.35; n^* = 0.05; \mu^* = 0.06; h_1^* = 0.1; h_2^* = 0.2; y^{p*} = 1.0; x^* = 2.0; \beta_w^* = .6; \beta_p^* = 1; \kappa_w^* = 0.5; \kappa_p^* = 0.5; \beta_\pi^* = 3; \alpha_\pi^* = 0.5; i^* = 0.5; \beta_k^* = 1.0; \bar{V}^* = 0.8; \beta_e = 0; \beta = 0; \beta_\epsilon = 0; \alpha_\epsilon = 0.5; \gamma_c = .5; \gamma = 1.5; \gamma_c^* = .5; \gamma^* = 1.5; m_{shock} = 1.1$

inflation dynamics of the home country from the ones observed abroad (with less flexible wages). Yet once the countries are coupled with each other, as indicated by the parameter set shown in footnote 26, cycle phase lengths become by and large synchronized in the upper graph (though not their amplitudes), while we can see in the lower time series comparison that phase length stay in a ratio of 2 to each other when only the significant peaks are taken into account. There are thus various possibilities for phase synchronization to be taken into account and to be explored further in future studies of the considered dynamics.



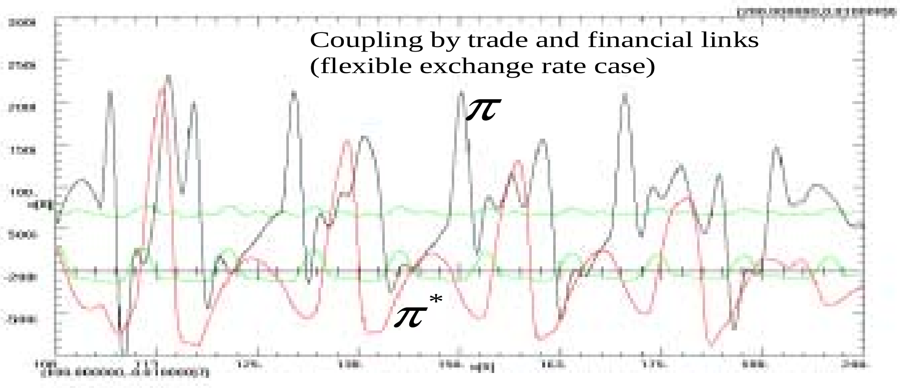


Figure 7: Phase synchronization in a fixed exchange rate system and its loss under flexible exchange rates.[27]

In the figure 7 (top) we show how cycles for countries that are interacting with respect to trade (in a fixed exchange rate system) are to some extent synchronized (with respect to the longer phase length in country 2). This synchronization gets lost to some extent in the case of a flexible exchange rate system ($\beta_e = \beta = \beta_\epsilon = 0.5$), and this in a way that makes the then still occurring persistent fluctuations (bottom figure) much more pronounced than

[27]The parameters of this simulation run are as follows: $s_c = 0.8; \delta = 0.1; t_c = 0.35; g = 0.35; n = 0.05; \mu = 0.057; h_1 = 0.1; h_2 = 0.2; y^p = 1.0; x = 2.0; \beta_w = 2; \beta_p = 3; \kappa_w = 0.5; \kappa_p = 0.5; \beta_\pi = 3; \alpha_\pi = 0.5; i = 0.5; \beta_k = 1.0; \bar{V} = 0.8; s_c^* = 0.8; \delta_2 = 0.1; t_c^* = 0.35; g^* = 0.35; n^* = 0.05; \mu^* = 0.05; h_1^* = 0.1; h_2^* = 0.2; y^{p*} = 1.0; x^* = 2.0; \beta_w^* = 1.95; \beta_p^* = .5; \kappa_w^* = 0.5; \kappa_p^* = 0.5; \beta_\pi^* = 3; \alpha_\pi^* = 0.5; i^* = 0.5; \beta_k^* = 1.0; \bar{V}^* = 0.8; \beta_e = 0; \beta = 0; \beta_\epsilon = 0; \alpha_\epsilon = 0.5; \gamma_c = 0.5; \gamma = 1; \gamma_c^* = .5; \gamma^* = 1; m_{shock} = 1.1$

they were in the fixed exchange rate case (top figure). Cycle interaction in the real and the financial part thus may make such interacting economies fairly volatile.[28]

