The Interval Digital Images Processing

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Abstract: - In this work we extended the classical notion of digital image, in the which each pixel has as degree of intensity an exact value for the interval digital image one, where each pixel possesses an interval intensity that can be seed as an approximate of the real intensity joint with the width of the error of this approximation. In this sense, we define the arithmetics and logical operation on interval digital images. Also we show the first evidences of how to use the interval mathematics together with the logic and fuzzy sets (theory interval fuzzy) to develop segmentation methods and classification of images, where the nature fuzzy is inherent to the problem and the precision is important.

Key-Words: Images Processing, Interval Mathematics, Interval Images Processing and Interval Images

1. Introduction

According to Gonzáles [04] an image refers to a bidimensional luminous function, denoted by *f(x,y),* where the value or width of *f* in the coordinates space *(x,y)* gives the intensity (brightness) of the image in that point. To be adapted for the computational processing, the image needs to be digitalyzed as much spacialy as in width. The digitalizacion of the space coordinates *(x,y)* it is denominated sampling of the image and the digitalizacion of the width quantification it is called of gray levels.

This stage happens approximating a continuous image for samples equally spaced in the form of a matrix, in that each element of the matrix is denominated of pixel (picture element). In other words, it happened in this phase it is turn discreet twice, a relative one to the bi-dimensional space of the image being turned at a matrix, other relative one the intensity of gray tones of the pixel. Logically, to convert a continuous image (seen in the nature) in a digital image it is naturally a stage of approaches and consequently generator of approach mistakes, for best that is the resolution (the degree of perceptible details) the digital image will never correspond the existent in the nature. Fuzzy Sets concepts [16], [17] and Intervals Mathematics [06], [07] are interlaced at this time. The quantification of the digital image as well as, the information of how good or bad it was the conversion in this processing it is a problem fuzzy. For your time, the improvement that we can obtain using the Interval Mathematics for representation of the pixel in a continuous way (an interval is a continuous space) it is right, in the sense that, we will be the sure of we be in a space whose great, that not always it is viable computationally to find, it is contained.

In this work we intended to provide some basic elements to develop a theory for digital processing of interval images, where an interval image refers to a bidimensional luminous function, denoted by *f(x,y)*, in that an interval assumes the width of *f* in the space coordinates *(x,y)* giving the intensity (brightness) of the image in that point in relation to a coefficient of tolerance that determines the difference among the superior and inferior limit of the interval. The digital images, to be appropriate for the computational processing, need to be digitalized as much as spacially than in width. The digitalization of the coordinates space *(x,y)* it is denominated sampling of the image and the quantification in gray levels it happens in a continuous way, in an interval reducing the loss in this discreetilization process. These intervals can be obtained through a software that turns the value of each pixel digitalized into an interval. The difference between the superior limit and the inferior, will depend on the relationship with that the analyzed pixel will have with the image, in a fuzzy area, for instance, this difference will be larger than in a non-fuzzy area.

2. Interval Mathematics

The quality of the result in scientific computation depends on the knowledge and control of the mistakes in the computation. Conventional algorithms, called punctual algorithms, compute an answer and, the times, an estimate of the mistake. However, the user cannot get an exact answer without the aid of a rigorous analysis of the mistakes, which is extensive, costly and not always viable, this way, the obtaining of a numeric solution for a real problem, applying the traditional numeric methods, it usually leads to approximate results. An other hand, interval techniques can be programmed in computers so that the computation possesses a rigorous one and it completes analysis of the mistake in the result.

The Interval Mathematics is a mathematical theory originated in the decade of the 60 [06] with the objective of answering subjects of accuracy and efficiency that appear in practice of the scientific computation and in the resolution of numeric problems.

Let $\mathbb{R}$ be the set of the real numbers, and be $x_1, x_2 \in \mathbb{R}$, such that $x_1 \le x_2$. Then, the set $\{x \in \mathbb{R} / x_1 \le x \le x_2\}$ it is an interval of real numbers or simply an interval, and it will be denoted by $\mathbf{X = [x_1, x_2]}$. The points of the set of the intervals of real numbers will be denoted by letters Latin capital letters. On the intervals set can be several defined operations only based on their end points. For instance,

$[x_1; x_2] + [y_1; y_2] = \{ z \in \mathbb{R} / z=x+y \text{ for some } x\in [x_1; x_2] \text{ and } y \in [x_1; x_2] \}$, then:
$[x_1; x_2] + [y_1; y_2] = [x_1 + y_1; x_2 + y_2]$

