Endogenous Fluctuations in a Cournotian Monopolistic Competition Model with Free Entry and Market Power Variability*

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Abstract

This article aims to show that one can link imperfections of competition to the occurrence of endogenous fluctuations. We consider a two sector model in which a perfectly competitive final good sector uses inputs that are produced in a Cournot monopolistic competition market. We show that when inputs are not perfect substitutes, and the depreciation rate of capital is sufficiently small, Neimark bifurcations are susceptible to emerge. This is a consequence of additional variability in the dynamical system generated by the dependence of the markup on the number of firms. This number changes over time because firms can enter and exit the market without costs. Moreover, a fixed cost in the technology ensures that the number of active firms at a given date is finite provided that the elasticity of substitution between inputs is bounded from above.

JEL classification: E32, D43

Keywords: Cournot competition, Endogenous Fluctuations

1 Introduction

The goal of this paper is to emphasize on the role of nonlinearities induced by imperfect competition in the goods market on equilibrium dynamics. Cournot competition has been analyzed by d'Aspremont et al. [d'Aspremont et al., 1995] who detected two channels through which imperfect competition can contribute to explaining endogenous fluctuations:

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increasing returns and the variability of market power. This second characteristic of imperfect competition is the one we focus on in this paper. Indeed, we wish to show that market power variability is a factor that can favor the emergence of equilibrium trajectories not reduced to the steady state. Several sources of variability of the markups can be put forward. In Gali [Gali, 1994] it results from modifications in the composition of the aggregate demand. Rotemberg and Woodford [Rotemberg and Woodford, 1991] propose several scenarios (standard monopolistic competition, customer market and implicit collusion) that lead them to link the markups variations with variations in the level of sales and in the present value of expected profits.

The key ingredient in this paper is the variation of the number of firms that results from free entry and exit together with fixed costs in the production function. We study a model la Woodford characterized by Cournot monopolistic competition on the inputs market. More precisely, we suppose that inputs are produced by a continuum of sectors, each sector being composed of a finite number of firms. This number is supposed to vary and is determined by the zero profit condition under the free entry hypothesis. The technology is characterized by a fixed cost, implying that the number of active firm is always finite provided that firms have a positive market power. Unlike Cazzavillan et al.'s model [Cazzavillan et al., 1997], which is characterized by a constant markup and focuses on the effects of increasing returns, market power is here variable because of the free entry hypothesis. We show that this variability induces larger possibilities of dynamic behavior compared to the perfect competition case. Indeed, if the depreciation rate of capital is weak enough (but remains strictly positive) Neimark bifurcations emerge when the imperfection of competition (given in this model by the ratio between fixed costs and the elasticity of substitution) varies between zero and an upper bound beyond which the system does not admit any steady state. On the contrary, the perfect competition case is characterized by local determinacy of the steady state, regardless of the parameter values[3].

The structural assumptions of the model are presented in the following section. Then, dynamics in the neighborhood of steady states are studied in section 3.

2 The Model

We consider a two sector economy. The first sector is perfectly competitive and produces a final good that will be used for consumption and investment. The intermediate sector contains a continuum of factor markets, the factors being imperfect substitutes. The elasticity of substitution between two goods is the same regardless of the goods considered.

In the first two subsections we derive the behavior of the two categories of consumers, workers and capitalists. The last subsection is devoted to the analysis of producers' behavior.

2.1 Workers

The present model is based upon Grandmont et al [Grandmont et al., 1998] which is a variant of Woodford's model [Woodford, 1986] allowing for factor substitutability. Three kinds of agents are considered: workers, firms and capitalists.

There is a continuum of workers represented by the [0,1] segment. They are strictly identical, so we can restrict the study to a representative worker. In Woodford's model workers and capitalists are both infinitely lived agents. Both hold capital but only workers supply labor. Hence, they have an additional liquidity constraint in the sense that they can't spend their wage during the period where it is earned. Woodford shows that capital stock is entirely held by capitalists at the equilibrium and that the level of consumption and labor chosen by workers is the same as in an OLG framework where old agents don't work. Here, we impose the OLG structure as a starting point in order to simplify the presentation. But it should be kept in mind that the initial structure is one of infinite lived agents[4], although we shall mention young and old agents to simplify the exposure.

Money is the only asset that the representative worker uses to transfer his earnings between periods. We assume that the quantity of money is held constant and equal to $M > 0$. The agent's goal will be to determine his optimal labor supply L_t (and the associated consumption level C_{t+1}) by trading off between the utility he derives from his consumption and the one procured by his leisure time, or, equivalently the disutility of working. Expected prices P_{t+1} and nominal wages w_t are taken as given. The consumer's program is the following:

$$\begin{array}{c} \max \ \dfrac{C_{t+1}^a}{a} - \dfrac{L_t^b}{b} \\ \text{s.t.} \left\{ \begin{array}{l} M \leq w_t L_t \\ P_{t+1} C_{t+1} \leq M \end{array} \right. \end{array} \tag{1}$$

with $0 < a < 1$ and $b > 1$.

The first order condition is given by the equalization of the marginal rate of substitution to the price ratio:

$$\frac{L_t^{b-1}}{C_{t+1}^{a-1}} = \frac{w_t}{P_{t+1}}$$

Together with the intertemporal budget constraint, this gives:

$$C_{t+1} = (L_t)^{\theta} \tag{2}$$

with $\theta = b/a > 1$.

2.2 Capitalists

We consider the limit case of Woodford's model [Woodford, 1986] where capitalists hold the whole capital stock and do not consume[5].

