AN INVERSE PROBLEM FOR THE HEAT TRANSFER IN ELECTRIC CABLES

DANIELA CARSTEA
Industrial Group CFR, Craiova
ION CARSTEA
University of Craiova, str. Decebal 5, 200072 Craiova
ROMANIA

Abstract: - The paper presents the optimal control problem of the heat transfer in the steady-regime, using boundary commands, whose positions are known. In this paper we limit our discussion to the case of bounday control of a high-voltage direct-current electrical cable. The necessary conditions for optimality are obtained by a variational approach. The command variable is the temperature of the cooling fluid of the cable but in our software we included the case with the fluid speed as a command.

In this paper the cooling-fluid temperature (environment temperature) is the command variable. The optimal command is determined by a gradient technique. A numerical model is developed using the finite element method for state and co-state equations.

We developed a CAD product in programming language C for 2D problems. Some examples illustrate the use of our product in the heat transfer problems. A comparison of our product with similar software in this area is done for analysis problems.

Key-Words: - CAD, finite element method; inverse problem; heat transfer; electrical cables.

1 Introduction

In electrical power transmission and distribution, insulated power cables are widely used. The performance on power carrying capacity is determined by the heat dissipation towards the ambient medium.

An elliptical equation (steady-state problem) or a parabolic equation (unsteady-state problem) can describe the heat transfer by conduction. In the heat transfer in electrical devices two aspects of the problem appear:

- An analysis of the temperature distribution with imposed boundary conditions (specified temperature, convective and radiation flow).
- Optimal control of the heat transfer, either distributed or boundary commands.

The first aspect is treated in most works and consists in determining of the temperature distribution in the parts of the system when the geometry of the system and the thermal load are known (that is the internal heat sources and the boundary conditions are given). The second aspect is more complex because it requires controlling the heat transfer that is to determine the values of certain variables called the commands so that the system has a desired evolution. More, we seek those commands, called optimal commands, that lead to the best evolution with respect a known criterion.

In this section we treat the elliptical systems, practically the optimal control in the steady state, in space 2D and in space 3D (only for axisymmetric field). The boundary commands are easy implemented because the boundary is an interface between environment and the controlled system, although the advanced technologies permit implementation of the distributed commands. An algorithm for this case was implemented in our software package where the distributed command is the heat source (the excitation current). Some previous works were presented with this software product.

2 Problem Formulation

The general class of the problems dealt with this paper is governed by the following differential equation:

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial u}{\partial y}\right) + f = 0 \qquad (1)$$

with specified boundary conditions. In (1) f is a known function that represents internal heat sources-the Joule-Lenz effect and eddy-current losses.

The boundary conditions are:

$$u(x,y)\,|_{C1} = u0(x,y) \qquad (2)$$

$$\left(\frac{\partial u}{\partial n} + q\right)|_{C2} = 0 \qquad (3)$$

$$\left[\frac{\partial u}{\partial n} + \alpha(u - w)\right]|_{C3} = 0 \qquad (4)$$

$$[\frac{\partial u}{\partial n} + g(u,w)]\,|_{C4} = 0 \qquad (5)$$

where: u (x, y) is the temperature in the domain Ω C R^2 and C =C_1 U C_2 U C_3 U C_4 is the boundary of the domain. In (2) u_0 is a known function (Dirichlet's condition) and in (3) we have a Neumann's condition with q -the flux on the boundary. On the boundary C_3 we have a convective condition (4) with α the convection coefficient and w the ambient temperature (a command variable). On the boundary C_4 we have a mixed-condition (as for example a convection and radiation condition), with g a known function. In (1) k_x, k_y are the thermal conductivities in the directions of the axes of the co-ordinates system Oxy. In conditions (3)-(5), ∂/∂n is the directional derivative normal to the boundary C.