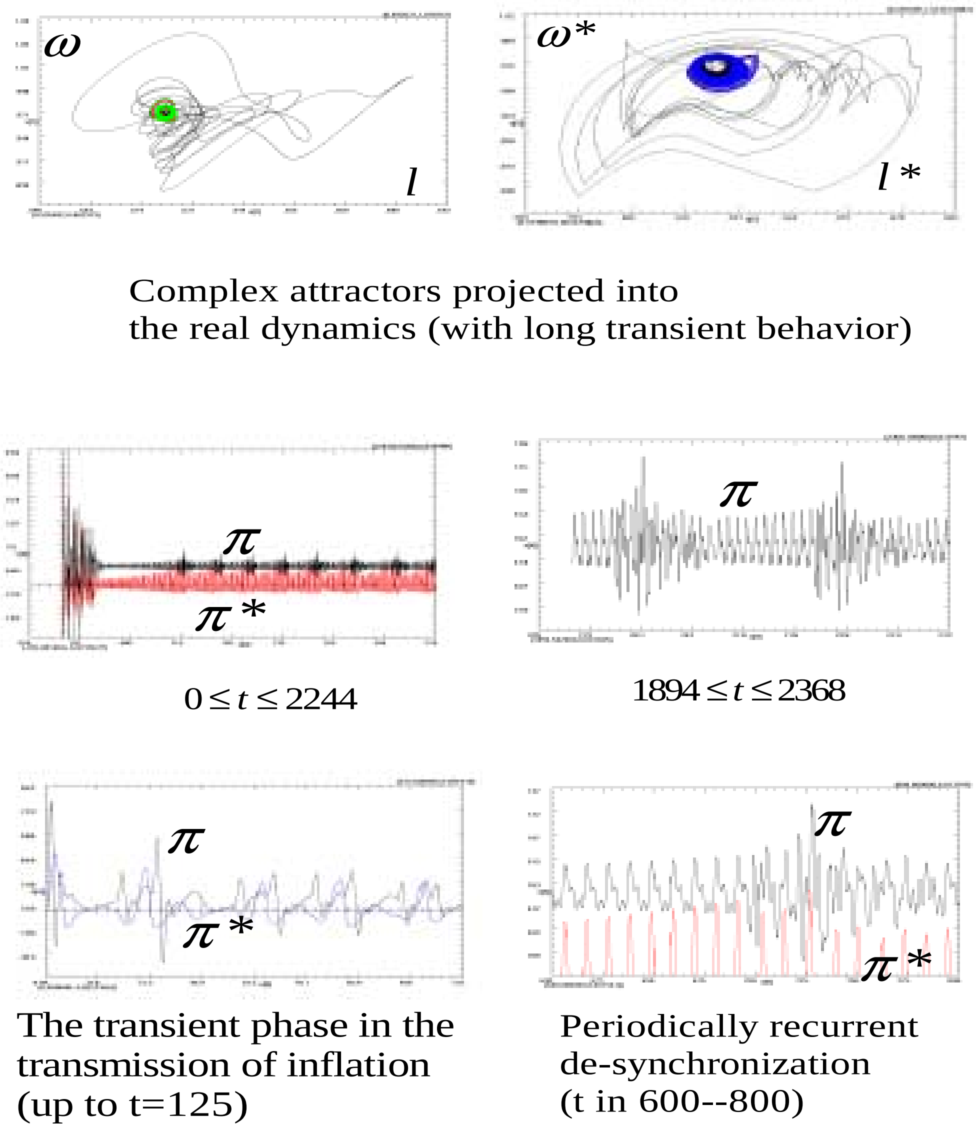


Figure 8: Complex dynamics with recurrent loss of phase synchronization under flexible exchange rates.[29]

Note with respect to figure 7 that countries are still very similar in their parameter values, both with a kink in their money wage Phillips curve which however becomes operative only

[28]Fixed and flexible exchange rate regimes are compared in Baxter and Stockman (1989), Gerlach (1988) and Greenwood and Williamson (1989). A two-country analysis for a fixed exchange rate regime that is very much in the spirit of the model used here is provided in Asada (2003). There the case of fixed exchange rates is considered on its own level and not just by setting a certain parameter in a flexible exchange rate regime equal to one.