Denoting δ the rate of capital depreciation and ρ_t the real interest rate at date t, we have the following capital accumulation equation:

$$K_t = (1 - \delta + \rho_t) K_{t-1} \tag{3}$$

where it should be reminded that capital good is the same composite good as the consumption good[6].

Investment decisions, in this very simplified context, provide the second source of dynamics in this model. We see easily from (3) that a non autarkic steady state exists only if the depreciation rate is strictly positive. The study of entrepreneurs decisions and the definitions of equilibria will allow us to rewrite equations (2) and (3) in terms of the sole variables K and L.

2.3 Firms

The composite good used for consumption and investment is produced on a perfectly competitive sector using a continuum of intermediate goods. Each of these intermediate goods is produced according to a technology using capital and labor on an imperfectly competitive market. More precisely, this market is characterized by a small number (greater or equal to 1) of productive firms who behave strategically by conjecturing the residual demand from the final sector enterprises. Hence, we speak about Cournotian monopolistic competition[7] to describe the two main features of this model: the great number (continuum) of intermediate goods (which relates to the Dixit Stiglitz model of monopolistic competition) and the Cournotian aspect of strategic behaviors on each market of intermediate good.

We first derive the production factors demands by studying final sector firm's behavior, then we solve the intermediate sector profit maximization.

2.3.1 Final Good Sector

The final sector firms use all inputs[8] $x_{jt}, j \in [0,1]$ produced by the intermediate sector in order to produce an amount y_t of composite good that will be sold to the capitalists and the consumers at price p_t^y. This good is produced according to a C.E.S. technology on a perfectly competitive market, that is:

$$y_t = F(x_{jt})_{j\in[0,1]} = \left(\int_0^1 x_{jt}^{\frac{\sigma-1}{\sigma}} dj\right)^{\frac{\sigma}{\sigma-1}} \tag{4}$$

A representative final sector firm's program is given by:

$$\max_{(y_t,x_t)\in\mathbb{R}_+\times\mathbb{R}_+^{[0,1]}} \ p_t^y y_t - \int_0^1 p_{jt}x_{jt}dj$$

In a first step, the entrepreneur seeks to minimize his total expenditure for a given output level. We have therefore:

$$\begin{aligned} E(y_t) \equiv \quad & \min_{x_{jt}\in\mathbb{R}_+^{[0,1]}} \left(\int_0^1 p_{jt}x_{jt}dj\right) \\ & \text{s.t.} \ \left(\int_0^1 x_{jt}^{\frac{\sigma-1}{\sigma}} dj\right)^{\frac{\sigma}{\sigma-1}} \geq y_t \end{aligned}$$

From this, we deduce the input demand as a function of the desired output level:

$$x_{jt}^* = \left[\frac{p_{jt}}{P_t}\right]^{-\sigma} y_t \tag{5}$$

with:

$$P_t = \left(\int_0^1 \left(p_{jt}^*\right)^{1-\sigma} dj\right)^{\frac{1}{1-\sigma}}$$

the associated expenditure level is then:

$$E(y_t) \equiv \int_0^1 p_{jt}x_{jt}^* dj = P_t y_t$$

Therefore, the optimal level of production, for each firm of the final sector, is given by:

$$y_t^* \in \arg\max_{y_t} \ p_t^y y_t - P_t y_t$$

This program only has a positive solution if $p_t^y = P_t$. In that case, the global level of production

$$Y_t \equiv \int_0^1 y_{it} di$$

is determined by the final good market equilibrium.

2.3.2 Intermediate sector

Inputs are produced on a market consisting of a continuum of sectors indexed by $j \in [0,1]$, each sector containing a finite number n_j of enterprises who behave as Cournot competitors. This situation has already been referred to as monopolistic Cournot competition (see d'Aspremont and al [d'Aspremont et al., 1995]). Thus, firms take into account, in their program, the effect of the price of the good they produce on the residual demand they're facing, other firms being supposed to produce the same amount $\overline{x}_{jt}$. This variation is included in their objective function by replacing in the total revenue, prices by the following inverse demand function[9]:

$$p_{jt}\left(x_{jt}\right) = P_t \left(\frac{x_{jt}}{y_t}\right)^{-\frac{1}{\sigma}} = P_t \left(\frac{x_{jt}^h + \left(n_j - 1\right)\overline{x}_{jt}}{y_t}\right)^{-\frac{1}{\sigma}} \tag{6}$$

Production of a given enterprise h in sector j is carried out using amounts of labor L_{jt}^h and of capital $K_{j(t-1)}^h$ [10] according to a Cobb Douglas technology with a fixed cost ϕ expressed in terms of input units:

$$x_{jt}^h = G\left(K_{j(t-1)}^h, L_{jt}^h\right) = \left(K_{j(t-1)}^h\right)^{\alpha}\left(L_{jt}^h\right)^{1-\alpha} - \phi$$

Each firm seeks to maximize its own profits taking as given other firms prices, i.e. the general level price since each sector has a zero statistical weight. Moreover, each entrepreneur suppose that all his production will be sold, so we have the following program:

$$\underset{\left(K_{j(t-1)}^h, L_{jt}^h\right)\in\mathbb{R}_+\times\mathbb{R}_+}{Max} p_{jt}\left(x_{jt}^h + \left(n_j - 1\right)\overline{x}_{jt}\right)\ G\left(K_{j(t-1)}^h, L_{jt}^h\right) - w_t L_{jt}^h - r_t K_{j(t-1)}^h$$