The mathematical model of the heat equation in space 2D, also is met in axisymmetric field, where the equation (1) becomes [3,5]:

$$\frac{\partial}{\partial r}(k_r r\frac{\partial u}{\partial r}) + \frac{\partial}{\partial z}(k_z r\frac{\partial u}{\partial z}) + rf = 0 \qquad (6)$$

In a convective control, w can be chosen as a command variable [1]. We consider a functional cost by the form:

$$J(w) = c_0 \int_\Omega (u - u_D)dxdy \qquad (7)$$

with: c_0 - a given positive coefficient; u_D-an imposed internal temperature distribution.

The functional cost has a practical significance: it penalises the deviations of the temperature in the domain from the imposed standard (u_D). On the boundary C_3 U C_4 we apply a command w ε L^2 (C) - the space of the integrable-squared functions, with g a known function. The boundary command w can be the temperature of the cooling medium that is we have a convective control like in (3) where the coefficient α is supposed constant or depends by the boundary temperature. In another practical case, the command w is the speed of the cooling medium (like in the oil-immersed transformer), and g has the form

$$g(u, w)=\alpha(w)(u-u_\infty)$$

where u_∞ is the temperature of the cooling medium (supposed a constant). The dependence of α by w must be known but unfortunately this is a difficult task. It is determined from experimental data and is expressed using nondimensional parameters as Nusselt and Reynolds numbers.

The problem of the optimal control consists in the minimisation of the functional (7), that is we seek a command w^* ε W (an admissible set) such that:

$$J(w^*) \le J(w) \quad \forall w \in W \qquad (8)$$

in the condition (1), with specified boundary conditions (2)-(4).

Frequently, the set of admissible commands is by the form

$$W = \{w \in L_2(\Omega) : w_{\min} \le w \le w_{\max}\} \qquad (9)$$

3 Necessary conditions for ptimality

We transform the constrained optimal control problem into an unconstrained problem through the introduction of adjoint function Φ. We define the augmented cost-functional by [1,2]:

$$L = J(w) + \int_\Omega \Phi(x,y)[\frac{\partial}{\partial x}(k_x\frac{\partial u}{\partial x}) + \frac{\partial}{\partial y}(k_y\frac{\partial u}{\partial y}) \qquad (10)$$
$$+ f]dxdy$$

Necessary conditions for optimality are derived by a variational approach. It is considered a variation δw in the command w that introduces a variation δL. From the first variation of L, results the adjoint equation [5]:

$$\frac{\partial}{\partial x}(k_x\frac{\partial \Phi}{\partial x}) + \frac{\partial}{\partial y}(k_y\frac{\partial \Phi}{\partial y}) + 2c_0(u - u_D) = 0 \qquad (11)$$

with boundary conditions:

$$\frac{\partial \Phi}{\partial n}|_{C2} = 0$$
$$[\frac{\partial \Phi}{\partial n} + \alpha\Phi]\,|_{C3} = 0 \qquad (12)$$
$$[\frac{\partial \Phi}{\partial n} + \Phi\frac{\partial g}{\partial u}]\,|_{C4} = 0$$

The gradient of the cost-functional is [5]:

$$J'(w) = -\Phi\frac{\partial g}{\partial w}|_{C4} \qquad (13)$$

The gradient method can be employed to obtain the optimal command w^* (or the method of gradient projection for the constrained problem).

4 A numerical model

For obtaining the optimal command w^*, the gradient method can be used with good results, especially for the unconstrained commands. For this case the gradient method proceeds as follows [4,5]:

[1]. *make an initial guess of the command w_0, and set the iterations counter to zero;*

[2]. *solve the state equation (1) with conditions (2)-(5);*

[3]. *solve the adjoint equation (11) with the conditions (12);*

[4]. *compute the new command:*

$$w_{n+1} = w_n - s.J'(w_n) \qquad (14)$$

[5]. *Repeat the steps 2^0-4^0 until subsequent changes in J are less than a preset criterion.*

