Image Quality Indices Based on Fuzzy Discrimination Information Measures

IOANNIS K. VLACHOS, GEORGE D. SERGIADIS
Aristotle University of Thessaloniki, Faculty of Technology
Department of Electrical & Computer Engineering, Telecommunications Laboratory
GR–54124, Thessaloniki, GREECE

Abstract:–Quality measures play an important role in the field of image processing. Such measures are commonly used to assess the performance of different algorithm that are designed to perform a specific image processing task. In this paper we propose two novel indices for image quality assessment based on the notion of discrimination information between two fuzzy sets. Two different definitions for the discrimination information have been used. In order to calculate the proposed quality indices two approaches were evaluated, one with application of the indices directly to the pixels of the image and the other using the fuzzy set corresponding to the normalized histogram of the image. A comparative study of the proposed indices is performed by investigating their behavior using images with different types of distortions, such as impulsive "salt & pepper" noise, additive white Gaussian noise, multiplicative speckle noise, blurring, gamma distortion, and JPEG compression.

Key-words:–Image quality assessment, Discrimination information measures, Fuzzy cross-entropy, Fuzzy sets

1 Introduction

Image quality assessment is of great importance in digital image processing. There are two basic categories of quality or distortion measures [1]. The first category involves mathematically defined measures such as the mean squared error (MSE), the signal-to-noise ration (SNR), the peak signal-to-noise ration (PSNR) and others. The second category contains measure that take into account the properties of the human visual system.

Fuzzy sets theory [2] has been successfully applied to several image processing and computer vision problems. The extensive use of fuzzy logic in digital image processing is based on the ability of fuzzy sets to model the ambiguity and vagueness often present in digital images. In addition, fuzzy sets theory provides a solid mathematical framework for incorporating expert knowledge into digital image processing systems. In [3] measures that express the similarity between fuzzy sets were used for image comparison in terms of their normalized histograms.

In our work we propose two mathematically defined quality indices based on the notion of discrimination information between fuzzy sets. These indices turn out to be efficient for assessing the quality of images, by measuring the degree of discrimination, in terms of informational content, between the reference and the distorted image. Two different approaches for the calculation of the indices are presented and a comparison between them is carried out with different types of distorted images.

2 Discrimination Information Between Fuzzy Sets

2.1 Fuzzy Cross-Entropy

Let us consider two non-empty fuzzy sets A and B defined on the same universe X, with membership functions μ_A and μ_B respectively. Using fuzzy sets notation, A and B are defined as follows:

$$A = \left\{ \left(x, \mu_A(x)\right) \middle| x \in X \right\}, \text{ where } \mu_A : X \to [0,1] \ , \quad (1)$$

$$D(A,B)=\sum_{i=1}^{n}\{2-(1-\mu_A(x_i)+\mu_B(x_i))e^{\mu_A(x_i)-\mu_B(x_i)}-(1-\mu_B(x_i)+\mu_A(x_i))e^{\mu_B(x_i)-\mu_A(x_i)}\} \quad (2)$$

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and

$$B=\{(x,\mu_B(x))|x\in X\}, \text{ where } \mu_B: X\to[0,1] \, . \quad (3)$$

In [4] the *fuzzy cross-entropy* between two fuzzy sets was defined.

$$E(A,B)=\sum_{i=1}^{n}\{\mu_A(x_i)\ln\frac{\mu_A(x_i)}{\frac{1}{2}(\mu_A(x_i)+\mu_B(x_i))} + (1-\mu_A(x_i))\ln\frac{1-\mu_A(x_i)}{1-\frac{1}{2}(\mu_A(x_i)+\mu_B(x_i))}\} \quad (4)$$

where n is the cardinality of the universe X, that is $n=\text{Card}(X)$.

