NUMERICAL SIMULATION OF COUPLED FIELDS IN THE SOLID DIELECTRICS

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Abstract - This paper deals with the heat generated by ohmic losses in High-Voltage (HV) solid imperfect dielectrics. An algorithm is proposed for the solution of coupled electric and heat dissipation problems in solid dielectrics of the HV electric cables

A lumped-parameter model is used for the electric field computation and a distributed-parameter model is used for the thermal field distribution. The heat source and the conductivity of the solid dielectric (insulation) accomplish the couplage between the two fields.

Keywords - Finite element method; DC cables; Coupled-problems.

1 Introduction

It is well known that in any engineering problem we use mathematical models that must be solved.

An analytical solution is possible only in particular cases. The numerical solution is a way to obtain accurate results and to study in laboratory conditions many operating regimes.

The aim of this paper is to describe a computational model to the accurate computation of the electric and thermal fields in co-axial cables. Accurate calculation of these fields is a very important aspect of transmission line design, communication and other aspects.

In our paper we consider high-voltage coaxial cable that is employed from relatively low distribution voltages up to 500kV. Such cable consists a metallic conductor and one or more insulation layers. We consider a parallel-plane model for the field analysis. The analysis domain is a quarter of the cross section of the cable. We propose a new algorithm and limit our presentation to one stage: resistive distribution of the electric field.

The problem is described by a coupled thermal-electric set of equations [2]. The equations are coupled because most of the heat sources are the effects of the electric field and the material properties are temperature-dependent. In our target examples, the coupling between the two fields is a material property as the electrical conductivity and/or the heat sources. The scope of this paper is to present some aspects of computational aspects in simulation of coupled electric and heat dissipation problems in insulation of the electric cables.

A coupled model for electrical current and its thermal effect involves development of mathematical models for the two distinct physical fields but the solving of these models must be done simultaneously. Our models are based on Maxwell equations and heat conduction [1].

Our case refers to the fields taking into account ohmic insulation losses, that is we consider the leakage current in the insulation. Normally, the power generated through the whole insulation per meter of cable is small compared to the power generated by conductor. However, the effect of the ohmic insulation losses may be not neglected at higher stresses and ambient temperatures.

The interaction of the electric and thermal fields is obviously. Thus, the leakage current heats the insulation due to the ohmic insulation losses so that the temperature of insulation will rise. The higher temperature of the insulation will increase the electrical conductivity of the insulation. The higher conductivity causes a higher leakage current and this current will heat more the insulation. This process repeats until either equilibrium is reached or an instable situation appears [4].

In other words, the electric field may be influenced very much by this phenomenon. An accurate computation of the electric field distribution can be obtained taking into account the effect of the ohmic insulation losses. Consequently, the electric field distribution can be computed in an iterative way.

More, the temperature drop of the insulation generated by the leakage current must be added to the well-known temperature drop which is the result of the conductor losses.

2 Problem formulation

In our discussion we consider a co-axial cable plotted in the figure 1. For a length more large than the cross section we can use a parallel-plane model (2D model) for the electric and thermal fields. A DC cable can operate in different regimes so that we limit our discussion at the case of application of the step voltage.

Mathematical model for the thermal field in insulation is the conduction equation [4]:

$$\frac{\partial}{\partial x}(\lambda_x \frac{\partial T}{\partial x}) + \frac{\partial}{\partial y}(\lambda_y \frac{\partial T}{\partial y}) + q = (\gamma c)\frac{\partial T}{\partial t} \quad (1)$$

with: T (x, y, t) - temperature in the point with co-ordinates (x, y) at the time t; λ_x, λ_y – thermal conductivities; γ - specific mass; c – specific heating; q – heating source.

The equation (1) is solved with a known initial condition $T(x,y,t_0)=T_0(x, y)$ and with specified boundary conditions.

It is obviously that there is a natural coupling between electrical and thermal fields. A direct couplage is obviously: the heat source q in (2) depends on the current density J in insulation. Another influence is the heat flux on the boundary conductor-insulation due to the ohmic losses in conductor.