The length of the step ***s*** is determined by a one-dimensional search technique. Recent developments allow replacing the step length rule by a trust region method. In the application program developed by the authors, it was used the following rule: an initial value for s is chosen and the functional-cost is calculated and if its value isn't less than the old value, the length of the step is divided to two and this procedure continues until the monotony of the functional is satisfied. The disadvantage of this rule is that requires an iterative method to determine s at each iteration. The steps 2^0 and 3^0 of the algorithm imply the solution of the state and adjoint equations. The finite element method was used to obtain approximate solutions in finite dimensional subspace. Using Galerkin's method with linear triangular elements, we obtain the element equation [3]:

$$([K]^e + [K_h]^e)\{u\}^e = \{f\}^e + \{v\}^e - \{q\}^e - \{g\}^e$$

with: K^e - the thermal stiffness matrix by dimension 3 X3; K_h^e- the influence matrix for boundary heat generation convection, by dimension 3 X 3; f^e- the column vector of internal heat generation; v^e- the influence vector for ambient temperature near convecting boundaries; q^e- the influence vector for external heat flow; g^e- the command vector for the boundary commands.

Finally, by assembling the element equations, results an algebraic equations system. The adjoint equation (9) and cost-functional are discretized in the same manner.

5 Applications

We consider an infinitely long coaxial cable with a stranded inner conductor carrying the direct current. A step voltage is applied and a steady-state regime is considered. This problem can be treated as a two-dimensional problem. The current density is a constant and this assumption is valid in the analysis and synthesis of electrical devices where the current density J is a specified constant in conductors and zero elsewhere. This inherent approximation becomes more and more valid as we use smaller and smaller triangles. In the alternating current, the skin effect appears but in the most practical systems the conductor is stranded (that is made up several tightly wound strands of conductor insulated from each other) so as to force the currents to flow through the entire cross section of the conductor and thereby utilise the material better. Hence the validity of assuming uniform density as in direct current systems can simplify the computation. This assumption can lead at some practical applications. For such a system it has seen that the governing equation is (1). With the origin of the co-ordinates system in the center of the cable, only a part of the entire domain is used. The convective command w is applied on the shield of the cable. The functional cost is by the form (7). We considered an averaged value of the gradient so that we can obtain a sub-optimal command.

Example 1. A coaxial cable with a non-uniform current density and two insulation layers.

In the figure 1 the analysis domain is presented. The geometrical dimensions are: conductor radius - 15 mm, the outer radius of the first layer - 30 mm, and the outer radius of the second layer - 50 mm. The resistivity of the copper was considered at the temperature 75 ^{0}C and equal to $1.78.10^{-8}$ Ω/m.

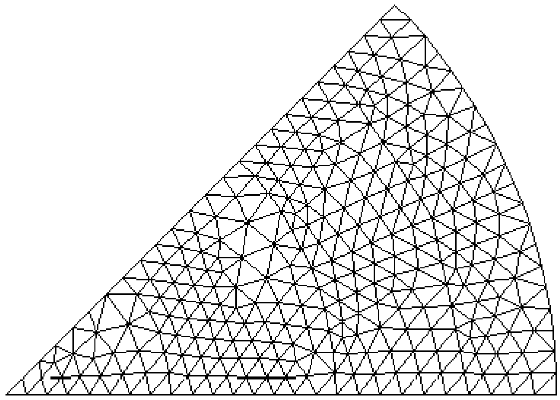

Figure 1. Analysis domain for a large-power cable

The physical properties are: thermal conductivities k_x=k_y=385 W/m.^{0}C in the copper and equal to k1=0.14 W/m.^{0}C and k2=0.175 W/m.^{0}C in the insulation layers; α=12 W/m^2.^{0}C and the current density is $5.0.10^{-5}$ A/mm^2. The minimum value of J(w) was found to be equal to 5.604 for c_0=0.0001. The iterations number is 181 with the initial value of the command equal to 40^{0}C. The optimal command is 63.95 ^{0}C for u_D=75 ^{0}C. In numerical simulation it is considered a medium value of the gradient on the boundary, that is in the formula (11) the command w, at each iteration step, is a constant (a frequent case in industry where we consider an average value of the command variable). Any case may be treated in the same manner (for example, a piecewise command or a local command).