[29]The parameters of this simulation run are as follows: $s_c = 0.8; \delta = 0.1; t_c = 0.35; g = 0.35; n = 0.05; \mu = 0.057; h_1 = 0.1; h_2 = 0.2; y^p = 1.0; x = 2.0; \beta_w = 2; \beta_p = 3; \kappa_w = 0.5; \kappa_p = 0.5; \beta_\pi = 3; \alpha_\pi = 0.5; i = 0.5; \beta_k = 1.0; \bar{V} = 0.8; s_c^* = 0.8; \delta^* = 0.1; t_c^* = 0.35; g^* = 0.35; n^* = 0.05; \mu^* = 0.05; h_1^* = 0.1; h_2^* = 0.2; y^{p*} = 1.0; x^* = 2.0; \beta_w^* = 1.95; \beta_p^* = .5; \kappa_w^* = 0.5; \kappa_p^* = 0.5; \beta_\pi^* = 3; \alpha_\pi^* = 0.5; i^* = 0.5; \beta_k^* = 1.0; \bar{V}^* = 0.8; \beta_e = 2; \beta = 1; \beta_\epsilon = 1; \alpha_\epsilon = 0.5; \gamma_c = 0.5; \gamma = 1; \gamma_c^* = .5; \gamma^* = 1; m_{shock} = 1.1$

in country 2 due to the fact that the steady state exhibits zero inflation there. In this country, we can observe therefore prolonged recessions where wage inflation is zero, but not price inflation, as the top figure in 7 shows. Country 1 exhibits a much higher price adjustment speed and only slightly higher wage adjustment speed and is thus less volatile in the fluctuations of the inflationary climate series shown, since price flexibility, but not wage flexibility, is stabilizing in the parameter range of the present case (as can be shown by eigenvalue diagrams). Yet, due to the operation of the kink in country two, fluctuations there are also much less volatile then they would be if some wage deflation would have been allowed for.

In figure 8 we provide an example of a complex attractor in our two-country setup. Projected into the $l, \omega-$phase subspaces these attractors appear (in the top figure) – after a long transient phase – more or less as fairly simple quasi-periodic motions, a periodicity that however goes hand in hand with slight increases in amplitude until there is an outbreak of more irregular fluctuations as shown in the middle of figure 8. At the bottom in figure 8 we finally show the fluctuating inflationary climates in the transient period after the expansionary monetary shock, applied in all our figures, with little phase synchronization over the first 125 hundred years and to the right we show how phase synchronization gets lost in periods where irregularities and amplitudes increase. Note here that the figure bottom right only shows the upswings in the foreign economy while the longer periods where there is some price, but no wage deflation, are not shown explicitly.

In figure 9 we consider again the case of no steady state inflation, now projecting the limit cycle then obtained into various subspaces of the 10D phase space. We note first of all that the steady state would be unstable in the absence of floors to money wages (here given by the assumption of complete inflexibility downwards). In the first four panels in figure 9 we see that real and monetary cycles are fairly different in the two countries, due to the much higher wage-price flexibility in the country 1. Real wages and labor intensity are basically negatively correlated as the next two panels then show and this also holds for the monetary sector as the panels at the bottom indicate.

Yet more important than these findings are subsequent numerical findings shown in figures 9a. Top left we again show that the kink in the money wage PC rapidly gives rise to stable limit cycle behavior, while the darker area in the middle of the figure shows the behavior of the dynamics without the kink. These dynamics is on the one hand not as volatile as the one with the kink, but on the other hand not viable over the very long horizon (roughly 1300 years in this simulations run). Really striking however is that very small variations in the growth rate of the money supply at home or abroad have dramatic consequences on the dynamic outcome of the model. In the place of the limit cycle top left (just discussed) we get the recurrent fluctuations directly below it when the growth of the domestic money supply is changed from 0.05 to 0.051 while the dynamics is very close to the steady state in between the shown irregular fluctuations (shown for a time horizon of 2300 years). Eigenvalue diagrams indeed confirm a very sensitive behavior of the maximum eigenvalue close to the growth rate of the money supply where there is zero steady state inflation.

In the opposite situation where μ^* is changed from 0.05 to 0.051 we by contrast get convergence to the steady state within the first 150 years, but finally economic breakdown (after 700 years) due to a very small positive root of the dynamics. This breakdown can be delayed a bit if also the growth rate of domestic money supply is changed to 0.051, giving rise to a second outburst as shown, but not to viability in the very long run.

