Maneuvering Target Tracking

Farhad Besharati   Nasrollah Eskandari   Faezeh Besharati
Young Researchers Club, Azad University Tehran Markaz
P.O. Box 14395/515, Tehran, IRAN

Abstract
This paper proposes three exponentially correlated acceleration approaches for accuracy and computational complexity. These models are Singer model, Bar-Shalom and Fortmann's model. Simulation results show that the Singer models and the Bar-Shalom and Fortmann models, each a six state estimate model, require approximately the same number of flops. The Bar-Shalom and Fortmann model requires more flops due to the size of the Q and G matrices. The Sklansky model is a four state estimator and requires about 2/3 of the number of flops of the Singer model.

1 introduction

Tracking a maneuvering target involves filtering and prediction in order to track the target. Filtering refers to estimating the state vector at the current time, based upon all past measurements. Prediction refers to estimating the state at a future time; we shall see that prediction and filtering are closely related [1]. One of the most commonly used technique for target tracking is the discrete Kalman filter developed by Rudolf Kalman. The Kalman filter is used to filter past measurements and predict where a target will be in the future. This target location prediction is then used to point a sensor in order to track the target. An error covariance matrix is maintained as part of the normal computation process of the Kalman filter. This error covariance matrix can be considered as a measure of uncertainty of the kinematic state of the target. The tracking of maneuvering targets may be complicated by the fact that acceleration may not be directly observable or measurable. Additionally, apparent acceleration can be induced by a variety of sources including human input, autonomous guidance, or atmospheric disturbances. Several approaches to tracking maneuvering targets have been proposed in the literature and can be divided into two categories. One approach is to model the maneuver as a random process. The other approach assumes that the maneuver is not random and that it is either detected or estimated in real time. Both assume a rectilinear model of target track. The random process models generally assume one of two statistical properties, either white noise or an autocorrelated noise. The multiple-model approach is generally used with the white noise model while a zero-mean, exponentially correlated acceleration approach is used with the autocorrelated noise model. The nonrandom approach uses maneuver detection to correct the state estimate or a variable dimension filter to augment the state estimate with an extra state component during a detected maneuver [2]. The exponentially correlated acceleration model approach is one of the approaches most widely used to track maneuvering targets. This paper examines and compares three exponentially correlated acceleration approaches for accuracy and computational complexity. They include the Singer model in polar coordinates, the Sklansky model (not an exponentially correlated acceleration), and Bar-Shalom and Fortmann's model.

2Singer Model Using Polar Coordinates

Singer [8], [7], developed a model that incorporates the maneuver capability of a target that is both simple and suitably represents the maneuver characteristics. The Singer model for manned maneuvering targets assumes that a target usually moves at constant velocity and that turns, evasive maneuvers, and accelerations due to atmospheric disturbances can be viewed as perturbations of the constant velocity trajectory. These accelerations are termed target maneuvers and are correlated in time with the previous time or the next time increment. That is to say that if a target is maneuvering at time t, it is likely to be maneuvering at time t+ τ assuming that τ is sufficiently small. Singer [8] states that a lazy turn will give correlated inputs for up to one minute, evasive maneuvers due to radar detection, terrain features, or preprogrammed maneuvers will provide correlated inputs for 10 to 30 seconds, and atmospheric turbulence for only 1 to 2 seconds. Due to this time dependence, the maneuvers are neither additive nor Gaussian. Singer's probability density function for a target's maneuvers are shown in Figure 1. A target can [8]:

- Accelerate (maneuver) at its maximum rate, ± Amax with a probability of Pmax

- No maneuver with a probability of P0, or
- Maneuver between - Amax and + Amax according to the uniform distribution shown in Figure 1.