As we did for the final sector, we split resolution of this program in two steps. In the first step, firms minimize their total expenditure under a given production level constraint:

$$\begin{aligned} CT\left(\overline{x}_t\right) &= \underset{\overline{x}_t}{Min}\ w_t L_{jt}^h + r_t K_{j(t-1)}^h \\ s.c.\ G\left(K_{j(t-1)}^h, L_{jt}^h\right) &= \left(K_{j(t-1)}^h\right)^{\alpha}\left(L_{jt}^h\right)^{1-\alpha} - \phi \geq \overline{x}_t \end{aligned}$$

The first order conditions, in connection with the production constraint, allow us to compute the labor and capital demands:

$$\begin{cases} \left(K^h_{j(t-1)}\right)^* = \left(\frac{1-\alpha}{\alpha}\frac{r_t}{w_t}\right)^{\alpha-1}(\overline{x}_t+\phi) \\ \left(L^h_{jt}\right)^* = \left(\frac{1-\alpha}{\alpha}\frac{r_t}{w_t}\right)^{\alpha}(\overline{x}_t+\phi) \end{cases} \tag{7}$$

as well as the total expenditure:

$$CT(\overline{x}_t) = \frac{w_t}{1-\alpha}\left(\frac{1-\alpha}{\alpha}\frac{r_t}{w_t}\right)^{\alpha}(\overline{x}_t+\phi)$$

the second step then consists to determine the optimal level of production, which is defined as follows:

$$\left(x^h_{jt}\right)^* \in \underset{\overline{x}_t\in\mathbb{R}_+}{Arg\ Max}\ \{p_{jt}(\overline{x}_t+(n_j-1)\overline{x}_{jt})\ \overline{x}_t - CT(\overline{x}_t)\}$$

The first order condition gives:

$$p_{jt}\left[1+\frac{\partial p_{jt}}{\partial \overline{x}_t}\frac{\overline{x}_t}{p_{jt}}\right] = Cm(\overline{x}_t) = \frac{w_t}{1-\alpha}\left(\frac{1-\alpha}{\alpha}\frac{r_t}{w_t}\right)^{\alpha} \tag{8}$$

Combining (7) and (8) and restricting our attention to the symmetrical equilibria, where $n_{jt}=N_t$, $K^h_{j(t-1)} = K_{t-1}/N_t$, $L^h_{jt} = L_t/N_t$ and $p_{ht}=P_t$, we obtain the two conditions:

$$\begin{aligned} \rho_t &\equiv \frac{r_t}{P_t} = \alpha\left[1-\frac{1}{N_t\sigma}\right]k^{\alpha-1}_{t-1} \\ \omega_t &\equiv \frac{w_t}{P_t} = (1-\alpha)\left[1-\frac{1}{N_t\sigma}\right]k^{\alpha}_{t-1} \end{aligned} \tag{9}$$

where k_{t-1} denote the capital-labor ratio in period t.

In the sequel of this paper, the symbol λ will denote the inverse of the margin factor:

$$\lambda = 1-\frac{1}{\sigma N_t}$$

Under the free entry assumption, the number of firms will adjust until there are no more profit opportunities. Ignoring integer constraints, we can therefore determine the number of enterprises in period t using the zero profit condition of the intermediate sector:

$$\frac{\Pi_t}{P_t} = \frac{K^{\alpha}_{t-1}L^{1-\alpha}_t}{N_t}\left\{1-\alpha\left[1-\frac{1}{N_t\sigma}\right]-(1-\alpha)\left[1-\frac{1}{N_t\sigma}\right]\right\}-\phi = 0$$

yielding a number of firms equal to:

$$N_t = \sqrt{\frac{K_{t-1}^{\alpha} L_t^{1-\alpha}}{\Phi\sigma}}$$

and eventually, the margin factor inverse that is given by:

$$\lambda\left(K_{t-1}, L_t\right) = 1 - \sqrt{\frac{\Phi}{\sigma K_{t-1}^{\alpha} L_t^{1-\alpha}}} \tag{10}$$

3 Equilibria and Dynamics of the model

3.1 Money Market Equilibrium

At each date, the money market is cleared if the real balance is equal, on the one hand, to the level that young agents wish to save $\omega_t L_t$ and, on the other hand, to the consumption expenditure of the older agents. Since we assume that the quantity of money is constant over time, the equilibrium condition on the money market can be written, in real terms as follows:

$$\omega\left(K_{t-1}, L_t\right) L_t = \frac{M}{P_t} = C_t = \left(L_{t-1}\right)^{\theta} \tag{11}$$

The equilibrium of the goods market is then ensured through Walras' law.

3.2 Intertemporal Equilibrium and Local Dynamics

Denote Z_t the aggregate gross production function:

$$Z_t = K_{t-1}^{\alpha} L_t^{1-\alpha} \tag{12}$$

Including the money market equilibrium condition in the first dynamical equation of our system, (2), we obtain a homogeneous dynamical system in terms of K_{t-1} and Z_t. We then define a perfect foresight intertemporal equilibrium as a sequence $\{K_{t-1}, Z_t\}_{t=1\ldots+\infty}$ such that in each period, there is a general temporary equilibrium that is, a sequence that verifies:

$$\begin{cases} K_t = \left(1 - \delta + \rho_t\right) K_{t-1} = \left(1 - \delta\right) K_{t-1} + \alpha\lambda(Z_t)Z_t \\ \left(1 - \alpha\right) \lambda(Z_{t+1})Z_{t+1} = \left(\left(\frac{K_{t-1}^{\alpha}}{Z_t}\right)^{\frac{1}{\alpha-1}}\right)^{\theta} \end{cases} \tag{13}$$

where the notation $\lambda(.)$ has been abusively kept for the degree of competition index, despite of the change of coordinates.