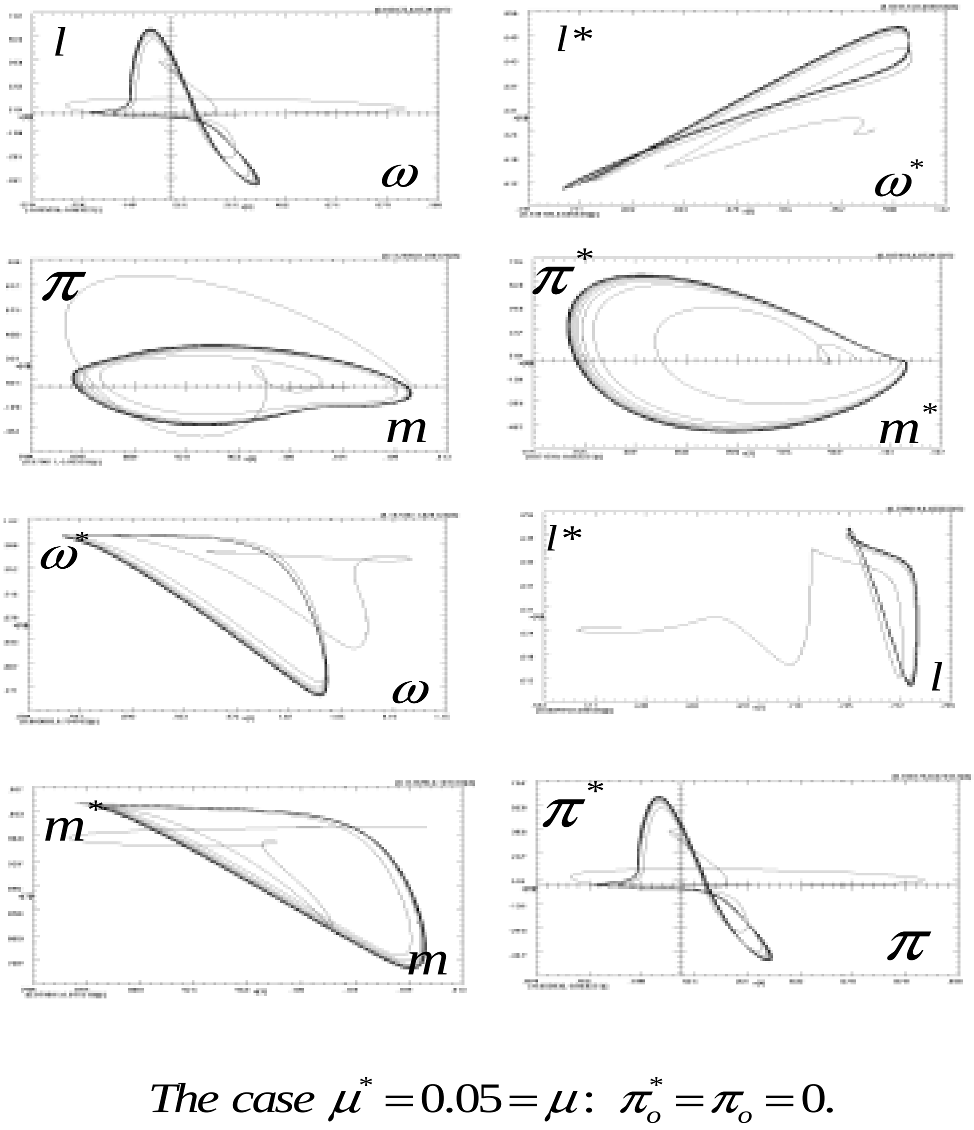


Figure 9: No steady state inflation and limit cycle projections.[30]

Figure 9a supplements figure 9 in the way just discussed and is of course based on the same parameter values as figures 9. It shows finally in its bottom panels cases of very minor steady state deflation. When there is steady state deflation in the domestic economy (with its high speeds of adjustments in the wage price module of the model) we now get convergence to the steady state, in the bottom left panel again confronted with the dark area of the dynamics when the kink is removed from them. In the case of deflationary policy in the foreign economy we however get instability both with and without the kink,

[30]The parameters of this simulation run are as follows: $s_c = 0.8; \delta = 0.1; t_c = 0.35; g = 0.35; n = 0.05; \mu = .05; h_1 = 0.1; h_2 = 0.2; y^p = 1.0; x = 2.0; \beta_w = 3; \beta_p = 3; \kappa_w = 0.5; \kappa_p = 0.5; \beta_\pi = 3; \alpha_\pi = 0.5; i = 0.5; \beta_k = 1.0; \bar{V} = 0.8; s_c^* = 0.8; \delta^* = 0.1; t_c^* = 0.35; g^* = 0.35; n^* = 0.05; \mu^* = 0.05; h_1^* = 0.1; h_2^* = 0.2; y^{p*} = 1.0; x^* = 2.0; \beta_w^* = 1; \beta_p^* = 1; \kappa_w^* = 0.5; \kappa_p^* = 0.5; \beta_\pi^* = 3; \alpha_\pi^* = 0.5; i^* = 0.5; \beta_k^* = 1.0; \bar{V}^* = .8; \beta_e = 2.0; \beta = 1.2; \beta_\epsilon = 1.0; \alpha_\epsilon = 0.5; \gamma_c = 0.5; \gamma = 1.0; \gamma_c^* = 0.5; \gamma^* = 1.0; m_{shock} = 1.1$

