Nonlinear electromagnetic wave propagation in isotropic and anisotropic antiferromagnetic media

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We study the nonlinear propagation of electromagnetic wave [EMW] in an isotropic charge-free antiferromagnetic medium by considering the two-sublattice model at low energy configurations. It is found that in the case of isotropic medium the dynamics is governed by a perturbed MKdV equation. It is also found that the magnetic field component of the EMW decelerates and the shape is distorted. Further the magnitude of the magnetic field is getting damped as time progressed. The above results are due to the interlocking of the adjacent antiparallel spins in the antiferromagnetic medium. In the case of an anisotropic medium the EM wave propagation is governed by the completely integrable derivative NLS equation which possess soliton solutions.

I. Introduction

There has been increased interest in the recent times in the study of nonlinear systems which are integrable and show regular behavior in the form of solitons. In this context ferromagnet with different magnetic interactions are inherently nonlinear in nature. It is found that the effect of nonlinearity in the ferromagnet with bilinear exchange interaction leads to localized structures called magnetic solitons [1]. The reason for the magnetic solitons in ferromagnetic medium is the integrable nature of the Landau-Lifshitz (LL) equation governing the spin dynamics in ferromagnet. The other class of interesting integrable nonlinear system is the electromagnetic wave (EMW) propagation in optical fibers. Hasegawa and Tappert [2] showed theoretically that an optical pulse in a dielectric fiber medium during propagation form an envelope soliton due to Kerr effect in the medium which was experimentally demonstrated by Mollenauer etal [3]. Another class of nonlinear system is the antiferromagnetic media. Though nonlinear spin dynamics of ferromagnetic systems have been studied extensively [1, 4–10], the study of antiferromagnetic spin systems is still in the infant stage, because, unlike in ferromagnets, in the case of antiferromagnets the adjacent spins are aligned antiparallel to each other and hence the system is treated as a two sublattice model and thus the dynamics is governed by highly nontrivial coupled nonlinear partial differential equations. Very recently some progress has been made in understanding the spin dynamics of the one dimensional classical continuum isotropic antiferromagnetic systems [11–15]. Interesting finite energy solutions including multi-solitons have been obtained in these systems only in certain limiting cases [11]. Also, when the adjacent antiparallel spins are locked together its dynamics is identified interms of twists and other classes of spin configurations [12–15].

In a very different context the interaction of EMW in ferromagnetic medium has become very important especially in relation to ferrite devices at microwave frequencies such as ferrite loaded wave guides [16]. This problem was extensively studied from the linear view point by linearizing the equation describing the spin dynamics in ferromagnets [17]. However, this study was not able to explain the rich and the interesting nonlinear dynamics observed in the ferromagnetic resonance experiments such as saturation of the main ferromagnetic resonances (FMR), absorption of energy in the FMR etc. at high power levels. This was explained by several authors by studying the exact nonlinear equation (for details refer [18]. Also, this problem of propagation of EMW in a ferromagnetic medium in the context of dispersion less propagation was studied and it was found that the magnetic field component of the EMW is modulated in the form of solitons as it propagates in a ferromagnetic medium and the magnetization excitations in the medium are governed by soliton modes [19–23]. In this context antiferromagnetic materials are equally important as they also exhibit long range order, with the exchange interaction leading to an antiparallel ordering of spins in the minimum energy configuration. Hence, the study of propagation of EMW in antiferromagnetic medium is also equally interesting and the same can be studied along the lines of ferromagnetic medium. This topic of problem is found to have potential applications in the areas of magneto optical recording, switching etc. [24]. In the present paper, we study EMW propagation in an antiferromagnetic medium, by solving the spin equation coupled with Maxwell's equations using a reductive perturbation method. In section II, we present the mathematical model for the antiferromagnetic system and construct the spin equation for the problem and the Maxwell's equations for EMW propagation. We study EMW propagation in isotropic as well as anisotropic antiferromagnetic medium using a reductive perturbation method in section III and conclude the results in section IV.

II. Antiferromagnetic model and EWW equation

The Heisenberg model of the effective Hamiltonian for an anisotropic antiferromagnet with N-spins in an inhomogeneous and time dependent external magnetic field $\mathbf{H}(\mathbf{r},\mathbf{t})$ can be written as

$$H=\sum_i\left[-J\mathbf{S}_i\cdot\mathbf{S}_{i+1}+A(S_i^x)^2-\hat{\gamma}\mathbf{S}_i\cdot\mathbf{H}\right]. \qquad (1)$$