We will restrict the study of the dynamics to the neighborhood of the steady states. Formally, a pair $\{K, Z\}$ is a steady state for the system (13) if it is a solution of the system of equations:

$$\begin{cases} K = \left(1 - \delta + \alpha\lambda\left(Z\right)\dfrac{Z}{K}\right)K \quad \Longleftrightarrow \quad \lambda\left(Z\right)Z = \dfrac{\delta}{\alpha}K \\ \left(1-\alpha\right)\lambda\left(Z\right)Z = \left(\left(\dfrac{K^{\alpha}}{Z}\right)^{\frac{1}{\alpha-1}}\right)^{\theta} \end{cases}$$

Conditions that ensure that the existence of at least one steady state are given in appendix and can be summarized as follows:

Proposition 1 *Denote $\zeta = \phi/\sigma$ a measure of the imperfection of competition.*

- *There is a strictly positive value ζ^*, given by*

$$\zeta^* = \left(B\left(1-2A\right)\right)^{\frac{1}{A}}\left(\frac{2A}{2A-1}\right)^{2A}$$

  with

$$A = \frac{\left(1-\alpha\right)\left(1-\theta\right)}{1-\alpha(1-\theta)} \quad \text{and} \quad B = \frac{\delta}{\alpha}\left(\frac{\alpha}{\delta(1-\alpha)}\right)^{\dfrac{1-\alpha}{1-\alpha(1-\varepsilon\chi)}}$$

  such that

  - *there is no steady state for $\zeta > \zeta^*$.*
  - *there is exactly one steady state $\left(K\left(\zeta^*\right), Z\left(\zeta^*\right)\right)$ for $\zeta = \zeta^*$*
  - *there are two steady states for $0 \leq \zeta < \zeta^*$, $\left(K_1\left(\zeta^*\right), Z_1\left(\zeta^*\right)\right)$ and $\left(K_2\left(\zeta^*\right), Z_2\left(\zeta^*\right)\right)$ with:*

$$K_1\left(\zeta\right) < K_2\left(\zeta\right), \text{ and}$$
$$Z_1\left(\zeta\right) < Z_2\left(\zeta\right)$$

    for all ζ in $[0, \zeta^*[$.

In particular, when ζ goes to zero, the lower steady state converges towards the autarkic steady state.

As shown in the proof of the proposition, the steady states are determined by the equation $S\left(Z\right) \equiv \left(1 - \sqrt{\zeta/Z}\right) Z^A = B$, where B is a positive constant. The graph of the function $S(Z)$ for different possible values of ζ is given by figure 1.

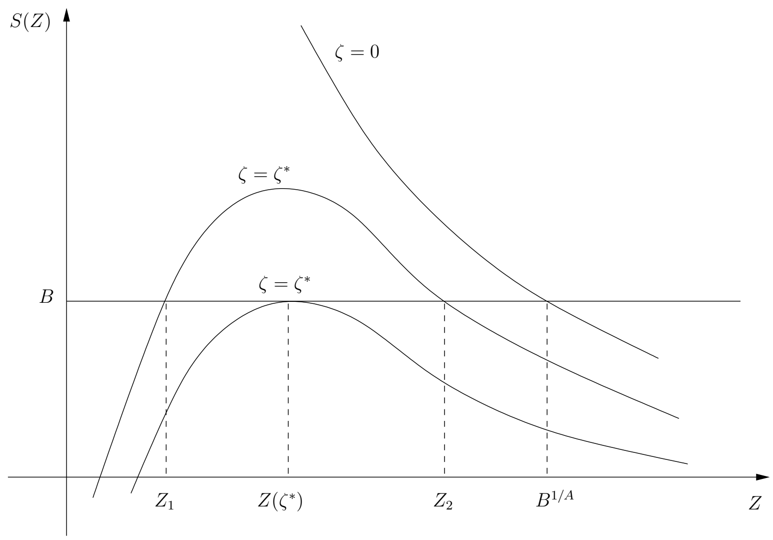

Figure 1: Determination of The Steady State

Corollary 1 *The high steady state Pareto dominates the low one.*

Proving this statement is quite straightforward. Indeed, since firms make zero-profits, entrepreneurs are indifferent between the two steady states. Capitalists prefer the high steady state since it is associated with a higher capital stock. It remains to show that the higher steady state is also preferred by workers. To see this, note that utility increases along the workers' offer curve, so we just have to check that, for a given degree of competition ζ, $L_2(\zeta) > L_1(\zeta)$. From the equations defining the steady states, we have the relation:

$$L = \left[\delta \frac{1-\alpha}{\alpha} K\right]^{\frac{1}{\theta}} \tag{14}$$

so the steady state level of labor is an increasing function of capital and the desired property follows.

Once the existence of steady states is established, the question of the behavior of the system in their vicinity arises. Using Hartman and Grobman's theorem, this problem can be reduced to the study of the linearized version of the system. Linearization around a non degenerate steady state provides the following result:

$$\begin{bmatrix} dK_t \\ dZ_{t+1} \end{bmatrix} = \begin{bmatrix} 1-\delta & \delta\left(1+\varepsilon^{\lambda}\right)\dfrac{K}{Z} \\ \dfrac{\alpha}{\alpha-1}\dfrac{Z}{K}\dfrac{\theta}{1+\varepsilon^{\lambda}} & \dfrac{\theta}{(1-\alpha)\left(1+\varepsilon^{\lambda}\right)} \end{bmatrix} \begin{bmatrix} dK_{t-1} \\ dZ_t \end{bmatrix} \tag{15}$$

where

$$\varepsilon^{\lambda} = \frac{\partial\lambda}{\partial Z}\frac{Z}{\lambda}.$$

The local stability properties of the system are then determined by studying the eigenvalues associated to the Jacobian matrix. Two cases can be considered: the case where firms have no market power (corresponding to zero fixed costs or perfect substitutability) and the case where markup is strictly positive and varies over time because of entries and exits of firms.