$[x_1; x_2] \times [y_1; y_2] = \{ z \in \mathbb{R} / z=x \times y \text{ for some } x\in [x_1; x_2] \text{ and } y \in [x_1; x_2] \}$, then:
$[x_1; x_2] \times [y_1; y_2] = [\min\{x_1y_1, x_1y_2, x_2y_1, x_2y_2\}; \max\{ x_1y_1, x_1y_2, x_2y_1, x_2y_2\}]$

The set of the real intervals has associate several consolidated partial orders, however none of them is total. Therefore, the maximum and minimum notions, necessaries for this work, are not the canonical ones. They are defined in the following way [11], [12] and [13]:

$\text{Max}([x_1;x_2],[y_1;y_2])=[\max(x_1,y_1);\max(x_2,y_2)]$
$\text{Min}([x_1;x_2],[y_1;y_2])=[\min(x_1,y_1);\min(x_2,y_2)]$

An important concept for this work is it of interval matrix. We say that A is an interval matrix of m by n order if A is a matrix with m lines and n columns, where each element aij of A is an interval. We can also defined operations among intervals matrixs in a similar way as are defined to real matrixs, but using the interval arithmetic instead of the real arithmetic.

3. Interval Digital Images

An interval image digital is an interval matrix $\mathcal{A}$ of $m \times n$ order that represents a spacially image discreetilized, obtained through a device of acquisition of images and digitalized by a interval digitalizer that turns each gray tone into an interval whose difference among the superior and inferior limit is an acceptable value in relation to the place in the space.

Each a_{ij}, in $\mathcal{A}$ is denominated interval pixel, because your value is an interval which define the variation Γ of the luminous intensity of this interval pixel.

3.1. Neighborhood-of-4 of an interval pixel

Let *p* be an interval pixel which *(x,y)* coordinate. The set neighborhood-of-4 of *p*, denoted $N_4(p)$, it is formed by the interval pixels of coordinates (x+1,y),(x-1,y),(x,y+1),(x,y-1), i.e.: $N_4(p)=\{(x+1,y),(x-1,y),(x,y+1),(x,y-1)\}$

3.2 Neighborhood-of-8 of a interval pixel p

Let *p* be an interval pixel which *(x,y)* coordinate. The set neighborhood-of-8 of *p*, denoted $N_8(p)$, it is formed by the interval pixels of coordinates (x+1,y), (x-1,y), (x,y+1), (x,y-1), (x+1,y+1), (x+1,y-1), (x-1,y+1) and (x-1,y-1), i.e.:: $N_8(p)=\{(x+1,y),(x-1,y),(x,y+1),(x,y-1),(x+1,y+1),(x+1,y-1)\ (x-1,y+1),(x-1,y-1)\}$

Each interval pixel is the an unit of distance of (x,y), and some of the neighbors of p will be out of the digital image if (x,y) is in the border.

We will define now, an important concept for treatment of interval images, the connectivity, this concept links with the neighborhood and gray levels and it is constantly used for establishment of the borders of objects and components of areas in an image. We will modify here, the natural definition of connectivity for us to adapt the interval situation.

3.3. Connectivity among the interval pixels p and q

Let V=[a,b] be an interval that defines the variation of gray levels that satisfies a certain similarity criterion.

a) **Connectivity-of-4**: $p,q \subseteq V$, are connected-of-4 if $q \in N_4(p)$.

b) **Connectivity-of-8**: $p,q \subseteq V$, are connected-of-8 q $\in N_8(p)$.

c) **Connectivity-of-m**:$p,q \subseteq V$, are connected-of-m if:

i) $q \in N_4(p)$, or

ii) $q \in N_8(p)$ and the set $N_4(p) \cap N_8(q)=\varnothing$

3.4. Distances in Interval Images

Let $I_p=[p_1,p_2]$ and $I_q=[q_1,q_2]$ be the intervals intensities of the pixels p and q, respectively. Moore's distance [07] among these interval intensities it is given by the function $D(I_p,I_q)=Max(|p_1-q_1|,|p_2-q_2|)$. Already the distance of Acióly-Bedregal [01] it is given by the function $Q(I_p,I_q)=0$ if $q_1 \le p_1 \le p_2 \le q_2$ and $Q(I_p,I_q)=Max(q_1-p_1,p_2-q_2)$ otherwise. Moore's distance satisfies the classic notion of metric [03] the one of Acióly-Bedregal already satisfies the notion of quasi-metric.

The Euclidiana's, "Quarteirão" and Plaid distance notions, typical of processing of digital images, are based on the location of the pixel and not in your intensity. Thus, your respective distances for the case of interval images are the same.