Here $\mathbf{S}_i$ represents the spin angular momentum operator which in the classical limit can be replaced by three component vectors $\mathbf{S}_i=(S_i^x,S_i^y,S_i^z)$ and the external magnetic field $\mathbf{H}(\mathbf{r},t)=(H^x,H^y,H^z)$ in our problem is infact the magnetic field component of the propagating EMW. In Eq.(1), J represents the exchange integral which take only values less than zero, A is the anisotropy parameter and $\hat{\gamma}=g\mu_B$ where g is the gyromagnetic ratio and μ_B is the Bohr magneton. The dynamics of spins can be considered from a classical point of view by treating spin as a dynamical variable and suitable canonical equations can be obtained in analogy with the spinless nonrelativistic particle. Therefore, for the given spin Hamiltonian, we can write down the classical equations of motion for the spin vectors $\mathbf{S}_{a,i}$ and $\mathbf{S}_{b,i-1}$ in the continuum limit as

$$\frac{\partial\mathbf{S}_a(\mathbf{r},t)}{\partial t}=\mathbf{S}_a\wedge[2J(\mathbf{S}_b+\lambda\cdot\nabla\mathbf{S}_b - 2AS_a^x\mathbf{n}+\hat{\gamma}\mathbf{H}], \qquad (2)$$

$$\frac{\partial\mathbf{S}_b([\mathbf{r}-\lambda],t)}{\partial t}=\mathbf{S}_b\wedge[2J(\mathbf{S}_a-\lambda\cdot\nabla\mathbf{S}_a + 2AS_b^x\mathbf{n}+\hat{\gamma}\mathbf{H}], \qquad (3)$$

$$\mathbf{S}_a=(S_a^x,S_a^y,S_a^z),\mathbf{S}_b=(S_b^x,S_b^y,S_b^z),{\mathbf{S}_a}^2=1={\mathbf{S}_b}^2.$$

While writing the equations of motion (II) for the two sublattices, we have considered the positive and negative x directions as the easy axes of magnetization for the sublattices a and b respectively. Eqs.(II) in the classical continuum limit describe the spin dynamics in the a and b sublattices of the anisotropic antiferromagnet in the presence of an external inhomogeneous and time dependent magnetic field which in our problem is the magnetic field component of the propagating EMW. Eqs.(II) are analogous to the Landau-Lifshitz equation in ferromagnets [25]. Now adding and subtracting Eqs.(2) and (3) and defining two unit vectors $\mathbf{M}$ and $\mathbf{M}'$ which we call as the staggered and total magnetization vectors in the form [12–14] $\mathbf{M}=\frac{(1-\rho^2)^{\frac{-1}{2}}}{2S}(\mathbf{S}_a-\mathbf{S}_b)$, $\mathbf{M}'=\frac{1}{2S\rho}(\mathbf{S}_a+\mathbf{S}_b)$, where $\mathbf{M}=(M^x,M^y,M^z)$ and $\mathbf{M}'=(M'^x,M'^y,M'^z)$, $\mathbf{M}\cdot\mathbf{M}'=0$ and $\rho^2=\frac{1}{2}(1+\frac{S_a\cdot S_b}{S^2})$, we obtain

$$\begin{aligned}\frac{\partial\mathbf{M}}{\partial t}&=4JS\rho(\mathbf{M}\wedge\mathbf{M}')+\frac{2J\lambda S\rho^2}{(\sqrt{1-\rho^2})}(\mathbf{M}'\wedge\frac{\partial\mathbf{M}'}{\partial x})\\&-2JS\lambda\sqrt{1-\rho^2}(\mathbf{M}\wedge\frac{\partial\mathbf{M}}{\partial x})+\hat{\gamma}(\mathbf{M}\wedge\mathbf{H})\\&-\frac{2AS}{\sqrt{1-\rho^2}}\left[(1-\rho^2)M^x\mathbf{M}+\rho^2M'^x\mathbf{M}'\right]\wedge\mathbf{n}, \qquad (4)\\ \frac{\partial\mathbf{M}'}{\partial t}&=2JS\lambda\left\{\sqrt{1-\rho^2}\frac{\partial}{\partial x}(\mathbf{M}\wedge\mathbf{M}')\right\}+\hat{\gamma}(\mathbf{M}'\wedge\mathbf{H})\\&-2AS\sqrt{1-\rho^2}(M^x\mathbf{M}'+M'^x\mathbf{M})\wedge\mathbf{n}. \qquad (5)\end{aligned}$$

On substituting $\mathbf{M}$ and $\mathbf{M}'$ in Eqs.(II) and assuming that the low energy configurations correspond to $|\mathbf{S}_a-\mathbf{S}_b|\approx 2S$ and $|\mathbf{S}_a+\mathbf{S}_b|\approx 0$, when $\rho\ll 1$, after suitable rescaling and redefinition of parameters, the dynamics is dominated by the equation

$$\frac{\partial\mathbf{M}}{\partial t}=\mathbf{M}\wedge\{\hat{\gamma}\mathbf{H}-J\lambda\nabla\mathbf{M}-2AM^x\mathbf{n})\}, \qquad (6)$$