The figure 2, the construction of which does not depend of the specific dynamical system under study, indicates, for the different possible values of the trace and the determinant, the corresponding nature of the steady state in terms of stability properties[11].

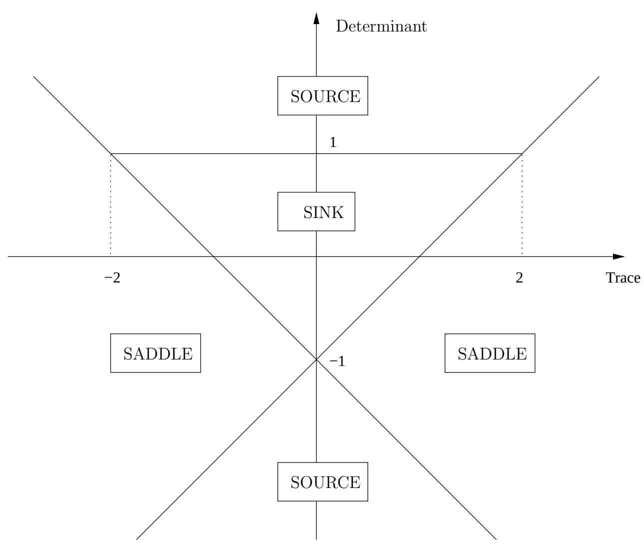

Figure 2: The Trace-Determinant Diagram

Using equations (10), (12) and the notation $\zeta = \phi/\sigma$, we have:

$$\varepsilon^{\lambda} = \frac{1}{2}\frac{\sqrt{\frac{\zeta}{Z}}}{1-\sqrt{\frac{\zeta}{Z}}} \tag{16}$$

Trace and Determinant of the transition matrix in (15) are given by:

$$\begin{aligned} Tr &= 1-\delta+\frac{\theta}{\left(\varepsilon^{\lambda}+1\right)(1-\alpha)} \\ Det &= \frac{\theta}{\left(\varepsilon^{\lambda}+1\right)(1-\alpha)}\left[1-\delta+\delta\alpha\left(\varepsilon^{\lambda}+1\right)\right] \end{aligned}$$

The expression $\left(\varepsilon^{\lambda}+1\right)$ being the only one that depends upon the parameters ϕ and σ, we can consider that, for a given level of the other parameters, these last two equations define a parameterized curve depending on the parameter $\zeta = \phi/\sigma$. In fact, by operating the substitution of ζ in the determinant equation by its expression in terms of the trace, we see that this curve is more precisely a line the equation of which is:

$$Det = (1-\delta)Tr + \delta\left(\frac{\alpha\theta}{1-\alpha}+2-\delta\right)-1 \tag{17}$$

When ζ goes to zero, we converge towards the perfect competition case, that is a saddle point equilibrium.

According to proposition 1, for ζ sufficiently close to zero, the dynamical system admits two stationary equilibria. Focusing on the variable Z, we have $Z_1(\zeta) < Z_2(\zeta)$ for all ζ in $[0, \zeta^*[$.

A problem arises in that the steady states can't be computed explicitly[12]. In order to provide a local analysis, we have to compute the limits of the solutions Z_1 and Z_2 when ζ respectively goes to zero and ζ^*, defining in this way bounds between which the steady states may vary.

The tables 1 and 2 on the last page (the construction of which is detailed in appendix) summarize the evolutions of trace and determinant as ζ varies between 0 and ζ^*. The subscripts 1 and 2 refer respectively to the low and high steady state.

We can deduce the following proposition:

ζ	0		ζ^*
Z_1	0	$\nearrow$	$\left(B\left(1-2A\right)\right)^{\frac{1}{A}}$
$\varepsilon^{\lambda}\left(Z_1\right)$	$+\infty$	$\searrow$	$-A$
$Tr\left(Z_1\right)$	$1-\delta$	$\nearrow$	$1-\delta+\dfrac{\theta}{\left(1-A\right)\left(1-\alpha\right)}$
$Det\left(Z_1\right)$	$\dfrac{\alpha\delta\varepsilon^{\chi}}{1-\alpha}$	$\nearrow$	$\dfrac{\theta}{\left(1-A\right)\left(1-\alpha\right)}\left[1-\delta+\delta\alpha\left(1-A\right)\right]$

Table 1: Behavior of trace and determinant for the low steady state

ζ	0		ζ^*
Z_2	$B^{\frac{1}{A}}$	$\searrow$	$\left(B\left(1-2A\right)\right)^{\frac{1}{A}}$
$\varepsilon^{\lambda}\left(Z_2\right)$	0	$\nearrow$	$-A$
$Tr\left(Z_2\right)$	$1-\delta+\dfrac{\theta}{1-\alpha}$	$\searrow$	$1-\delta+\dfrac{\theta}{\left(1-A\right)\left(1-\alpha\right)}$
$Det\left(Z_2\right)$	$\dfrac{\theta}{1-\alpha}\left[1+\delta\left(\alpha-1\right)\right]$	$\searrow$	$\dfrac{\theta}{\left(1-A\right)\left(1-\alpha\right)}\left[1-\delta+\delta\alpha\left(1-A\right)\right]$

Table 2: Behavior of trace and determinant for the high steady state

Proposition 2 *The dynamical system (13) undergoes a Neimark bifurcation as the market power, as measured by ζ, is increased from 0 to ζ^* under the condition:*

$$0<\delta<\min\left\{\frac{1-\alpha}{\alpha\theta},\frac{\alpha\theta}{1-\alpha}\right\}$$

Moreover, when ζ goes through ζ^*, a saddle-node bifurcation emerges, meaning that the two steady states coalesce in ζ^* and that the system has no $\zeta>\zeta^*$.