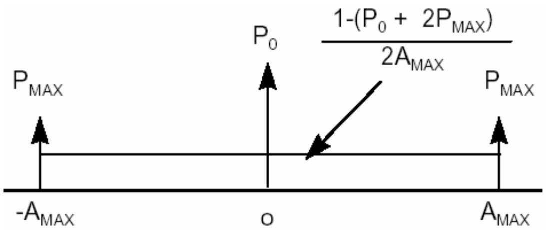


Figure 1: Target maneuver probability density function [7]

In order to use this model in an optimal filter such as a Kalman filter, the maneuver noise needs to be whitened. Singer [7] uses a procedure analogous to the whitening procedure developed by Wiener and Kolmogorov. The whitening processes are done by augmenting the state vector to include the maneuver variables and expressing them recursively in terms of white noise.

The target maneuver model is in polar coordinates and given by the state equation

$$X_{k+1} = \Phi X_k + G W_k$$

Where

$$X_k = \begin{bmatrix} r_k & \dot{r}_k & u_{r,k} & \theta_k & \dot{\theta}_k & u_{\theta,k} \end{bmatrix}^T$$

$w_k = \begin{bmatrix} w_{1,k} & w_{2,k} \end{bmatrix}^T$

$$\phi = \begin{bmatrix} 1 & T & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & \rho & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & T & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & \rho \end{bmatrix} \qquad G = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

T= sampling period

$e^{\alpha T}$ or $\cong 1 - \alpha T$ IF αT is small

$$Q_K = E\left[W_k \quad W_k^T \right] = \begin{bmatrix} \sigma^2{}_{M_1}(1-\rho) & 0 \\ 0 & \sigma^2{}_{M_2}(1-\rho) \end{bmatrix}$$

$$\sigma^2{}_{M_1} = \frac{A^2{}_{\max} T^2}{3}(1 + 4P_{\max} - p_0)$$

$$\sigma^2{}_{M2} = \frac{A^2{}_{\max} T^2}{3R^2}(1 + 4P_{\max} - p_0)$$

R=target range

The measurement equation is given by

$Z_k = H\, X_k + V_k$

Where

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$R_k = \begin{bmatrix} \sigma^2{}_{r,k} & 0 \\ 0 & \sigma^2{}_{\theta,k} \end{bmatrix}$$

The standard filter equations for state estimation extrapolation, error covariance extrapolation, Kalman gain matrix computation, state estimate update, and error covariance updates are then applied. The filter is initialized based on the first two observations with the state estimate given by

$$\hat{X}_2 = \begin{bmatrix} Z_2(1) & \frac{1}{T}(Z_2(1) - Z_1(1)) & 0 & Z_2(2) & \frac{1}{T}(Z_2(2) - Z_1(2)) & 0 \end{bmatrix}^T$$

and the nonzero elements of the updated error covariance matrix, $\mathbf{P}^+{}_2$, defined as

with σ_{M_1} calculated in (4) and

$P_{11} = \sigma^2{}_r \qquad P_{44} = \sigma^2{}_\theta$

$$P_{22} = \sigma^2{}_{M_1} + \left(\frac{2\sigma^2{}_r}{T^2} \right)$$

$$P_{55} = \sigma^2{}_{M_2}(1) + \left(\frac{2\sigma^2{}_\theta}{T^2} \right)$$

$P_{33} = \sigma^2{}_{M_1} \qquad P_{66} = \sigma^2{}_{M_2}(1)$

$$P_{12} = P_{21} = \frac{\sigma^2{}_r}{T^2} \qquad P_{45} = P_{54} = \frac{\sigma^2{}_\theta}{T^2}$$

$P_{23} = P_{32} = \rho\sigma^2{}_{M_1} \qquad P_{56} = P_{65} = \rho\sigma^2{}_{M2}(1)$

with σ_{M_1} calculated in (4) and

$$\sigma^2{}_{M_2}(1) = \frac{\sigma^2{}_{M_1}}{Z^2{}_1(1)}$$

.3Sklansky Model

The Sklansky model is a Cartesian coordinate, constant velocity tracking algorithm that does not model acceleration to generate position and velocity estimates of maneuvering targets [10].