The above equation represents the dynamics of magnetization in a classical continuum Heisenberg anisotropic antiferromagnetic medium in the presence of an inhomogeneous time dependent external magnetic field when the adjacent antiparallel spins are locked together. The isotropic limit of Eqs.(II) in the absence of any external field has been solved and twist-like excitations have been found [13–15]

The dynamics of electromagnetic field in a material medium is described by Maxwell's equations which in the absence of stationary and moving charges can be written as [26]

$$\nabla\cdot\mathbf{E}=0,\qquad \nabla\cdot\mathbf{B}=0, \qquad (7)$$

$$\nabla\wedge\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t},\qquad \nabla\wedge\mathbf{H}=\epsilon_0\frac{\partial\mathbf{E}}{\partial t}. \qquad (8)$$

Here the fields $\mathbf{H}(\mathbf{r},t)=(H^x,H^y,H^z)$, $\mathbf{B}(\mathbf{r},t)=(B^x,B^y,B^z)$ and $\mathbf{E}(\mathbf{r},t)=(E^x,E^y,E^z)$ have the usual meaning of the magnetic field, magnetic induction and electric field respectively and ϵ_0 is the dielectric constant of the medium. The above three field in antiferromagnetic medium is connected by the relation [26] $\mathbf{B}=\mu_0[\mathbf{H}+\mathbf{M}]$, where μ_0 is the permeability of the medium. Now, taking curl on both sides of the second of Eq.(8) and using the first of (8) and the relation connecting $\mathbf{H},\mathbf{B}$ and $\mathbf{M}$ and after little algebra we obtain

$$\frac{\partial^2}{\partial t^2}[\mathbf{H}+\mathbf{M}]=c^2\left[\nabla^2\mathbf{H}-\nabla(\nabla.\mathbf{H})\right]. \qquad (9)$$

Here $c=\frac{1}{\sqrt{\mu_0\epsilon_0}}$ is the velocity of propagation of the EMW in the antiferromagnetic medium. Eq.(9) describes the propagation of EMW in antiferromagnetic medium. Thus, the set of coupled equations (6) and (9) completely describe the propagation of EMW in an anisotropic antiferromagnetic medium when the adjacent antiparallel spins are locked under low energy configurations.

III. EMW in antiferromagnetic medium

In order to find the nature of propagation of EMW in the antiferromagnetic medium, we now have to solve the set of coupled equations (6) and (9). However, the nonlinear character of Eq.(6) makes the problem of solving them difficult and therefore we restrict ourselves to one dimension. We are interested in finding soliton solutions to these equations so that one can have lossless propagation of EMW in the medium in the form of electromagnetic solitons. Infact, we are going to study the nonlinear modulation of the slowly varying EM plane wave of small but finite amplitude in the antiferromagnetic medium into soliton using a reductive perturbation method developed by Tanuiti and Yajima [27]. Very recently, the reductive perturbation method has been successfully used to study the electromagnetic soliton propagation in different ferromagnetic media by the present authors and also by few others [19–23]. Experience suggests that the magnetization and the magnetic fields have to be expanded uniformly and nonuniformly respectively in the case of isotropic and anisotropic media. Also an examination of the dispersion relation for the plane EMW propagation suggests us to introduce different coordinate stretchings in the case of isotropic and anisotropic media. This has made us to treat the propagation of EMW in isotropic and anisotropic media separately.

A. Isotropic medium

We first consider the propagation of EMW in an isotropic antiferromagnetic medium and therefore try to solve the set of coupled equations (6) and (9) in one dimension in the isotropic limit by setting $A = 0$. We expand the magnetization and the magnetic field about an undisturbed uniform state specified by $\mathbf{M_0}$ and $\mathbf{H_0}$ as

$$\mathbf{M} = \mathbf{M}_0 + \varepsilon\mathbf{M}_1 + \varepsilon^2\mathbf{M}_2..., \tag{10}$$
$$\mathbf{H} = \mathbf{H}_0 + \varepsilon\mathbf{H}_1 + \varepsilon^2\mathbf{H}_2.... \tag{11}$$