4. Arithmetic Operations between Interval Pixels

To proceed we will define several arithmetic and logical operations among interval pixels. Let K be the degenerate interval [k,k] where k is the largest possible intensity of an image. Let p and q be interval pixels whose intervals of intensities of gray levels are $I_p=[p_i,p_s]$ and $I_q = [q_i,q_s]$, respectively.

4.1 Sum between Interval pixels

The sum, s=p+q, is an interval pixel, that possesses intensity I_s equal to the sum of the intensities of the interval pixels p and q, that is, $I_s= [Min(p_i+q_i,k),Min(p_s+q_s,k)]$.

4.2. Subtraction between Interval pixels

The difference, m=p-q, is an interval pixel that possesses intensity I_m equal to the difference of the intensities of the interval pixels p and q, that is, $I_m= [Max(p_i-q_i,0),Max(p_s-q_s,0)]$.

4.3. Multiplication between Interval pixels

The product, t=p×q is an interval pixel that possesses intensity I_t equal to the product of the intensities of the interval pixels p and q, that is, $I_t=[Min(p_i \times q_i,k); Min(p_s \times q_s,k)]$.

4.4. Division between Interval pixels

The quotient, d=p/q is an interval pixel that possesses intensity I_d equal to the quotient of the intensities of the pixels p and q, that is $I_d=[p_i/q_s;p_s/q_i]$.

5. The Logical Operations

The logical operations under interval binary images, that is interval images in each pixel has the intesity [0;0] or [1;1], will be analogous to used in the traditional images. The results obtained they will also be similar to the traditional ones, once, to the we define the interval image, we maintained the space position of each interval pixel, just modifying the form of presentation of your gray tones (defined now as intervals), will be used for tasks such as a mask, detection of characteristics and analysis in ways and in operations guided to the neighborhood. The processing of the neighborhood is accomplished through masks (filters, windows), which modify the value of an interval pixel in function of your own gray level and the one of your neighbors. We will define now, the logical operations under interval images non binary, for so much, not more we will analyze your space position, but your intensity. We will use this technique for us to make sobreposicion of images, refinements, etc.

5.1. Disjunction of Interval Images

Let $\mathcal{A}= a_{ij}$ and $\mathcal{B}= b_{ij}$ be interval images of m × n order. The disjunction of these images is the matrix $\mathcal{C}= c_{ij}$ of m × n order, built pixel by pixel choosing that of larger intensity. i.e, $\mathcal{A} \vee \mathcal{B} = \mathcal{C}$,, where c_{ij}=**Max**[a_{ij}, b_{ij}] for all *i,j.*

5.2. Conjunction of Interval Images

Let $\mathcal{A}= a_{ij}$ and $\mathcal{B}= b_{ij}$ be interval images of m × n order. The conjunction of these images is the matrix $\mathcal{C}= c_{ij}$ of m × n order, built pixel by pixel choosing that of smaller intensity. i.e, $\mathcal{A} \wedge \mathcal{B} = \mathcal{C}$,, where c_{ij}=**Min**[a_{ij}, b_{ij}] for all *i,j.*

5.3. Negation of Interval Images

Let $\mathcal{A}= a_{ij}$ and $\mathcal{K}= k_{ij}$ be interval images of m × n order, where k_{ij} = K. the negation of $\mathcal{A}$ it is the matrix $\neg\mathcal{A}= \neg a_{ij}$ of m × n order, built pixel by pixel choosing the complement with respect to K. i.e. $\neg\mathcal{A}= \mathcal{K}\text{-}\mathcal{A}$, where $\neg a_{ij}$= kij- a_{ij} for all i,j.

6. Segmentation Of Fuzzy-Interval Regions

The objective of the segmentation in the digital processing of the image is to decompose an image in areas that are significant with respect to a private application [08]. A good segmentation facilitates the interpretation process. However, the task of sketching the limit of an object in digital processing of images is a difficult task and recent studies indicate that a great method doesn't still exist for a correct segmentation.

Many of the basic concepts of image analysis, for instance, the concept of an extremity or a song or a relationship among areas are imprecise definitions, because the areas in an image cannot always be very defined, uncertainties can appear inside of every process of "vision" of the computational system.

Prewitt [09] it was the first to suggest the use of the logic fuzzy in the treatment of images. However as the theory of the Interval Fuzzy sets, [05], [10] and [11], [12, [13]] and [15] works with degrees of interval pertinence instead of punctual to describe the degree of the specialists certainty on certain subject, we believed that is more appropriate for our interval approach of images.