though the kink makes the dynamics viable over a much longer horizon than in the case of no kink in the money wage Phillips curve. We stress finally that the cycle length in the show time series is approximately ten years and that this phase length tends to become longer the more sluggish wages (and prices) become.

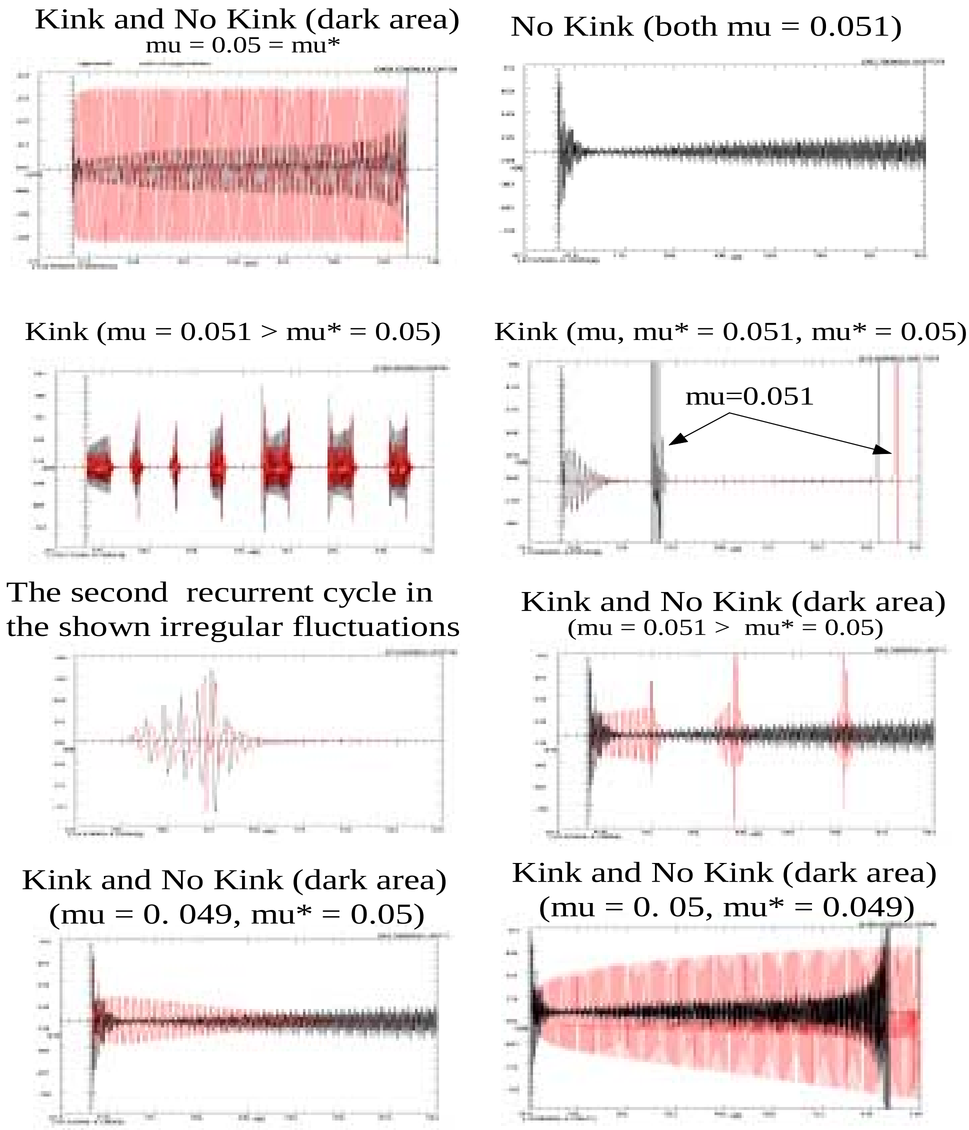


Figure 9a: Steady state inflation and the generation of irregular time series patterns (here shown for the inflation rate π).[31]