The target motion is described by

$$X_{n+1} = X_n + T\dot{X}_n + \frac{1}{2}T\ddot{X}_n + \ldots$$

$$\dot{X}_{n+1} = \dot{X}_n + T\ddot{X}_n$$

where

$X_n =$ target position

$\dot{X}_n =$ target velocity

T = time interval between observations

$\ddot{X}_n =$ target acceleration

The state space representation of the Sklansky model is given by

$$X_{k=1} = \Phi_k X_k + G_k a_k$$

$$Z_k = HX_{k=1} + V_k$$

where

$$\Phi_k = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$X_k = \begin{bmatrix} X & \dot{X} & Y & \dot{Y} \end{bmatrix} =$$

$$\begin{bmatrix} Xposition & Xvelocity & Yposition & Yvelocity \end{bmatrix}^T$$

$$G = \begin{bmatrix} \frac{T^2}{2} & 0 \\ T & 0 \\ 0 & \frac{T^2}{2} \\ 0 & T \end{bmatrix}$$

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$a_k = \begin{bmatrix} u_x(k) & u_y(k) \end{bmatrix}^T$ =random acceleration in the x and y coordinate respectively

V_k = scalar random measurement noise with Q ~ N(0 ,1)

4 Bar-Shalom and FortmannModel

Another exponentially correlated acceleration model based on the Singer Model is presented by Bar-Shalom and Fortmann [2]. They use a linear shaping filter to augment the Kalman filter.

The continuous-time state equation and measurement model is

$$\dot{X}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -\alpha & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & -\alpha \end{bmatrix} X(t) + W(t)$$

$$Z(t) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} X(t) + V(t)$$

X(t)=

$$\begin{bmatrix} X(t) & \dot{X}(t) & \ddot{X}(t) & Y(t) & \dot{Y}(t) & \ddot{Y}(t) \end{bmatrix}^T$$

The discrete-time state equation corresponding to (24) with sample interval T is

$$X(K+1) = FX(K) + W(K)$$

Where

$F = e^{AT} =$

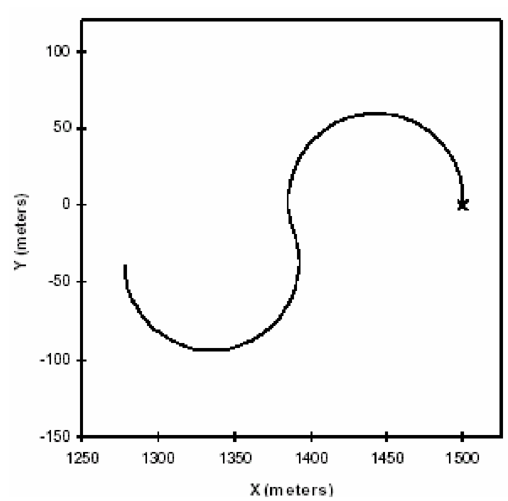
$$\begin{bmatrix} 1 & T & (\alpha T - 1 + e^{-\alpha T})/\alpha^2 & 0. & 0 & 0 \\ 0 & 1 & (1 + e^{-\alpha T})/\alpha^2 & 0 & 0 & 0 \\ 0 & 0 & e^{-\alpha T} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & T & (\alpha T - 1 + e^{-\alpha T})/\alpha^2 \\ 0 & 0 & 0 & 0 & 1 & (1 + e^{-\alpha T})/\alpha^2 \\ 0 & 0 & 0 & 0 & 0 & e^{-\alpha T} \end{bmatrix}$$