Proof. The idea of the proof is to compute the trace and determinant in both limit cases where $\zeta=0$ (perfect competition) and $\zeta=\zeta^*$. Then, we use the fact that they move continuously on a line when ζ is made to vary, to show that a Neimark bifurcation must unfold[13].

First, we give conditions for the low steady state to be locally indeterminate in the perfectly competitive situation:

$$\begin{aligned}\lim_{\zeta\to 0}D_1 &> \lim_{\zeta\to 0}T_1-1\\ \lim_{\zeta\to 0}D_1 &< 1\end{aligned}$$

Expressed in terms of the parameters, these conditions can be rewritten as:

$$\begin{aligned}\frac{\alpha\theta}{1-\alpha} &> -1\\ \frac{1-\alpha}{\alpha\theta} &> \delta\end{aligned}$$

The second condition will be satisfied provided that the capital rate of depreciation is small enough and the first one is verified for all admissible values of the parameters.

Secondly, in the perfect competition case the high steady state is a saddle. Indeed $Det(Z_2) < Tr(Z_2) - 1$ iff:

$$\frac{\theta}{1-\alpha}\left[1-\delta\left(\alpha+1\right)\right] < \frac{\theta}{1-\alpha} - \delta$$

or equivalently,

$$\frac{\alpha+1}{1-\alpha}\theta > 1$$

which is always verified.

Traces and determinant associated to both steady states move on a line with slope $(1-\delta)$ as ζ is increased and coalesce when $\zeta = \zeta^*$.

We can then define a segment in the trace-determinant coordinates, for which one end is associated to a saddle equilibrium, the other to a sink, and such that all points correspond to feasible equilibria. A sufficient condition for a Neimark bifurcation to emerge as ζ goes from zero to ζ^* is then that the determinant associated with a trace of 2 is higher than 1 (see figure 3 on the following page).

Using equation (17), we see that this condition is verified if and only if:

$$\delta^2 - \frac{\alpha\theta}{1-\alpha}\delta < 0$$

which is ensured for:

$$\delta < \frac{\alpha\theta}{1-\alpha}$$

The second claim, that a saddle-node bifurcation occurs in $\zeta = \zeta^*$, is just a reformulation of proposition 1 in terms of bifurcation theory.

A saddle-node bifurcation corresponding to an eigenvalue of modulus 1, we must have the relationship[14]:

$$Det\left(\zeta^*\right) = Tr\left(\zeta^*\right) - 1.$$

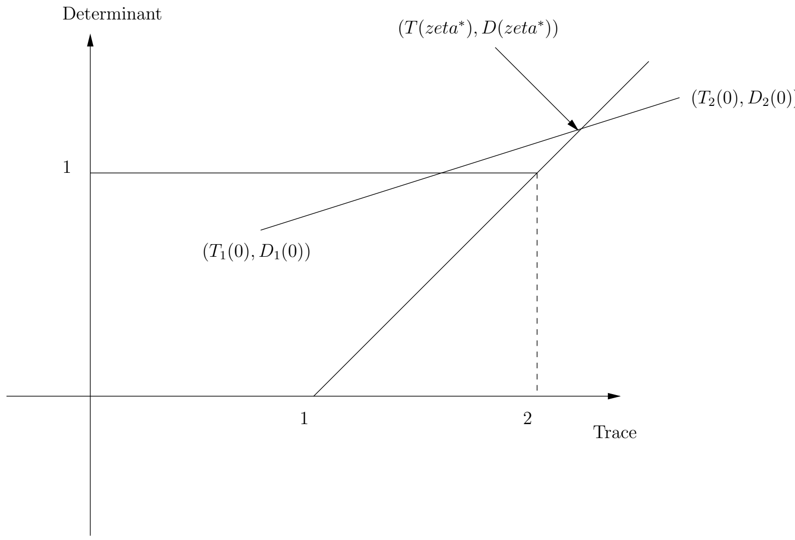

Figure 3: Evolution of the Steady States with ζ

This is indeed verified, when we replace A by its expression, in the expressions of $Det\,(\zeta^*)$ and $Tr\,(\zeta^*)$. ■

As a result of bifurcation theory, we know that, when a Neimark bifurcation emerges, an invariant closed curve appears around the steady state, for all values of the bifurcation parameter in a sufficiently close left or right neighborhood of the bifurcation value. Studies of the dynamics restricted to this closed curve revealed that there were a countable infinity of periodic orbits and an uncountable infinity of aperiodic orbits (see [de Vilder et al., 1999] for an overview of dynamics associated with a Neimark bifurcation). Two cases, the supercritical and the subcritical ones, are distinguished according to whereas the invariant closed curve is an attractive or repulsive one, and are illustrated in the figure 4 on the next page. Discriminating between these two cases would necessitate to investigate the second order approximation of the system, and this task isn't achieved here. Economic interpretations of the Neimark bifurcation can be found in [Kind, 1999].