Without loss of generality we assume that the one dimensional plane waves propagate along the x-direction and therefore assume that $\mathbf{H}$ and $\mathbf{M}$ can be treated as functions of $(x - vt)$ alone where v is the speed of the wave. We also introduce slow variables to separate the system into rapidly varying and slowly varying parts by stretching the time and the newly introduced wave variable as $\tau = \varepsilon^3 t$ and $\xi = \varepsilon(x - vt)$, where ε is a small parameter. Also we make the lattice spacing to be very small by rescaling $\lambda = \varepsilon\lambda$ and $\lambda_x = \lambda_y = \lambda_z = \lambda$. We now substitute Eqs.(III A) and use the newly introduced slow variables in the one dimensional (say x) component equations of (6) and (9) and collect the coefficients of different powers of ε, and solve the resultant equations. On solving the equations at $O(\varepsilon^0)$, we obtain $H_0^x = -M_0^x = 0$, $H_0^y = kM_0^y$, $H_0^z = kM_0^z$, where $k = \frac{v^2}{c^2-v^2}$. Similarly on solving the equations at $O(\varepsilon^1)$, after using the solutions at $O(\varepsilon^0)$ we obtain $H_1^x = -M_1^x, H_1^y = kM_1^y, H_1^z = kM_1^z$, and

$$\frac{\partial M_0^y}{\partial \xi} = \sigma M_0^z M_1^x, \tag{12}$$
$$\frac{\partial M_0^z}{\partial \xi} = -\sigma M_0^y M_1^x. \tag{13}$$

Here $\sigma = \frac{(1+k)}{v}$. Finally, at $O(\varepsilon^2)$, we obtain

$$H_2^x = -M_2^x, \tag{14}$$
$$\frac{\partial}{\partial \xi}\left[H_2^y - kM_2^y\right] = -\frac{\partial M_0^y}{\partial \tau}, \tag{15}$$
$$\frac{\partial}{\partial \xi}\left[H_2^z - kM_2^z\right] = -\frac{\partial M_0^z}{\partial \tau} \tag{16}$$

and

$$-v\frac{\partial M_1^x}{\partial \xi} = J\lambda\left[M_0^z\frac{\partial M_0^y}{\partial \xi} - M_0^y\frac{\partial M_0^z}{\partial \xi}\right] + \left[M_0^y\left(H_2^z - kM_2^z\right) - M_0^z\left(H_2^y - kM_2^y\right)\right], \tag{17}$$
$$\frac{\partial M_1^y}{\partial \xi} = \sigma\left[M_0^z M_2^x + M_1^z M_1^x\right], \tag{18}$$
$$-\frac{\partial M_1^z}{\partial \xi} = \sigma\left[M_0^y M_2^x + M_1^y M_1^x\right]. \tag{19}$$

While writing equations (III A), we have used the results of the previous orders of perturbation.

To proceed further we assume that the EMW is propagating obliquely at an angle θ with reference to the uniform fields in the medium. Hence we represent the unperturbed uniform magnetization $\mathbf{M}_0$ in terms of polar coordinates in a unit sphere. As the magnetization $\mathbf{M}_0$ is restricted to (y-z)-plane in the lowest order of perturbation (i.e.$M_0^x = 0$), we can choose the azimuthal angle ϕ as $\frac{\pi}{2}$ so that $\mathbf{M}_0$ is represented by

$$\mathbf{M}_0 = (0, \sin\theta(\xi), \cos\theta(\xi)). \tag{20}$$

In view of this, Eq.(12) can be rewritten as

$$M_1^x = -\frac{1}{\sigma}\frac{\partial\theta}{\partial\xi}. \tag{21}$$

which on using in Eq.(17) gives

$$-\alpha\frac{\partial^2\theta}{\partial\xi^2} = \beta\frac{\partial\theta}{\partial\xi} + \cos\theta\frac{\partial}{\partial\tau}\int_{-\infty}^{\xi}\sin\theta d\xi' - \sin\theta\frac{\partial}{\partial\tau}\int_{-\infty}^{\xi}\cos\theta d\xi'. \tag{22}$$

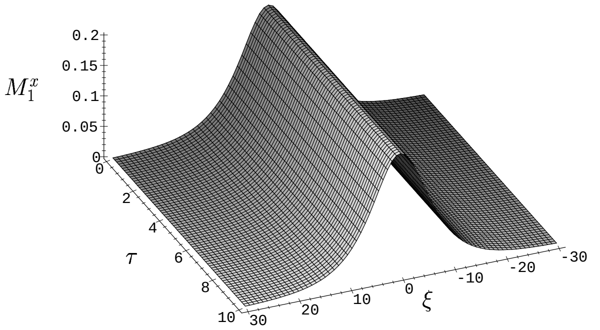

FIG. 1: Evolution of M_1^x for $\eta = 0.01$, $\alpha = 0.1$ and $\sigma = -2$.