Finally in figure 10 we show a situation where countries have now been differentiated from each other in most of their parameter values, not only in the wage price module. We indicate various types of phase synchronization, basically by the establishment of negative correlations and consider again the case of separated economies, of economies that are only

[31]The parameters of this simulation run are as follows: $s_c = 0.8; \delta = 0.1; t_c = 0.35; g = 0.35; n = 0.05; \mu = .05; h_1 = 0.1; h_2 = 0.2; y^p = 1.0; x = 2.0; \beta_w = 3; \beta_p = 3; \kappa_w = 0.5; \kappa_p = 0.5; \beta_\pi = 3; \alpha_\pi = 0.5; i = 0.5; \beta_k = 1.0; \bar{V} = 0.8; s_c^* = 0.8; \delta^* = 0.1; t_c^* = 0.35; g^* = 0.35; n^* = 0.05; \mu^* = 0.05; h_1^* = 0.1; h_2^* = 0.2; y^{p*} = 1.0; x^* = 2.0; \beta_w^* = 1; \beta_p^* = 1; \kappa_w^* = 0.5; \kappa_p^* = 0.5; \beta_\pi^* = 3; \alpha_\pi^* = 0.5; i^* = 0.5; \beta_k^* = 1.0; \bar{V}^* = .8; \beta_e = 2.0; \beta = 1.2; \beta_\epsilon = 1.0; \alpha_\epsilon = 0.5; \gamma_c = 0.5; \gamma = 1.0; \gamma_c^* = 0.5; \gamma^* = 1.0; m_{shock} = 1.1$

linked via trade (the fixed exchange rate case) and economies that have the usual financial links in addition.

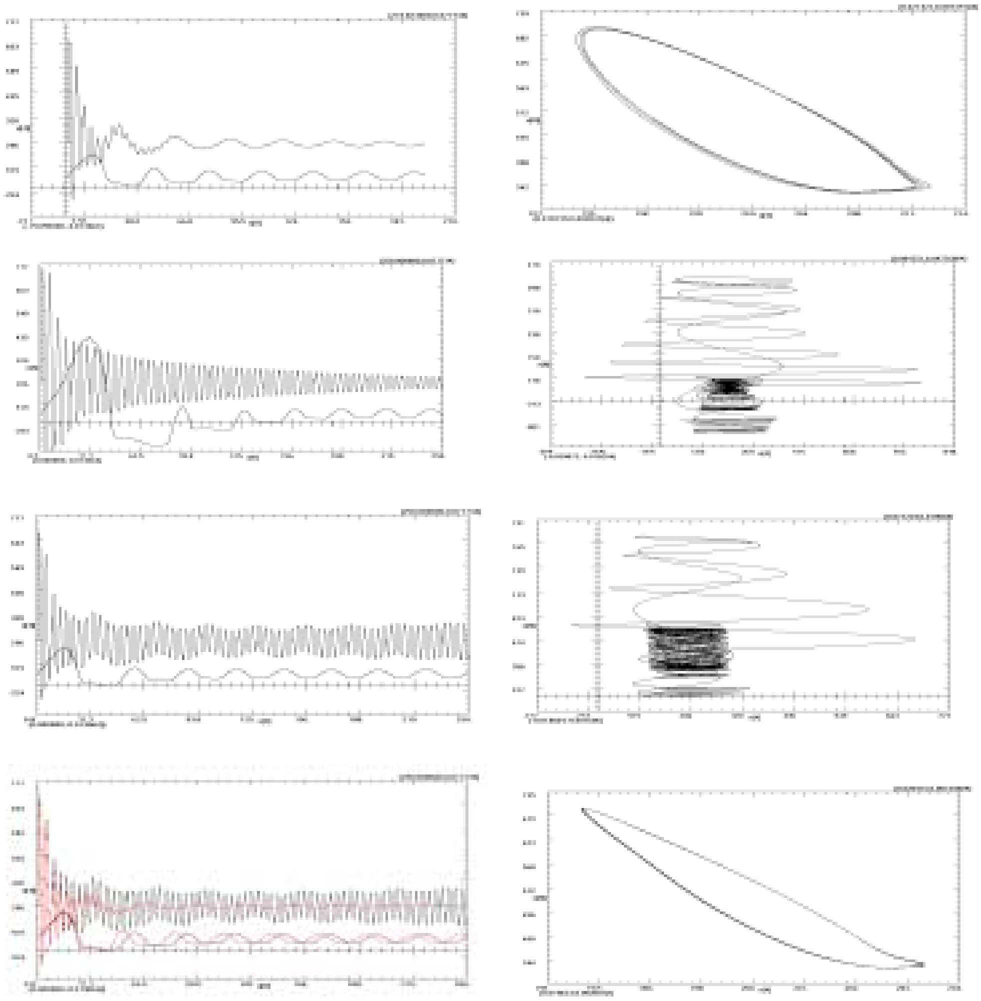


Figure 10: Interacting economies with fast vs. slow wage-price dynamics and further significant differences.[32]