For an imperfect insulation the electric field distribution can be obtained using a model based on Maxwell's equations considering a finite conductivity of the insulation of the cable. In a recent paper we presented this accurate approach but in some assumptions we can use a simplified model. The electric field in insulation can be computed using a lumped-parameter model. An accurate computation of the thermal field is based on the FEM using a distributed-parameter model described by the equation (1).

Mathematical model for the thermal field is the conduction equation (1) with the following boundary conditions:

- A Neumann's condition or Dirichlet's condition on the boundary conductor-insulation
- A convective condition on the boundary medium- insulation

In insulation the resistivity depends on the temperature by relation:

$$\rho(T) = \rho(T_0)[1 - \alpha(T - T_0)] \quad (2)$$

ρ (T_0) is the resistance at ambient temperature T_0 (usually +20 ^{0}C) and ρ (T) is the resistivity at the instantaneous temperature T.

Consequently, we have a non-linear problem so that only a numerical solution can be obtained in an iterative way.

A numerical model for the heat dissipation in insulation can be obtained using the finite element method (FEM).

We limit our study at the steady-state regime.

3 A computational model

There is a natural couplage between the two fields so that only an iterative procedure must be used for the numerical simulation of the temperature distribution in insulation.

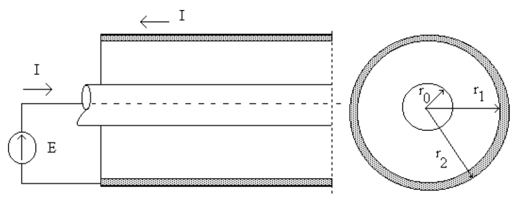

Fig. 1. The case in discussion

The algorithm in pseudo-code has the following structure [1]:

1. **Choose the initial value of the temperature**
2. ***Repeat***
   - *{Computations for electrical field}*
     - **Compute the resistivity ρ with (2)**
     - **Compute the electric field with (4)**
     - **Compute the insulation current I_{ins}**
   - *{Computations for thermal field}*
     - **Compute the heating source q by (3)**
     - **Solve the numerical model for the heat conduction using (1)**
3. ***Until*** the **convergence_test** is *TRUE*

We have an iterative process because the electric conductivity depends highly on temperature T so that the heat source q depends on the temperature. Consequently, the algorithm includes the loop *repeat-until*. The convergence test is selected so that the difference between two successive values of the thermal field must be less than a prescribed value imposed by the accuracy of the computation.

An accurate numerical model can be obtained both for the electric field distribution and thermal field using the finite element method (FEM).

4 A simplified model

Obviously the performance of any software product is estimated on the run-time and the memory of the host computer. Our goal is to reduce at the minimum both the run-time and the memory for the simulation program. An approach is to consider a lumped-parameter model for the electric field computation in the insulation.

In this assumption the heat source in insulation can be considered as being the ohmic losses in insulation, that is [4]:

$$q = \frac{I_{ins}^2}{(2\pi r)^2 \sigma} \qquad (3)$$

in which: q is the power generated per unit volume, I_{ins} is the leakage current (insulation current) per meter cable, and σ is the insulation conductivity.

The boundary conditions for the heat equation (1) are the following:

- A Neumann's condition on the boundary inner metal - insulation
- A convective condition on the boundary cable-ambient medium

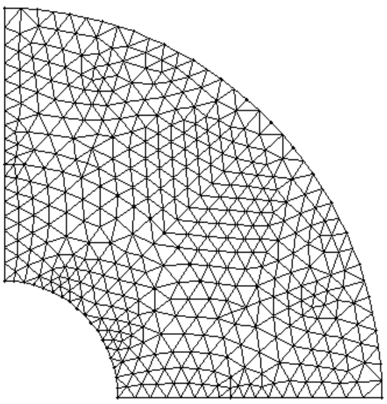

Fig.2 - Field domain (a quarter) and mesh

The Neumann's condition can be computed by the conductor losses in the case the cable was loaded before switching of the step voltage, that is the current in the cable has been raised long before and the temperature distribution in the cable is stable. In this case the value of the heat flux is computed with the relation:

$$p = \frac{P_{cond}}{2\pi r_0}$$

with P_{cond} - the ohmic losses per cable meter in the inner conductor as Joule-Lenz's effect.