Here $\alpha = \frac{v}{\sigma}$ and $\beta = J\lambda$. While writing Eq.(22) we have rescaled $\tau \to \frac{\sigma}{2k}\tau$. After differentiating Eq.(22) twice and using the results of the previous steps and integrating with respect to ξ and after many lengthy calculations, we obtain the following perturbed modified Korteweg-de-Vries (PMKDV) equation

$$\frac{\partial f}{\partial \tau}+\frac{3}{2}\alpha f^2\frac{\partial f}{\partial \xi}+\alpha\frac{\partial^3 f}{\partial \xi^3} = -\beta\left[\frac{\partial^2 f}{\partial \xi^2}+\frac{\partial}{\partial \xi}\left(f\int_{-\infty}^{\xi} f^2 d\xi\right)\right], \tag{23}$$

where $f = \frac{\partial \theta}{\partial \xi}$. It may be noted that when $J = 0$,(then $\beta = 0$) the right hand side of Eq.(23) vanishes thus reducing to the well known completely integrable modified Korteweg-de-Vries (MKDV) equation for which the N-soliton solutions have been found using the Inverse Scattering Transform (IST) method [28]. For instance, the one-soliton solution of the MKDV equation is written as [28].

$$f = 2\sqrt{\frac{\eta}{\alpha}}sech\left[\sqrt{\frac{\eta}{\alpha}}(\xi-\eta\tau)\right], \tag{24}$$

where η is a real constant. Knowing f, θ can be straight away calculated and further the components of magnetization $\mathbf{M}$ can be obtained from the Eqs.(20) and (21). Knowing $\mathbf{M}$ the magnetic field $\mathbf{H}$ can be evaluated and hence the magnetic induction $\mathbf{B}$ can be obtained from the linear relation connecting $\mathbf{H}$, $\mathbf{B}$ and $\mathbf{M}$. For instance, the x-component of magnetization at the $O(\varepsilon)$ is given by

$$M_1^x = \frac{-2}{\sigma}\sqrt{\frac{\eta}{\alpha}}sech\left[\sqrt{\frac{\eta}{\alpha}}(\xi-\eta\tau)\right]. \tag{25}$$

Fig.(2) shows the evolution of M_1^x for $\sigma = -2$, $\eta = 0.01$ and $\alpha = 0.1$ when $J = \beta = 0$. This implies that when the Zeeman energy dominates over the exchange energy (i.e. when $J \ll 1$ of the antiferromagnetic medium that normally happens at microwave frequency the magnetic field component of the EMW on interacting with the magnetization of the medium, generates excitation of magnetization in the form of soliton(EM spin soliton) in the medium and also the plane EMW is modulated in the form of soliton (EM soliton).

In the general case when $J \neq 0$ ($\beta \neq 0$), then we have the full PMKDV equation (23) which on solving will bring out the effect of perturbation due to spin-spin exchange interaction on the soliton during evolution. This can be done using a soliton perturbation theory [22, 29] based on the IST theory. The results show that a small structural difference will lead to slow variation of soliton parameters and distortion of the soliton shape. The one soliton solution of Eq.(23) under perturbation can be written in the form [22]

$$f = 2g_0(\tau)\left[sech(u) - W(u,\tau)\right], \tag{26}$$

where $u = 2g_0(\tau)\left[\xi - \phi_0(\tau)\right]$ and the parameters $g_0(\tau)$ and $\phi_0(\tau)$ are found from the relations

$$\frac{dg_0}{d\tau} = \frac{1}{2}\int_{-\infty}^{\infty} R\frac{du}{coshu}, \tag{27}$$

and

$$\frac{d\phi_0}{d\tau} = 4g_0^2 + \frac{1}{4g_0^2}\int_{-\infty}^{\infty} R\frac{udu}{coshu}. \tag{28}$$

Here R stands for the right hand side of Eq.(23). The correction to the soliton namely $W(u,\tau)$ is determined from a cumbersome expression which has the asymptotic forms

$$W = \frac{1}{32g_0^4}u^2\exp(-u)\int_{-\infty}^{\infty} R\frac{du}{coshu}, \quad u \to \infty, \tag{29}$$

$$W = \frac{1}{32g_0^4}2\sigma' u\int_{-\infty}^{\infty} Rdu, \quad u \to -\infty,. \tag{30}$$

where $\frac{1}{\sigma'} = 8\int g_0^2 d\tau$.

On evaluating the integrals in Eqs.(27) and (28) the velocity $v'(\tau) = \frac{d\phi_0}{d\tau}$ of the soliton is found to evolve as

$$v'(\tau) = v'(0)\left[1+\frac{4}{3}\beta\tau v'(0)\right]^{-1}, \tag{31}$$

where $v'(0)$ is the initial velocity of the soliton. From Eq.(31) we observe that the soliton is getting decelerated, and as an illustration we have shown the deceleration of M_1^x for $v'(0) = 1$ and $\beta = \frac{3}{4}$ in Fig(3). Eqs.(III A) give the variation of soliton shape by the asymptotic relations $W = (\frac{-\beta}{6g_0})u^2\exp(-u)$ as $u \to \infty$ and $W = \frac{-\beta\pi}{2g_0}\exp(\sigma' u)$ as $u \to -\infty$. Again, knowing f from Eq.(26), the value of θ can be found using the relation $f = \frac{\partial\theta}{\partial\xi}$ and then the magnetization of the medium and consequently the magnetic induction and the magnetic field component of the EMW can also be computed as done in the unperturbed case. For example the value M_1^x when ($J \neq 0$) is given by

$$M_1^x = \frac{-2}{\sigma}g_0(\tau)\left[sech(u) - W(u,\tau)\right], \tag{32}$$

Here u, g_0 and W takes the form as given below Eq.(22).