Fuzzy cross-entropy measures the *degree of discrimination* of the fuzzy set A from fuzzy set B. From (4) it is evident that fuzzy cross-entropy is not symmetric for A and B. Therefore, a symmetric discrimination information measure is defined based on fuzzy cross-entropy and is given by the following equation:

$$D(A,B)=E(A,B)+E(B,A) \, . \quad (5)$$

The discrimination information measure $D_E(A,B)$ possesses the following properties:

E1 $D_E(A,B)$ is symmetric with respect to the fuzzy sets A and B

E2 $D_E(A,B)$ attains its minimum value, that is $D_E(A,B)=0$, if and only if $A=B$

E3 $0\leqslant D_E(A,B)\leqslant 2n\ln 2$

$D_E(A,B)$ attains its maximum values when A and B are crisp sets.

2.2 Fuzzy Divergence Between Two Fuzzy Sets

In [5] Pal and Pal introduced an exponential entropy of a probability distribution based on the classical Shannon's definition of entropy. Based on this definition a fuzzy divergence measure between fuzzy sets A and B defined on the same universe X given by (2) was proposed in [6].

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D3 $0\leqslant D(A,B)\leqslant n(2-2e^{-1})$

$D(A,B)$ reaches its maximum value when A and B are crisp sets.

3 Image Quality Indices Using Discrimination Information of Fuzzy Sets

The main purpose of this paper is to exploit the definitions of the discrimination information between two fuzzy sets for image quality assessment. Smaller values of the indices indicate better quality of the image, due to small discrimination information between the reference and the distorted image. In order to calculate the information-based quality indices, two approaches were used: an image-based approach and a histogram-based approach.

3.1 Image-Based Approach

Let us consider an image A of size $M\times N$ pixels, having L gray levels g ranging from 0 to $L-1$. The image A can be regarded as an array of fuzzy singletons [7], [8]. Each element of the array denotes the membership value $\mu_A(g_{ij})$ of the gray level g_{ij}, corresponding to the (i,j)-th pixel, regarding to a predefined image property, such as brightness, edgeness, homogeneity, etc. For the definition of the proposed quality indices we consider the property "*brightness*" of the gray levels. Using fuzzy sets notation the image can be repre-

$$D_1^i = \frac{1}{2MN\ln 2}\sum_{i=0}^{M-1}\sum_{j=0}^{N-1}\{\mu_A(g_{ij})\ln\frac{\mu_A(g_{ij})}{\frac{1}{2}(\mu_A(g_{ij})+\mu_B(g_{ij}))} + (1-\mu_A(g_{ij}))\ln\frac{1-\mu_A(g_{ij})}{1-\frac{1}{2}(\mu_A(g_{ij})+\mu_B(g_{ij}))}\}$$
$$+\frac{1}{2MN\ln 2}\sum_{i=0}^{M-1}\sum_{j=0}^{N-1}\{\mu_B(g_{ij})\ln\frac{\mu_B(g_{ij})}{\frac{1}{2}(\mu_B(g_{ij})+\mu_A(g_{ij}))} + (1-\mu_B(g_{ij}))\ln\frac{1-\mu_B(g_{ij})}{1-\frac{1}{2}(\mu_B(g_{ij})+\mu_A(g_{ij}))}\} \quad (6)$$

$$D_2^i = \frac{1}{MN(2-2e^{-1})}\sum_{i=0}^{M-1}\sum_{j=0}^{N-1}(2-(1-\mu_A(g_{ij})+\mu_B(g_{ij}))e^{\mu_A(g_{ij})-\mu_B(g_{ij})} - (1-\mu_B(g_{ij})+\mu_A(g_{ij}))e^{\mu_B(g_{ij})-\mu_A(g_{ij})}) \quad (7)$$

$$D_2^h = \frac{1}{L(2-2e^{-1})}\sum_{g=0}^{L-1}\{2-(1-\tilde{h}_A(g)+\tilde{h}_B(g))e^{\tilde{h}_A(g)-\tilde{h}_B(g)} - (1-\tilde{h}_B(g)+\tilde{h}_A(g))e^{\tilde{h}_B(g)-\tilde{h}_A(g)})\} \quad (8)$$

sented as:

$$A = \{\mu_A(g_{ij})/g_{ij}|\ i=0,1,\ldots,M-1,\ j=0,1,\ldots,N-1\}\ . \quad (9)$$

The image A is fuzzified by dividing the gray level of each pixel by the largest intensity level of the image, as described by the following formula:

$$\mu_A(g_{ij}) = \frac{A(i,j)}{L-1}\ . \quad (10)$$

For two images A and B defined as in (9), the information-based quality indices D_1^i and D_2^i can be calculated using (6) and (7) in the case of the image-based approach.