This closes the numerical illustrations provided in this section for the case of two coupled KWG economies, where wage price dynamics is at the main focus of interest (besides income-distribution driven accumulation dynamics), but where the quantity dynamics of the KMG modelling framework are still absent in this formulation of full capacity growth.

[32] The parameters of this simulation run are as follows: $s_c = 0.8; \delta = 0.1; t_c = 0.35; g = 0.35; n = 0.05; \mu = 0.057; h_1 = 0.1; h_2 = 0.2; y^p = 1.0; x = 2.0; \beta_w = 2; \beta_p = 3; \kappa_w = 0.5; \kappa_p = 0.5; \beta_\pi = 3; \alpha_\pi = 0.5; i = 0.5; \beta_k = 1.0; \bar{V} = 0.8; s_c^* = 0.8; \delta^* = 0.1; t_c^* = 0.35; g^* = 0.35; n^* = 0.05; \mu^* = 0.05; h_1^* = 0.1; h_2^* = 0.2; y^{p*} = 1.0; x^* = 2.0; \beta_w^* = 1.95; \beta_p^* = .5; \kappa_w^* = 0.5; \kappa_p^* = 0.5; \beta_\pi^* = 3; \alpha_\pi^* = 0.5; i^* = 0.5; \beta_k^* = 1.0; \bar{V}^* = 0.8; \beta_e = 2; \beta = 1; \beta_\epsilon = 1; \alpha_\epsilon = 0.5; \gamma_c = 0.5; \gamma = 1; \gamma_c^* = .5; \gamma^* = 1; m_{shock} = 1.1$

6 Conclusions

In this paper we have extended the KWG approach to the dynamics of closed economies to the case of two interacting open economies. The model was introduced on the extensive form level by way of nine submodules, presenting the behavioral equations, the laws of motion and the budget equations of the sectors and markets. On the basis of simplifying assumptions we then derived the 10D core dynamics implied by the model. The uniquely determined interior steady state of the dynamics, its stability and its loss stability by way of Hopf-bifurcations was discussed in an economically intuitive, but mathematically informal way. Finally, in the case of local explosiveness of the dynamics around the steady state we have bounded them by an institutionally determined kink in the money-wage Phillips curve of the model (adding downward wage rigidity to it). This behavioral nonlinearity restricts the dynamics around the interior steady state to economically meaningful domains in many situations, a variety of which were investigated from the numerical point of view in the preceding section of the paper. These numerical simulations of the dynamics showed interesting features of more or less coupled oscillators and thus indicated that interesting dynamics may be obtained from the coupling of models of monetary growth of the KWG and other type when applied to the case of two interacting open economies.

Notation

The following list of symbols contains only domestic variables and parameters. Magnitudes referring to the foreign country are defined analogously and are indicated by an asterisk ($*$), while domestic and foreign commodities are distinguished by the indices 1 and 2, respectively. Real magnitudes are generally expressed in terms of the domestic good when composite commodities are considered. We use w and c as index to characterize magnitudes that refer to workers and pure asset holders respectively, while indices p, f and g refer to private households, firms and the government. Superscript d characterizes demand expressions, while the corresponding supply expressions do not have any index (in order to save notation). We use the superscript e to denote expected variables, while subscript e is used to denote the price of equities (the variable e is the nominal exchange rate).

A. Statically or dynamically endogenous variables:

Y	output
Y^p	potential output
Y^e	expected sales
Y^d	aggregate demand $C + I + \delta K + G$
Y_w^D, Y_c^D	disposable income of workers and asset-holders
$U_c = Y/Y^p$	rate of capacity utilization ($\bar{U}_c$ the NAIRU utilization rate)
L^d	employment
L	labor supply
$V = L^d/L$	rate of employment ($\bar{V}$ the employment–complement of the NAIRU)
$C = C_w + C_c$	private consumption
C_1	consumption of the domestic good (index 1: good