The technology in the area of images is always in growth and it possesses infinite applications. The use of interval fuzzy logic will be very appropriate to work with ambiguities and uncertainties in the image model. An area interval fuzzy (taking advantage of the definition of a interval fuzzy set) dependent of the order of assimilation of each pixel for each area. The definition of the areas Interval fuzzy are useful to retain uncertainties and not to happen propagation of mistakes for the next level in a system of computational vision. At this time they can be made premature decisions that later can be corrected. The postulated advantage is that the uncertainty can be suitable and a final decision can be made after other information they be available, for instance the incorporation of anatomical knowledge for classification of structures of the brain in a fuzzy system.

7. Conclusions

In this work we extended the notion of digital image, in the which each pixel has as degree of intensity an exact value for the interval digital image one, in which each pixel possesses an interval intensity that can be thought as an approximate of the real intensity with the size of the mistake of this approximatation. For being an introductory paper, we gives the inherent basic definitions to the digital processing of images from this new optics. The obtaining of these interval approximate of the intensities, can be obtained through a software that turns the value of each pixel digitalized into an interval, the difference between the superior and inferior limit, will depend on the relationship with that the analyzed pixel will have with the image, in such a way that in a fuzzy area, for instance, this difference is larger than in a non-fuzzy area.

In this work, we also gave the first evidences of how to use the interval mathematics together with the logic and fuzzy sets (theory interval fuzzy) to develop segmentation methods and classification of images, where the nature fuzzy is inherent to the problem and the precision is important. For example, a class of problems with this nature are the medical images.

Bibliography

[01] Acióly, B,M. & Bedregal, B.R.C. (1997) A Quasi-Metric Topology Compatible with Inclusion Monotonicity on Interval Space. In Reliable Computing 3:305-313, Kluwer Academic Publisher, Netherlands.

[02] K R, (1979). Digital Image Processing, Prentice Hall.

[03] Dugundji, J. (1966) Topology. Allyn and Bacon, New York.

[04] Rafael, C. & Woods, R.C. (1992). Digital Image Processing. Addison-Wesley Publishing Company, Inc.

[05] Kreinovich, V., Mukaidono, M. and Atanassov, K.(1999) From Fuzzy Values to Intuitionistic Fuzzy Values to Intuitionistic Fuzzy Intervals etc.: Can We Get an Arbitrary Ordering? Published in Notes on Intuitionistic Fuzzy Sets, Vol. 5, No. 3, pp. 11-18.

[06] Moore, R. E. (1966). Interval Analysis. Prentice Hall, New Jersey.

[07] Moore, R.E. (1979). Methods and Applications of Interval Analysis. SIAM Studies in Applied Mathematics, Philadelphia.

[08] Ong, Hong P and Rajapakse, Jagath C. (2001). Fuzzy-Region-Segmentation. IEEE. pp 1374-1379.

[09] Prewitt, J.M. (1970). Object Enhancement and Extraction in Picture Processing and Psychopictorics, B. S. Lipkin & A. Rosenfeld eds., Acad. Press, pp 75-149.

[10] Rocha, L.M, Kreinovich, V. (1996). Computing Uncertainty in Interval Based Sets. In Applications of Interval Computer. R.B. Kearfott and V. Kreinovich (Eds.). Kluwer Academic Press. pp. 337-380.

[11] Silveira, M.M.M.T. e Bedregal, B.R.C, (2001), Toward an Interval Fuzzy System. In: Procedings the 2001 International Conference on Artificial Intelligence. USA, June 2001.

[12] Silveira, M.M.M.T. e Bedregal, B.R.C, (2001), A Method of Inference and Defuzzification Fuzzy Interval. In: The 2001 Artificial Intelligence and Application. Spanish, Setember 2001.

[13] Silveira, M.M.M.T. e Bedregal, B.R.C.(2001) Uma Teoria de Conjuntos Fuzzy Intervalar. In: JCC, Chile. Novembro, 2001.

[14] M. M. M. T. Silveira, and B. R. C. Bedregal, (2001) Um Método de Simplificação de Conjuntos Fuzzy Usando Intervalos. In Second Congress of Logic Applied to Technology: Frontiers on Logic, Artificial Intelligence and Robotic, São Paulo, Brazil, November 2001. Vol. 2, pp. 247-254.

[15] Turksen, I. B. (1986). Interval value fuzzy sets based on normal form. Fuzzy Sets and Systems 20. pp. 191-210.

[16] Zadeh, L. (1965) Fuzzy Sets. Information Control 8.

[17] Zadeh,L.A. (1964), Yager,R.R. Fuzzy Sets, Neural Networks, and Soft Computing. VNR, New York.