3.2 Histogram-Based Approach

In the histogram-based approach, the histogram $h_A(\cdot)$ of the image is used in order to calculate the information-based quality indices. The histogram is transformed into a fuzzy set $\tilde{h}_A$ by dividing the values of the histogram by the maximum number of pixels with the same gray level [3]. Thus, the membership value $\tilde{h}_A(g)$ of the gray level g in the fuzzy set $\tilde{h}_A$, associated with the histogram of the image A, is given by:

$$\tilde{h}_A(g) = \frac{h_A(g)}{\max_g h_A(g)}\ . \quad (11)$$

Using the histogram of the image, instead of the image itself, to calculate the quality measures is faster and requires less computational resources.

For two images A and B, with corresponding histograms $h_A(\cdot)$ and $h_B(\cdot)$ respectively, the calculation of the quality measures D_1^h and D_2^h using the histogram-based approach is performed by the formulas of (12) and (8) respectively:

$$D_1^h = \frac{1}{2L\ln 2}\sum_{g=0}^{L-1}\{\mu_A(g)\ln\frac{\tilde{h}_A(g)}{\frac{1}{2}(\tilde{h}_A(g)+\tilde{h}_B(g))} + (1-\tilde{h}_A(g))\ln\frac{1-\tilde{h}_A(g)}{1-\frac{1}{2}(\tilde{h}_A(g)+\tilde{h}_B(g))}\}$$
$$+\frac{1}{2L\ln 2}\sum_{g=0}^{L-1}\{\tilde{h}_B(g)\ln\frac{\tilde{h}_B(g)}{\frac{1}{2}(\tilde{h}_B(g)+\tilde{h}_A(g))} + (1-\tilde{h}_B(g))\ln\frac{1-\tilde{h}_B(g)}{1-\frac{1}{2}(\tilde{h}_B(g)+\tilde{h}_A(g))}\} \quad (12)$$

4 Evaluation of the Information-Based Quality Indices

In order to evaluate the performance of the proposed information-based quality indices, we investigated their behavior using various test images that had undergone different types of distortions, such as impulsive "salt & pepper" noise, additive white Gaussian noise, multiplicative speckle noise, blurring, gamma distortion, and JPEG compression. Sample images that were used in the simulations are depicted in Fig. 5. Both the image-based and the histogram-based approaches were used in order to assess the performance of the proposed indices. The mean squared error (MSE) served as the common ground for their comparison. The amount of distortion, and therefore the MSE, was controlled by varying the parameters of the various types of distortions such as the density of the "salt & pepper" noise, the size of the averaging filter for blurring etc. For two images A and B of size $M \times N$ pixels, the *mean squared error* (MSE) is defined as follows:

$$MSE(A,B) = \frac{1}{MN} \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} |A(i,j) - B(i,j)|^2 \; . \tag{13}$$

The criteria we considered for evaluating the performance of the proposed quality measures were the sensitivity of the measures to degradations of the reference image and their ability to respond even to small changes of the MSE.

4.1 Behavior to Noise

The performance of the indices was evaluated in the presence of noise using both the image and the histogram-based approaches. The reference image was contaminated with different types of noise and the indices were plotted against MSE as illustrated in Fig. 1.

Fig. 1(a) depicts the performance of the quality indices when the reference image is contaminated with "salt & pepper" noise using the image-based approach. We can see that both D_1^i and D_2^i are linear-dependent on the MSE. Moreover, quality index D_2^i is more sensitive against MSE than D_1^i, but the dynamic ranges of both D_1^i and D_2^i are very compressed, ranging approximately between 0 and 0.17. On the contrary, the quality indices calculated using the histogram based approach do not depend linearly on the MSE as one can observe form Fig. 1(b). Especially in the case of the information-based quality index D_1^h its dynamic range is increased compared to the ones calculated using the histogram-based approach and the D_2^h quality index. For values of MSE approximately between 0 and 2000 the behavior of both D_1^h and D_2^h is similar. From Fig. 1(b) we can also observe that for values of MSE larger than 6000 D_2^h is practically insensitive to the increment of the MSE.