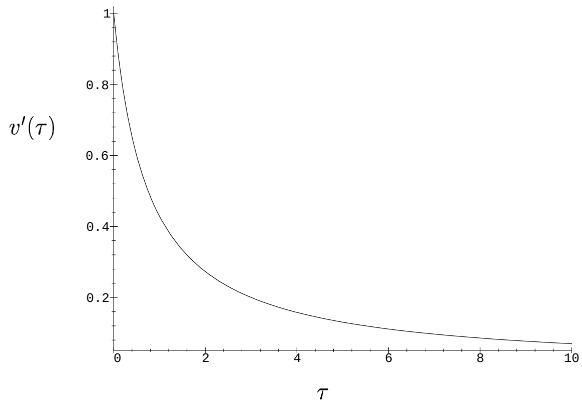

FIG. 2: Deceleration of M_1^x for $v'(0) = 1$ and $\beta = \frac{3}{4}$

B. Anisotropic Medium

Now, we study the propagation of EMW in the anisotropic antiferromagnetic medium by solving the full ($A \neq 0$) one dimensional version of the coupled equations (6) and (9). The anisotropic character of the medium suggests us to make a nonuniform expansion of the magnetization $\mathbf{M}$ and magnetic field $\mathbf{H}$. Since, the easy axis of magnetization of the anisotropic medium lies parallel to the direction of propagation (x-direction), we assume that at the lowest order of expansion the magnetization of the medium and the magnetic field lie parallel to the anisotropic axis and turn around to the $y - z$ plane at higher orders.

$$M^x = M_0 + \varepsilon M_1^x + \varepsilon^2 M_2^x + ..., \quad (33)$$

$$M^\alpha = \varepsilon^{\frac{1}{2}}\left[M_1^\alpha + \varepsilon M_2^\alpha + ...\right], \quad (34)$$

and

$$H^x = H_0 + \varepsilon H_1^x + \varepsilon^2 H_2^x + ..., \quad (35)$$

$$H^\alpha = \varepsilon^{\frac{1}{2}}\left[H_1^\alpha + \varepsilon H_2^\alpha + ...\right],. \quad (36)$$

where α= y, z and ϵ is the same perturbation parameter used in the isotropic limit. Also the magnetic field is expanded in the same way. It may be noted that along the direction of propagation of the EMW (i.e.) along x-direction the magnetization and the magnetic field have been expanded about uniform values M_0 and H_0 respectively. Before carrying out the analysis, for considering the slowly varying part of the EMW we introduce the slow variables $\xi = (x - vt)$ and $\tau = \varepsilon^2 t$, based on the nonuniform expansion which are different from the isotropic case.

We now substitute the expansions of $\mathbf{M}$ and $\mathbf{H}$ as given in Eqs.(III B) in the component form of the one dimensional version of Eqs.(6) and (9) and collect the terms proportional to different powers of ε. On solving the resultant equations at $O(\varepsilon^0)$, we obtain the relation $H_0 = -M_0, H_1^\alpha = kM_1^\alpha$, where $k \equiv (H_0/M_0) = \frac{v^2}{c^2-v^2}$. At $O(\varepsilon^1)$, after using the results at $O(\varepsilon^0)$ we finally obtain $H_1^x = -M_1^x$ and also

$$\frac{\partial}{\partial\xi}\left[H_2^\alpha - kM_2^\alpha\right] = -\left(\frac{\partial H_1^\alpha}{\partial\tau}\right), \quad (37)$$

$$\frac{\partial M_1^x}{\partial\xi} = \frac{-J\lambda}{v}\left[M_1^z\frac{\partial M_1^y}{\partial\xi} - M_1^y\frac{\partial M_1^z}{\partial\xi}\right] + \frac{1}{v}\left[M_1^y\int_{-\infty}^{\xi}\frac{\partial M_1^z}{\partial\tau}d\xi' - M_1^z\int_{-\infty}^{\xi}\frac{\partial M_1^y}{\partial\tau}d\xi'\right] \quad (38)$$

$$\frac{\partial M_1^y}{\partial\xi} = \frac{-J\lambda M_0}{v}\frac{\partial M_1^z}{\partial\xi} + \sigma M_1^x M_1^z + \frac{AM_0}{v}M_1^z - \frac{M_0}{v}\int_{-\infty}^{\xi}\frac{\partial M_1^z}{\partial\tau}d\xi', \quad (39)$$

$$\frac{\partial M_1^z}{\partial\xi} = \frac{J\lambda M_0}{v}\frac{\partial^2 M_1^y}{\partial\xi^2} - \sigma M_1^x M_1^y - \frac{AM_0}{v}M_1^y + \frac{M_0}{v}\int_{-\infty}^{\xi}\frac{\partial M_1^y}{\partial\tau}d\xi'. \quad (40)$$