Example 2. A coaxial cable with a constant temperature on conductor surface and two solid insulators.

In the figure 2 the analysis domain is presented. The physical properties are: thermal conductivities for insulators k1=0.14 W/m.^{0}C and k2=0.175 W/m.^{0}C; the temperature on the conductor surface is 100 ^{0}C; the convective coefficient is α=12 W/m^2 ^{0}C. The minimum value of J (w) was found to be equal to 0.1637 for c_0=$1.0.10^{-6}$. The iterations number is 93 with the initial command equal to 40 ^{0}C. The optimal command is 54 ^{0}C for u_D=75 ^{0}C.

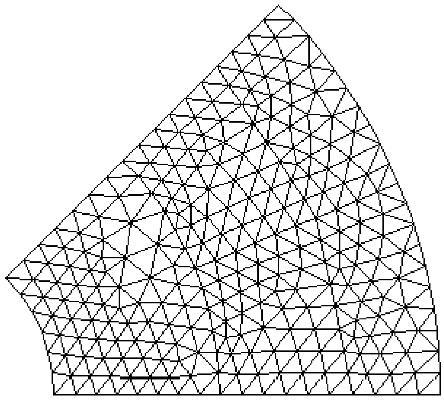

Fig. 2. Analysis domain with convective command

Example 3. A coaxial cable with a uniform temperature flux from conductor.

Another case involves a constant flux at conductor surface. The analysis domain is presented in the figure 2 but at the boundary conductor-insulation we have a heat flux with a constant value. The value of the flux is computed using the heat quantity developed by Joule-Lenz effect in conductor and the area of the interface conductor-insulation.

The boundary conditions for the heat equation (6) are the following:

- A Neumann's condition on the boundary inner metal - insulation
- A convective condition (command) on the boundary cable- ambient medium

The Neumann condition can be computed by the conductor losses in the case the cable was loaded before switching of the step voltage, that is the current in the cable has been raised long before and the temperature distribution in the cable is stable. In this case the value of the heat flux is computed with the relation:

$$q = \frac{P_{cond}}{2\pi r_0}$$

with P_{cond} - the ohmic losses per cable meter in the inner conductor, and r_0 – the external radius of the conductor.

The temperature of the conductor can be considered a constant (Dirichlet's condition) if the cable was loaded before switching of the step voltage or the cable is not loaded (the current in conductor is zero).

At the lead sheath we consider a convective condition by the form:

$$\frac{\partial u}{\partial n} = \alpha(u - w)$$

with w - the ambient temperature (command variable.

6 Conclusions

Because the similarities between the axisymmetric problem and the two-dimensional plane problem, only slight program modifications are necessary to handle one problem or the other. In industrial applications the convection coefficient α is not a constant but depends by a number of factors (the temperature, characteristics of the ambient medium etc.). This influence can be included in the program and we have applied this idea in our software. In other applications, the boundary command is the speed of the external fluid for cooling medium. The complexity of the problem increases and in many cases it is a non-linear problem.

A version of our software uses the database of the problem exported from the software product Quickfield [6].

References:

[1]. Cavin, R.K., Tandon, S.C. Howze, I.W., Necessary conditions for a class of smooth second-order distributed systems. In*: IEEE Transactions AC-21*, June 1976.

[2]. Cavin, R.K., Tandon, S.C. Distributed Parameter System Optimum Control Design via Finite Element Discretization. In: *Automatica*, vol.13, Pergamon Press, 1977.

[3]. Segerlind., L., J., *Applied Element Analysis*, John Willey and Sons,1984, USA.

[4]. Cârstea, I.T., Cârstea, D.P., The application programs for analysis and synthesis of thermal processes. In: *Modelling, Measurement, Control,* B, vol. 50, No.1, 1993, AMSE Press, France.

[5]. Cârstea, D., Cârstea, I., *CAD in electrical engineering. The finite element method.* Publishing House SITECH. 1999, Craiova, Romania.

[6]. Quickfield package. Tera Analysis Co.