In the case of additive white Gaussian noise the behavior of the indices calculated using both the image and the histogram-based approaches is similar to the one exhibited in the presence of "salt & pepper" noise as depicted in Figs. 1(c) and 1(d).

Finally, Figs. 1(e) and 1(f) demonstrate the behavior of the quality indices in the case of multiplicative speckle noise, which is similar to the one exhibited in the cases when the reference image was contaminated with "salt & pepper" and Gaussian noise.

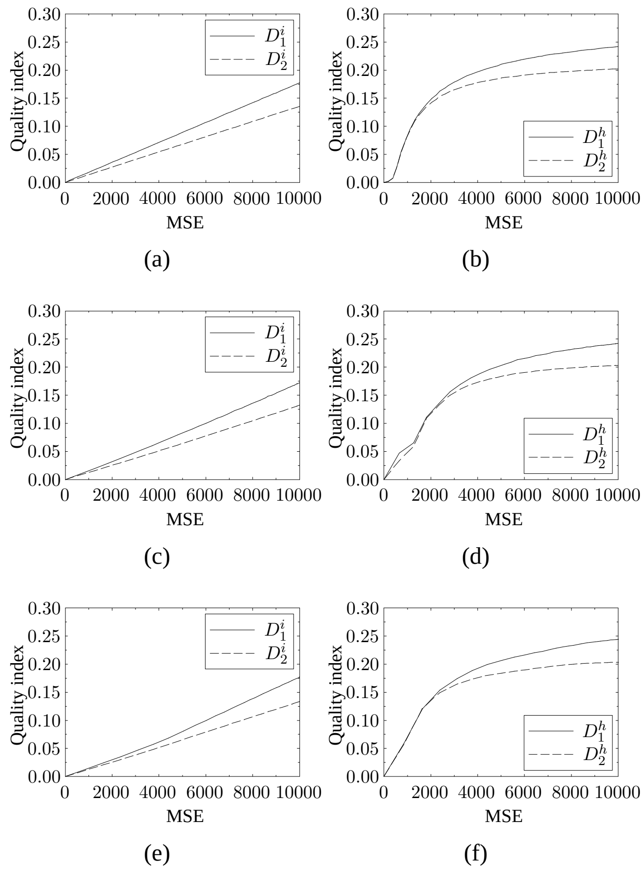
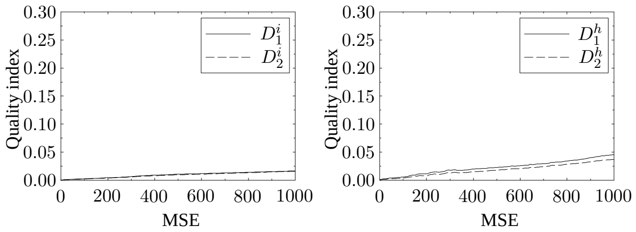


Figure 1: Behavior of the quality indices to noise. (a), (c) and (e) illustrate the quality indices using the image-based approach for images contaminated with impulsive "salt & pepper" noise, additive white Gaussian noise, and multiplicative speckle noise respectively. (b), (d) and (f) illustrate the quality indices using the histogram-based approach.

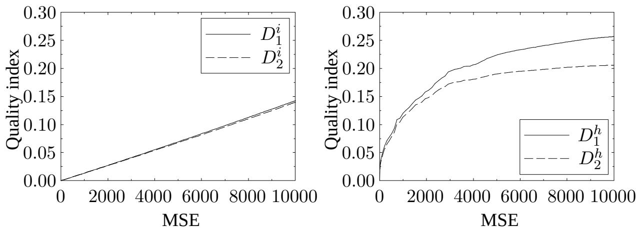
4.2 Behavior to Blurring

The proposed information-based quality indices were also evaluated under blurring. The reference image



(a) (b)

Figure 2: Quality indices behavior to blurring using (a) image-based and (b) histogram-based approaches.