Here we have rescaled $\tau \rightarrow \frac{\tau}{2k}$. In order to identify the above set of equations with more standard nonlinear partial differential equations, we define

$$\psi = (M_1^y - iM_1^z), \qquad |\psi|^2 = M_1^x. \quad (41)$$

The above definitions suggest that $M_1^x = {M_1^y}^2 + {M_1^z}^2$, which is in accordance with the nonuniform expansion we have made for the magnetization of the medium. After a single differentiation of Eqs.(39) and (40) and using the transformation $X = \xi + A\tau$, the resultant equations can be combined together and after some simple algebra we finally obtain

$$i\psi_\tau + \psi_{XX} + i\alpha'[|\psi|^2\psi]_X = 0, \quad (42)$$

where $\alpha' = \frac{-\sigma v}{M_0\sqrt{\frac{v}{M_0}+iJ\lambda}}$ and X is rescaled as $\frac{-1}{\sqrt{\frac{v}{M_0}+iJa}}X$. Here the suffices X and τ represent partial derivatives. It may be verified that on using the definitions (41), Eq.(38) can also be rewritten in the form of Eq.(42). From Eq.(42), it can be observed that even if the lattice parameter is made smaller by writing $\lambda \rightarrow \varepsilon\lambda$, the magnetization shows the same dynamics except for a change in the coefficient of the nonlinear term. Thus, Eq.(42) represents the dynamics of magnetization in an anisotropic antiferromagnetic medium at the first order of expansion when the EMW propagates through it. Eq.(42) is the well known completely integrable derivative nonlinear Schrödinger (DNLS) equation. The DNLS equation has been solved by Kaup and Newell [30] using IST and also solved by Liu etal using Hirota's bilinearisation procedure [31] and N-soliton solutions were obtained. For instance the one soliton solution can be written as

$$\psi = Psech(\zeta + A_0)tanh(\zeta + A_0). \quad (43)$$

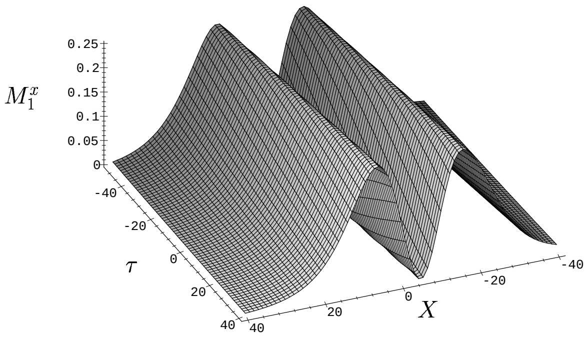

FIG. 3: Evolution of M_1^x for $\Omega_{1R} = 0.08$, $\Omega_{1I} = 0.003$, $\eta_{1R} = 0.02$ and $\alpha = 0.1$

where $\zeta = \Omega_{1R}(\Omega_{1R}X + \tau) + \eta_{1R}^{(0)}$, $\Omega_1 = \Omega_{1R} + i\Omega_{1I}$, is a complex constant where Ω_{1R} is the real part and Ω_{1I} is the imaginary part of Ω_1 respectively, and $\eta_{1R}^{(0)}$ is the real part of the complex constant η_1 associated with the soliton. $P = exp[i\Omega_{1I}(\Omega_{1I}\tau + X) + A]$ and $A_0 = \eta_{1R}^{(0)} + A$, $A = \frac{1}{2}ln(\frac{i\alpha'\Omega_1}{8\Omega_{1R}^2})$. Using Eq.(43) in Eqs.(42) we can find the components of magnetization of the medium at $O(\varepsilon^1)$. For example the magnetization component M_1^x is given as

$$M_1^x = sech^2(\zeta + A_0)tanh^2(\zeta + A_0). \qquad (44)$$

Here ζ takes the form as given below Eq.(43). Fig.(6) represents the evolution of M_1^x for the values of $\Omega_{1R} = 0.08$, $\Omega_{1I} = 0.003$, $\eta_{1R} = 0.02$ and $\alpha = 0.1$. Knowing the magnetization, the magnetic field components of the EMW can be written down straight away.