	originates from country 1 = domestic economy)
C_2	consumption of the foreign good (index 2: good originates from country 2 = foreign economy)
$S_p = S_w + S_c$	private savings
S_f	savings of firms (= Y_f, the income of firms)
S_g	government savings
$S = S_p + S_f + S_g$	total savings
I	intended (= realized) fixed business investment
N	stock of inventories
N^d	desired stock of inventories
$\mathcal{I}$	planned inventory investment (existing stock = N)
I^p	planned total investment $I + \mathcal{I}$
$I^a = I + \dot{N}$	actual total investment
$\Delta Y^e = Y^e - Y^d$	expectations error on the goods market
K	capital stock
M	money supply (index d: demand, growth rate μ_0)
B	domestic bonds, of which B_1 and B_1^* are held by domestic and foreign asset-holders respectively (index d: demand)
B^*	foreign bonds, of which B_2 and B_2^* are held by domestic and foreign asset-holders respectively (index d: demand)
E	equities (index d: demand)
W	real domestic wealth
R	stock of foreign exchange
$T = T_w + T_c$	real taxes
G	government expenditure
$Ex = X$	exports in terms of the domestic good
$Im = J^d$	imports in terms of the domestic good
$NX = Ex - Im$	net exports in terms of the domestic good
NFX	net factor export payments (in AUD)
NCX	net capital exports (in AUD)
$nx = NX/K = (a)$	net exports per unit of capital
Z	Surplus in the balance of payments (in AUD)
r	nominal rate of interest (price of bonds $p_b = 1$)
ρ^e	expected rate of profit (before taxes)
w	nominal wages
p	price level
$\omega = w/p$	the real wage
$u = \omega/x$	the wage share
p_c	consumer price index
π^e	expected rate of inflation
π_c^e	expected rate of change in the consumer price index
p_e	price of equities
e	exchange rate (units of domestic currency per unit of foreign currency: AUD/USD or €/$)
ϵ	expected rate of depreciation of the exchange rate e
ε	expected excess profitability
ε^m	expected medium-run excess profitability
$\eta = p/(ep^*)$	real exchange rate (Goods*/Goods)
T_c	taxes on domestic capital income
T_w	taxes on domestic wage income

X^w	excess capacity on the labor market
X^p	excess capacity on the goods market

B. Parameters of the model

$\bar{V}$	NAIRU-type normal utilization rate concept (of labor)
$\bar{U}_c$	NAIRU-type normal utilization rate concept (of capital)
δ	depreciation rate
$\bar{\mu}(=\mu)$	steady growth rate of the money supply
n_l	rate of natural growth
n_x	rate of productivity growth
$n = n_l + n_x$	natural growth rate augmented by productivity growth
i_1, i_2	investment parameters
h_1, h_2	money demand parameters
β_w	wage adjustment parameter
β_p	price adjustment parameter
β_{π^e}	inflationary expectations adjustment parameter
β_n	inventory adjustment parameter
β_{y^e}	demand expectations adjustment parameter
β_e	exchange rate adjustment parameter
β	disequilibrium measure in international capital flows
β_ϵ	adjustment parameter of exchange rate expectations
β_ε	adjustment parameter for the investment climate
α_{n^d}	desired inventory-output ratio
α	weights for forward and backward looking expectations
α_π	impact of technical analysis on expected inflation
α_ϵ	impact of chartists on expected exchange rate changes
κ_w, κ_p	Weights of short– and long–run inflation ($\kappa_w\kappa_p \neq 1$)
κ	$= (1-\kappa_w\kappa_p)^{-1}$
$y^p = Y/K$	output–capital ratio
$x = 1/l_y^e = Y/L^d$	output–labor ratio
τ_w, τ_c	tax rates on wage and interest income
s_c	savings–ratio (profits and interest)
s_w	savings–ratio (wages)
γ_w, γ_c	share of the domestic good in consumption of workers and capitalists
$g = G/K$	fiscal policy parameter
t_c	tax to capital ratio of workers (net of interest)
t_w	tax to capital ratio of asset holders (net of interest)
$j = J^d/Y$	import parameter
$p_b = 1$	price of domestic bonds
γ_c, γ	import function parameters
ξ	risk premium

C. Further notation

$\dot{x}$	time derivative of a variable x
$\widehat{x}$	growth rate of x
l', l_w	total and partial derivatives
$y_w = y'(l)l_w$	composite derivatives

$r_o, etc.$	steady state values ($\bar{r}$ parameter which may differ from r_o)
$y = Y/K, etc.$	real variables in intensive form
$m = M/(pK), etc.$	nominal variables in intensive form
$\nu = N/K$	inventory-capital ratio

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