Thus, Neumann's condition is:

$$\left.\frac{\partial T}{\partial n}\right|_{C1} = -p$$

with C1 – the boundary of the cable conductor and insulation.

The temperature of the conductor can be considered a constant (Dirichlet's condition) if the cable was loaded before switching of the step voltage or the cable is not loaded (the current in conductor is zero).

At the lead sheath we consider a convective condition by the form:

$$\left.\frac{\partial T}{\partial n}\right|_{C2} = h(T - T_\infty)$$

with h – the convective coefficient, T_∞ - the ambient temperature and C2 – the boundary of the cable and the external medium.

The spatial domain and the mesh are presented in the Figure 2. An accurate computation can be obtained if we include the inner conductor but the run-time of the program increases.

5 Discussions

The field distribution is a hyperbolic function if there is no temperature drop in insulation. With no temperature drop in insulation, the maximum value of the electric field strength is near the conductor (the curve E1 in Fig.3).

For large loads that lead to high temperatures in conductor, the field near the lead sheath may become higher than the highest field strength near the conductor. An analytical relation is proposed in the work [4]:

$$E(r,\Delta T) = U.\frac{k(\Delta T)}{r_1\left[1-\left(\frac{r_0}{r_1}\right)^{k(\Delta T)}\right]}.\left(\frac{r}{r_1}\right)^{k(\Delta T)-1} \qquad (4)$$

where: r_0 is the inner insulation radius, r_1 is the outer insulation radius, Δt is the temperature drop of insulation, and k (t) is defined as:

$$k(\Delta T) = \frac{\alpha.\Delta T}{\ln\frac{r_1}{r_0}}$$

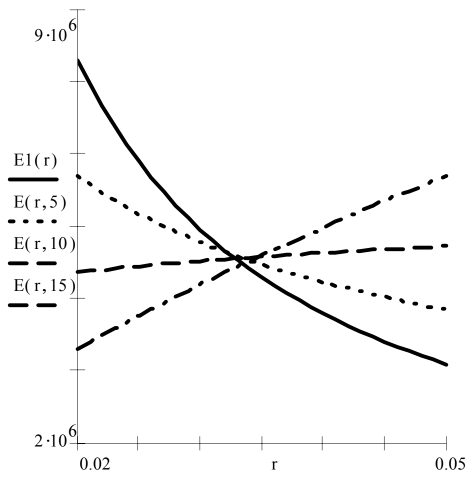

Fig. 3. Electric field for different temperature drops

In Fig.3 the electric field is plotted for different temperature drops in insulation. E1(r) represents the electric field in the cable without temperature drops in insulation, that is there is no load current. The heat source in this case is the leakage current in insulation. The temperature drop in insulation modifies the electrical field distribution, that is a load of cable (a load current) can influence the resistive field distribution.

In our examples the computations were performed on a 150 kV cable with r_0=20 [mm], r_1=50 [mm] and the temperature coefficient $\alpha=0.1^0$C.

The second stage of the numerical algorithm is the simulation of the thermal field. The heat source is the thermal effect of the current in the dielectric insulation and the load current of the cable (a heat source very important). In our model we consider that there is a constant heat flux on the interface conductor-insulation. The source of this flux is the Joule's effect of the load in the cable. For this stage we used a software product developed by the authors. The results were compared with the results obtained with Quickfield [5].

6 Conclusions

In our work we presented a computational model for a coupled problem in the solid dielectrics (a component of the high voltage DC cables). Our presentation was limited to steady-state regime although our software product includes the transient regime.

We considered an algorithm based on both explicit and implicit couplage between the two phenomena. The couplage is achieved both by the right term of the conduction equation (heat source) and electric conductivity of the material.

The non-linear mathematical model of the problem is solved in an iterative mode. At each iteration step, the physical properties of the insulation are computed using the temperature distribution computed by a distributed-parameter model.

Another version of our software product imports the database generated by Quickfield program [5].

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