IV. Conclusions

In this paper, we studied the propagation of EMW in an isotropic charge-free antiferromagnetic medium by treating the system as a two-sublattice model. This problem is analyzed using an uniform perturbation analysis by stretching the space and time variables and perturbing the fields. It is found that the dynamics of the magnetic field component of the EMW and the magnetization excitations are governed by perturbed modified KdV equation. This shows that as the magnetic field component of the EMW interacts with the magnetization of the antiferromagnetic medium, it is modulated in the form of soliton and as it further propagates in the medium the amplitude of the soliton decreases and gets damped. Further, the velocity of the soliton also decreases. It is also found that the shape of the soliton is also distorted. The magnetization excitations due to the interaction of the magnetic field component of the EMW is found have the similar excitations. Though the spin dynamics of this isotropic antiferromagnetic system is found to have soliton excitations as its isotropic ferromagnetic counterpart, the results of the EMW propagation in this medium is not similar to that in isotropic ferromagnetic medium, where the magnetic field component of the EMW and the magnetization of the medium are modulated in the form of solitons, This change in the dynamics of the electromagnetic field and the magnetization of the medium is attributed to the interlocking of the adjacent antiparallel spins in the two-sublattice of the antiferromagnet.

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[1] M. Lakshmanan, Phys. Lett. **A61**, 53 (1977).
[2] A. Hasegawa, Y. Kodama, *Solitons in Optical Communications*, Oxford University Press, Oxford, 1995.
[3] L. F. Mollenauer and R. H. Stolen, J. P. Gordon, Phys. Lett. **45**, 1095 (1980).
[4] K. Nakamura and T. Sasada, Phys. Lett. A **48**, 321 (1974)
[5] H. J. Mikeska and M. Steiner, Adv. Phys. **40**, 191 (1990).
[6] A. M. Kosevich, B. A. Ivanov, A. S.Kovalov, Phys. Rep. **195**, 117 (1990).
[7] M. Daniel, M. D. Kruskal, M. Lakshmanan and K. Nakamura, J. Math. Phys. **33**, 771 (1992).
[8] K. Porsezian, M. Daniel and M. Lakshmanan, J. Math. Phys. **33**, 1807 (1992).
[9] M. Daniel, K. Porsezian and M. Lakshmanan, J. Math. Phys. **35**, 6498 (1994).
[10] M. Daniel and R. Amuda, Phys. Rev. B **53**, R2930 (1996).
[11] B. V. Costa, M. E. Gourea and A. S. T. Pires, Phys. Rev. B **47**, 5059 (1993).
[12] R. Balakrishnan, A. R. Bishop, R. Dandoloff, Phys. Rev. Lett. **64**, 2107 (1990); Phys. Rev. B **47**, 3108 (1992).
[13] M. Daniel and A. R. Bishop, Phys. Lett. A **169**, 162 (1992)
[14] M. Daniel and R. Amuda, Phys. Lett. A **191**, 46 (1994)
[15] R. Balakrishnan and R. Blumenfeld, Phys. Lett. A **237**, 69 (1997); Phys. Letts. A **237**, 73 (1997).
[16] R. F. Soohoo, *Theory and Applications of Ferrites*, Prentice-Hall, London, 1960.
[17] A. D. M. Walker and J. F. Mckenzie, Proc. R. Soc. A, **339**, 217 (1985).
[18] R. W. Dawon, *Magnetism: A Treatise on Modern Theory and Materials*, (Eds.) G. T. Rado and H. Suhl, Academic Press, New York, 1963.
[19] I. Nakata, J. Phys. Soc. Jpn. **60**, 2179 (1991).
[20] H. Leblond, J. Phys. A: Math. Gen. **34**, 9687 (2001).
[21] M. Daniel, V. Veerakumar and R. Amuda, Phys. Rev. E **55**, 3619 (1997).

[22] V. Veerakumar and M. Daniel, Phys. Rev. **E57**, 1197 (1998).

[23] V. Veerakumar and M. Daniel, Phys. Lett. A **278**, 331 (2001)

[24] J. L. Simonds, Phys. Today **48**, 26 (1995).

[25] L. Landau and E. Lifshitz, Phys. Z. Sowjetunion **8**, 153 (1935).

[26] J. D. Jackson, *Classical Electrodynamics* (Wiley Eastern, Newyork, 1978).

[27] T. Taniuti and N. Yajima, J. Math. Phys. **10**, 1369 (1969).

[28] M. Wadati, J. Phys. Soc. Japan, **32**, 1681 (1972).

[29] V. M. Karpman and E. M. Maslov,Zh. EKSP. Teor. Fiz. **73**, 538 (1977).

[30] D. J. Kaup and A. C. Newell, J. Math. Phys. **19**, 798 (1984).

[31] S. L. Liu and W. Wang, Phys. Rev. E, **48**, 3054 (1993).