The discrete-time process noise covariance matrix **Q** is given by

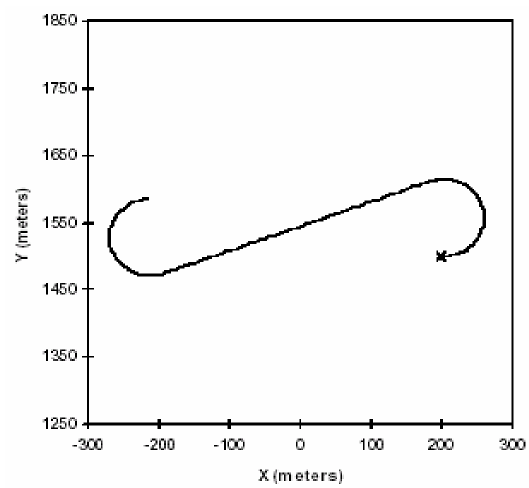
$$Q = 2\alpha\sigma^2{}_m \begin{bmatrix} \frac{T^5}{20} & \frac{T^4}{8} & \frac{T^3}{6} & 0 & 0 & 0 \\ \frac{T^4}{8} & \frac{T^3}{6} & \frac{T^2}{2} & 0 & 0 & 0 \\ \frac{T^3}{6} & \frac{T^2}{2} & T & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{T^5}{20} & \frac{T^4}{8} & \frac{T^3}{6} \\ 0 & 0 & 0 & \frac{T^4}{8} & \frac{T^3}{6} & \frac{T^2}{2} \\ 0 & 0 & 0 & \frac{T^3}{6} & \frac{T^2}{2} & T \end{bmatrix}$$

5 SIMULATION AND Model Comparisons

The three models described above where tested using Monte-Carlo simulations with 50 replications in order to compare the state estimation performance of each model. Two different target paths [11] were used in the simulations. The first was a target performing an S turn lasting 40 seconds and the second is also a S turn maneuver but with an straight segment between turns and lasts for 80 seconds. The target paths are shown in Figure 2 while Table 1 provides a summary of the maneuver parameters used in the simulations. Figure 2a is the simulated target path for the S turn without the straight segment and Figure 2b is the simulated target path for the S turn with the straight segment. The “x” denotes the starting position.



a) S turn without straight segment



b) S turn with straight segment

Figure 2: Simulated Target Paths

Table 1: Kalman Filter Simulation Parameter Summary

	Scenario	
	S turn without straight segment	S turn with straight segment
Initial x, y position	(1500 m, 0 m)	(200 m, 1500 m)
Initial polar position	$r = 1500$ m, $\vartheta = 0°$	$r = 1513$ m, $\vartheta = 82.4°$
Initial heading	90°	0°
Duration	40 sec	80 sec
Turn rate	10 m/s for 20 sec −10 m/s for 20 sec	10 m/s for 20 sec 0 m/s for 40 sec −10 m/s for 20 sec
Sample rate	$T = 0.5\ \mathrm{s}^{-1}$	$T = 0.5\ \mathrm{s}^{-1}$
Range measurement variance	$\sigma_r = 10\ \mathrm{m}^2$	$\sigma_r = 10\ \mathrm{m}^2$
Bearing measurement variance	$\sigma_\vartheta = 0.0001\ \mathrm{rad}^2$	$\sigma_\vartheta = 0.0001\ \mathrm{rad}^2$
Maximum acceleration	$A_{\max} = 1.745\ \mathrm{m/s}^2$	$A_{\max} = 1.745\ \mathrm{m/s}^2$
Forward velocity	$v_t = 10\ \mathrm{m/s}$	$v_t = 10\ \mathrm{m/s}$
Maximum turn rate	$r_{\max} = 0.1745\ \mathrm{rad/s}$	$r_{\max} = 0.1745\ \mathrm{rad/s}$
Mean number of changes	$\alpha = 0.05556\ \mathrm{s}^{-1}$	$\alpha = 0.05556\ \mathrm{s}^{-1}$

The remaining model specific parameters and initial error covariance matrices needed to perform the filter simulations are as follows:

Singer (Polar) Pmax = 0.1

P0 = 0.4 Q as defined in (4-4)

P initialized according to (4-7)

Sklansky $Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

P initialized with [100000 1000 100000 1000] along the main diagonal

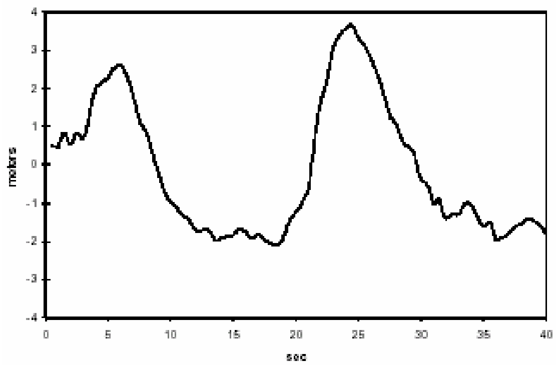
Bar-Shalom and Fortmann

$$\sigma_m = A_{\max} / 6$$

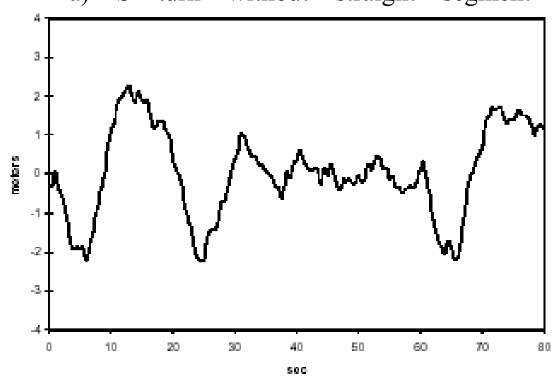
P initialized with

[100000 1000 1000 100000 1000 1000]

along the main diagonal All of the models performed exceedingly well with extremely small average position and velocity errors and RMS position and velocity errors regardless of target path used. Figure 3 and Figure 4 show the average range and bearing errors, respectively, for both target paths. The average range errors are less than ±4 meters for either target path while the average bearing error is between ±0.3°. The average range and bearing rate errors are show in Figure 5 and Figure 6 while the RMS range and bearing errors are shown in Figure 7 and the RMS range and bearing rate errors are shown in Figure 8. The average range rate error is between ±5 m/s and the average bearing rate is between ±0.4 deg/s. The RMS errors are 2-4 meters for range, 0.3-0.6 m/s for range rate, 0.5-2° for bearing and 0.05 deg/s for bearing rate.