As shown in the proof, the non autarkic steady state, in the perfect competition case, is always a saddle. Thus, under the perfect foresight assumption, trajectories in the neighborhood of the steady state will eventually converge and no fluctuations are to be expected[15].

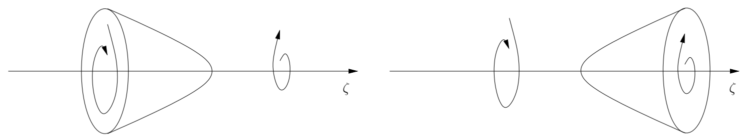

Figure 4: The Hopf Bifurcation

4 Conclusion

This paper sought to demonstrate that the introduction of imperfect competition in a general dynamic equilibrium model is not neutral. Indeed, when firms have no market power in this model, the nondegenerate steady state is of saddle type, i.e. there is a unique local trajectory eventually converging towards the steady state. On the contrary, when the imperfect competition index is strictly positive, both steady states are non autarkic and the low one (which correspond to the autarkic steady state in the perfect competition case) is indeterminate. As the market power is raised, if the rate of depreciation of capital is sufficiently small, the system undergoes a Neimark bifurcation followed by a saddle-node bifurcation, where both steady states coalesce and then vanish. The Neimark bifurcation implies that an invariant closed curve exists for parameter values close to the bifurcation value. Therefore, we have shown that dynamical behavior of the economy is more complicated in an imperfect competition framework than in its perfect competition counterpart. Fluctuations that are not explained by external shocks are possible in the long term. The source of variations of the economic activity is to be found in the free entry and exit of intermediate sector firms implying zero profits at the equilibrium. Imperfect substitutability and a fixed cost in the technology ensure that the number of active firms remains bounded. Moreover, this number varies procyclicaly while the markup evolution is countercyclical, in accordance with former empirical results (see e.g. [Portier, 1995]). The interpretation of that fact is that, in growth phases, the production of the final sector raise, increasing the demand of inputs to the intermediate sector. This implies that new firms will enter in each input market. In this Cournot framework, coordination between firms will be weaker so the markup will go down until it won't be any more profitable for some firms to stay on the market. The number of productive firm will then lower increasing coordination between the remaining ones and so on.

One appreciable feature of this model is to emphasize the dynamical effects of introducing imperfections in competition in a very standard framework, i. e. Cournot competition. Indeed, the recent literature concerning imperfect competition is characterized by a wide variety of hypotheses, with more complicated market structures, and more specific strategic decisions (e.g R&D investment decisions etc...). Such approaches have not been widely exploited in the dynamic general equilibrium framework and constitute therefore a possible track for future research concerning dynamical aspects of imperfect competition.

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Appendix

A Second Order Conditions of the Intermediate Sector

In the two-steps procedure of maximization, first order conditions can be written, dropping the time subscripts for notational convenience, as follows:

$$MR(\bar{x}) - MC(\bar{x}) = 0$$

where $MC(\bar{x})$ has been shown to be constant and

$$MR(\bar{x}) = p_j \left[1 + \frac{\partial p_j}{\partial \bar{x}} \frac{\bar{x}}{p_j} \right]$$

with

$$p_j = P \left(\frac{\bar{x} + (n_j - 1)\bar{x}_j}{y} \right)^{-\frac{1}{\sigma}}$$

So, the only thing to be done is to show that the marginal revenue function $MR(\bar{x})$ is decreasing.

$$\frac{\partial MR(\bar{x})}{\partial \bar{x}} = \frac{\partial p_j}{\partial \bar{x}} \left[1 + \frac{\partial p_j}{\partial \bar{x}} \frac{\bar{x}}{p_j} \right] + p_j \left[1 + \frac{\partial^2 p_j}{\partial \bar{x}^2} \frac{\bar{x}}{p_j} + \frac{\partial p_j}{\partial \bar{x}} \frac{\bar{1}}{p_j} \right]$$

which gives, at the symmetrical equilibrium, where $\bar{x} = \bar{x}_j$:

$$\frac{\partial MR(\bar{x})}{\partial \bar{x}} = \frac{\partial p_j}{\partial \bar{x}} \left[1 - \frac{1}{\sigma n_j} \right] - p_j \frac{1}{\sigma} \frac{n_j - 1}{n_j^2 \bar{x}}$$

So the marginal revenue is a decreasing function provided that the number of firms is at least higher or equal to one[1]6.

B Existence of the Steady State:

The steady state associated to the dynamical system (13), if it exists, is given by the solution of:

$$\begin{cases} \left(1-\sqrt{\frac{\zeta}{Z}}\right) Z = \frac{\delta}{\alpha} K \\ \left(1-\sqrt{\frac{\zeta}{Z}}\right) Z = \frac{1}{1-\alpha} L^{\theta} \\ Z = K^{\alpha} L^{1-\alpha} \end{cases} \tag{18}$$

Expressed in terms of capital stock and of production, (18) becomes:

$$\left(1-\sqrt{\frac{\zeta}{Z}}\right) Z = \frac{\delta}{\alpha} K$$
$$\left(1-\sqrt{\frac{\zeta}{Z}}\right) Z = \frac{1}{1-\alpha} K^{\frac{\theta\alpha}{\alpha-1}} Z^{\frac{\theta}{1-\alpha}}$$

from which we deduce:

$$K = \left(\frac{\alpha}{\delta\left(1-\alpha\right)}\right)^{\frac{1-\alpha}{1-\alpha(1-\theta)}} Z^{\frac{\theta}{1-\alpha(1-\theta)}} \tag{19}$$

which we substitute in the first equation to obtain:

$$\left(1-\sqrt{\frac{\zeta}{Z}}\right) Z^{\frac{(1-\alpha)(1-\theta)}{1-\alpha(1-\theta)}} = \frac{\delta}{\alpha}\left(\frac{\alpha}{\delta\left(1-\alpha\right)}\right)^{\frac{1-\alpha}{1-\alpha(1-\theta)}}$$