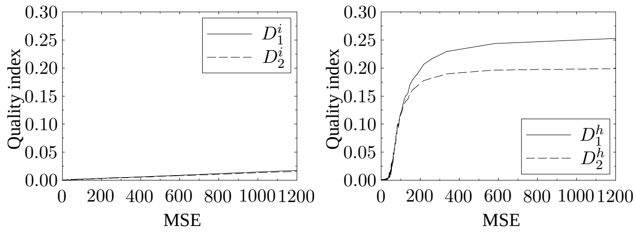
(a) (b)

Figure 3: Quality indices behavior to gamma distortion using (a) image-based and (b) histogram-based approaches.

was blurred using a circular averaging filter with variable radius R within a square window of size $2R + 1$ pixels. The indices were plotted against MSE as illustrated in Fig. 2. One can observe that both the indices calculated using image-based and histogram-based approaches show an approximately linear dependence to the MSE. Moreover, the dynamic range of the indices calculated using the histogram-based approach is slightly increased, especially in the case of D_1^h.

4.3 Behavior to Gamma Distortion

We have also adjusted the shape of the gamma curve of the reference image in order to alter its contrast. The gamma curve describes the relationship between the intensity levels of the reference and the resulting image. Figs. 3(a) and 3(b) depict the performance of the quality indices using the image-based and the histogram-based approaches respectively. From Fig. 3(a) we can see that both D_1^i and D_2^i have an approximately identical linear behavior against the MSE. Also they are characterized by a compressed dynamic range.



(a) (b)

Figure 4: Quality indices behavior to JPEG compression using (a) image-based and (b) histogram-based approaches.

On the contrary, using the histogram-based approach the dynamic range is increased, with the D_1^h index being more discriminative against small distortions of the reference image caused by the alteration of the gamma curve. Moreover, D_1^h shows similar behavior with D_2^h for values of MSE less than 1000. For values of MSE larger than 6000 D_2^h is practically constant and insensitive to the further increment of the MSE.

4.4 Behavior to JPEG Compression

The information-based quality indices were also tested against JPEG compression. Different levels of compression were used in order to obtain a set of compressed images with varying MSE. The indices were then calculated to assess the quality of the compressed images compared to the initial uncompressed reference image. From Fig. 4(a) that illustrates the quality indices calculated using the image-based approach, one can observe that both D_1^i and D_2^i are approximately linearly-dependent to the MSE and that their dynamic ranges are very compressed compared to those of the quality indices D_1^h and D_2^h calculated using the histogram-based approach and shown in Fig. 4(b). Specifically, once more D_1^h is more sensitive to small degradations of the reference image caused by the JPEG compression algorithm than D_2^h and its dynamic range is approximately between 0 and 0.25 for values of MSE ranging between 0 and 1200. Furthermore, D_2^h is practically independent of the increment of the MSE for values of MSE larger than 200. For smaller values the behavior of both the histogram-based quality indices D_1^h and D_2^h is practically identical.

Figure 5: Sample images used in the simulations. Images contaminated with (a) impulsive "salt & pepper" noise, (b) additive white Gaussian noise, (c) multiplicative speckle noise. (d) Blurred image, (e) gamma-distorted image, (f) JPEG-compressed image, and (g) reference image.

5 Conclusions

In this paper we have proposed two novel quality indices for image comparison based on the notion of discrimination information between fuzzy sets. Two approaches for calculating the indices were also presented. A detailed investigation of the behavior of the indices was carried out using different types of image distortions. Experimental results showed that the proposed quality indices calculated using the the histogram-based approach are characterized by a large dynamic range and therefore are capable of discriminate even small degradations of the reference image. The quality index that performed best was D_1^h, which was based on the definition of the fuzzy cross-entropy and was calculated using the histogram-based approach.

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