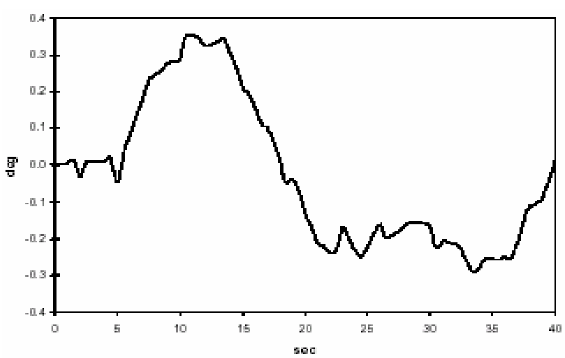


a) S turn without straight segment

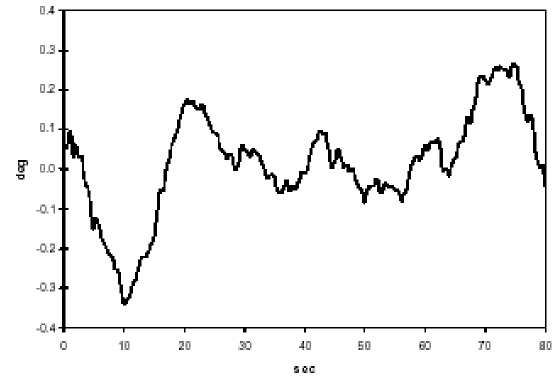


b) S turn with straight segment

Figure 3: Singer Model (Polar) Average Range Erro

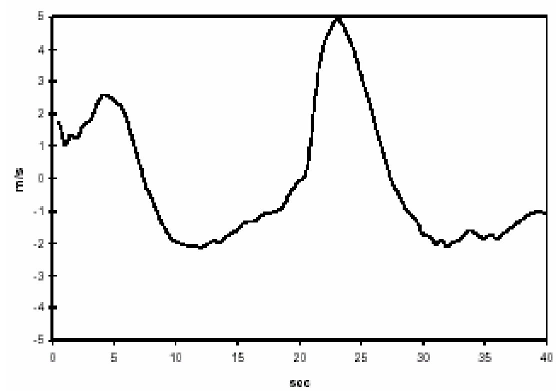


a) S turn without straight segment

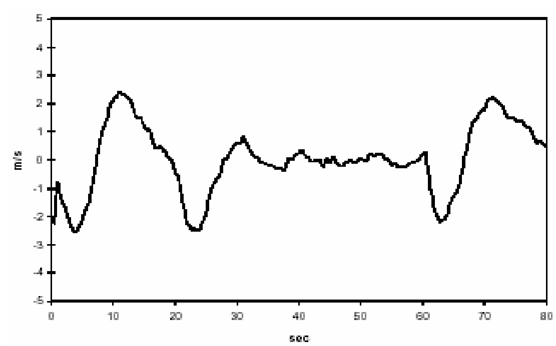


b) S turn with straight segment

Figure 4: Singer Model (Polar) Average Bearing Error

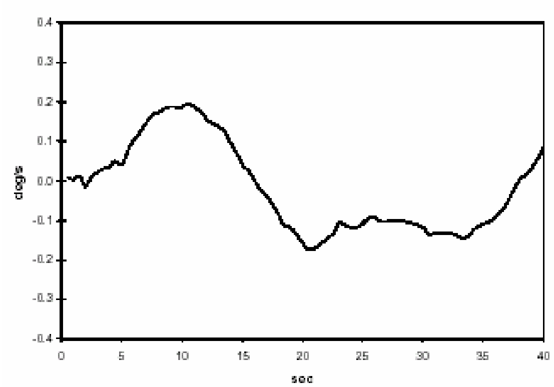


a) S turn without straight segment

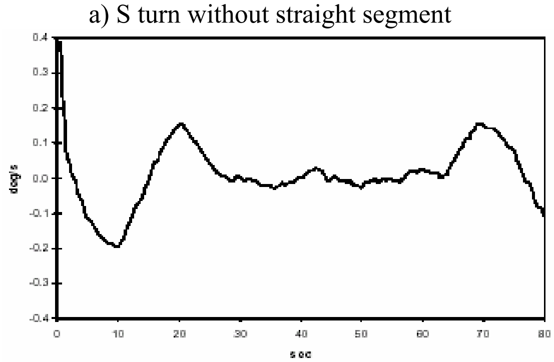


b) S turn with straight segment

Figure 5: Singer Model (Polar) Average Range Rate Error

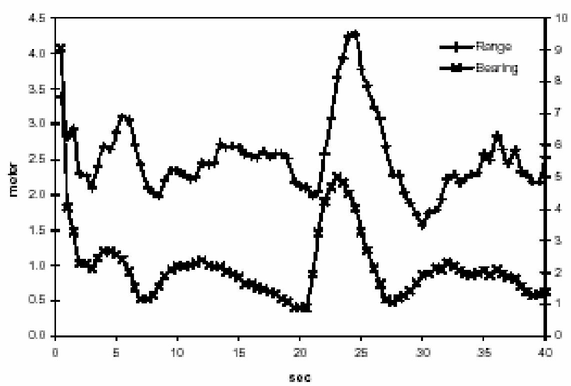


a) S turn without straight segment

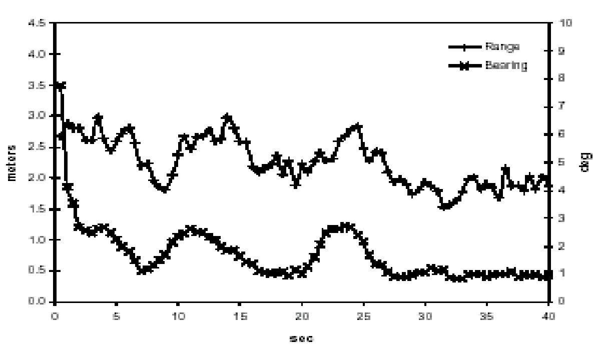


b) S turn with straight segment

Figure 6: Singer Model (Polar) Average Bearing Rate Error

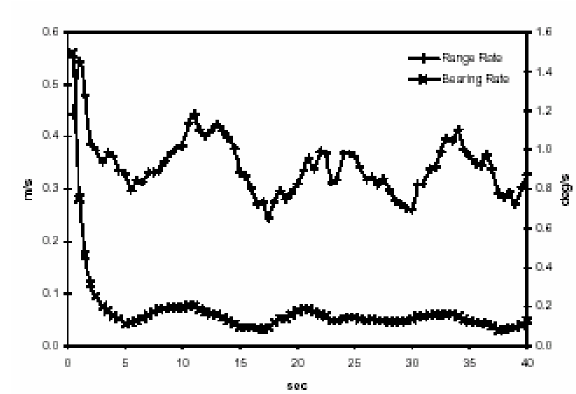


a) S turn without straight segment

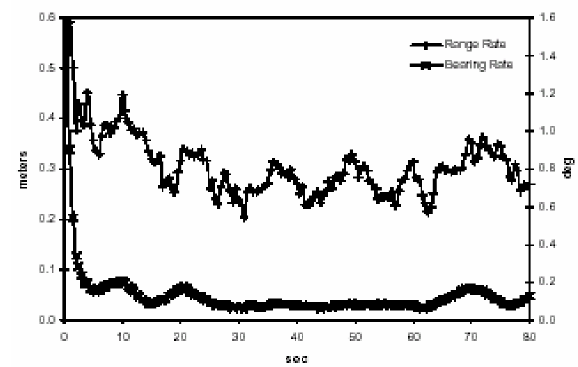


b) S turn with straight segment

Figure 7: Singer Model (Polar) RMS Range and Bearing Error



a) S turn without straight segment



b) S turn with straight segment

Figure 8: Singer Model (Polar) RMS Range Rate and Bearing Rate Error

Table 2: Maneuvering Target Model Complexity

Model	Number of flops
One-Siger(Polar)	2270
Sklansky	896
Bar-Shhalom & Fortman	2946

8 Summary

The exponentially correlated acceleration models appear to be valid and accurate models of target maneuvers as demonstrated above. All of the model, whether in polar, Cartesian, or polar converted to Cartesian provide very accurate position estimates. Besides state estimate accuracy, another consideration in choosing a maneuvering target tracking model is the computational complexity of the model. One such measure is the number of floating point operations (flops).

Table 2 shows the number of flops for one iteration of state estimate extrapolation, error covariance extrapolation, Kalman gain matrix computation, state estimate update and error covariance update for each model. The conversion of the measurement noise covariance matrix from polar to Cartesian coordinates only add an additional 32 flops. As can be seen, the Singer models and the Bar-Shalom and Fortmann models, each a six state estimate model, require approximately the same number of flops. The Bar-Shalom and Fortmann model requires more flops due to the size of the Q and G matrices. The Sklansky model is a four state estimator and requires about 2/3 of the number of flops of the Singer model. The flops were computed for comparable runs of each model averaged over 80 iterations of the update process using MATLAB.

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