We will use the following notations:

$$A \equiv \frac{\left(1-\alpha\right)\left(1-\theta\right)}{1-\alpha\left(1-\theta\right)} \; (< 0)$$
$$B \equiv \frac{\delta}{\alpha}\left(\frac{\alpha}{\delta\left(1-\alpha\right)}\right)^{\frac{1-\alpha}{1-\alpha(1-\theta)}} \; (> 0)$$

Knowing the production level, the capital stock is uniquely determined by the equation (19). So we have to determine under which conditions the equation:

$$S\left(Z\right) \equiv \left(1-\sqrt{\frac{\zeta}{Z}}\right) Z^{A} = B \tag{20}$$

has one or more solutions.

$S(.)$ is a function that has limits:

$$\lim_{Z\to 0} S(Z) = -\infty$$
$$\lim_{Z\to +\infty} S(Z) = 0$$

Moreover, we have:

$$S'(Z) = Z^{A-1}\left(A\left(1-\sqrt{\frac{\zeta}{Z}}\right)+\frac{1}{2}\sqrt{\frac{\zeta}{Z}}\right)\begin{cases} >0 \text{ if } \sqrt{\frac{\zeta}{Z}} < \frac{2A}{2A-1} \\ \leq 0 \text{ otherwise}\end{cases}$$

The function $S(Z)$ has then a maximum for $Z_{\max} = \zeta\left((2A-1)/2A\right)^2$, which is given by:

$$S(Z_{\max}) = \frac{\zeta^A}{1-2A}\left(\frac{2A-1}{2A}\right)^{2A} > 0$$

There is consequently a level Z such that condition (20) is fulfilled, if and only if:

$$\zeta \leq \left(B\left(1-2A\right)\right)^{\frac{1}{A}}\left(\frac{2A}{2A-1}\right)^2$$

The system defining steady states then admits generically two solutions (the uniqueness case being destroyed under slight perturbations).

C Construction of the tables 1 and 2

We wish to study the evolution of the trace and the determinant as ζ is varied between 0 and ζ^*..For this purpose, we have to distinguish the cases corresponding to each stationary solution of the dynamical system, i.e. Z_1 and Z_2 ($Z_1 < Z_2$). We can compute the expressions of the trace and the determinant in terms of the parameter ζ, by replacing ε^λ by its value:

$$\varepsilon^\lambda = \frac{1}{2}\frac{\sqrt{\frac{\zeta}{Z}}}{1-\sqrt{\frac{\zeta}{Z}}}$$

We will only specify the construction of the first table, the other one being straightforward. We have:

$$\lim_{\zeta\to 0} Z_1 = 0$$

The equation (20) can be rewritten:

$$\sqrt{\frac{\zeta}{Z}} = 1 - \frac{B}{Z^A}$$

from which we deduce ($A < 0$):

$$\lim_{\zeta \to 0} \sqrt{\frac{\zeta}{Z_1(\zeta)}} = 1$$

thus, the trace and determinant limits are:

$$\begin{array}{l} \lim_{\zeta \to 0} Tr(Z_1(\zeta)) = 1 - \delta \\ \lim_{\zeta \to 0} Det(Z_1(\zeta)) = \frac{\delta \alpha \varepsilon^{\chi}}{1-\alpha} \end{array}$$

Limits of the trace and the determinant as ζ goes to ζ^* can be obtained by a direct computation, using the fact that

$$Z(\zeta^*) = \zeta^* \left(\frac{2A-1}{2A} \right)^2 .$$

In order to conclude the description of the tables, it remains to study their variations according to the parameter ζ.

We obtain by a direct differentiation of the equation defining the steady state (20):

$$\frac{\partial Z}{\partial \rho} = \frac{Z}{2A\rho} \frac{1}{\sqrt{\frac{Z}{\rho}} - \frac{2A-1}{2A}}$$

which is positive if $Z > \rho \left(\frac{2A-1}{2A} \right)^2$, and negative otherwise.We have:

$$\frac{\partial Tr}{\partial \left(\sqrt{\frac{\zeta}{Z}} \right)} < 0 \quad \text{and} \quad \frac{\partial Det}{\partial \left(\sqrt{\frac{\zeta}{Z}} \right)} < 0$$

The sign of the derivative of ζ / Z with respect to ζ is obtained using the rules of elasticity computations:

$$\varepsilon_{\left(\frac{Z}{\zeta} \right) / \zeta} \equiv \frac{\partial \left(\frac{Z}{\zeta} \right)}{\partial \zeta} \frac{\zeta}{\left(\frac{Z}{\zeta} \right)} = \frac{1}{2A \left(\sqrt{\frac{Z}{\zeta}} + \frac{1-2A}{2A} \right)} - 1$$

thus

$$\frac{\partial \left(\sqrt{\frac{\zeta}{Z_1}} \right)}{\partial \zeta} < 0 \quad \text{and} \quad \frac{\partial \left(\sqrt{\frac{\zeta}{Z_2}} \right)}{\partial \zeta} > 0$$

The arrows in the first table follow directly. The second table is constructed by direct computation and will not be detailed here.