A Fundamental Frequency Estimation Method for Non-sinusoidal Signal Based on Numerical Differentiation

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Abstract– **An algorithm basing on numerical differentiation and central Lagrange interpolation with multi-points is presented for the fundamental frequency estimation of non-sinusoidal signals in this paper. The signal is sampled at a fixed sample frequency of 25600Hz with the unknown parameters, the frequency is estimated with 7-point consequences using the high-order differentiation at a high accuracy of 0.001% over a very wide range varying from 2Hz to 1MHz in at most 1 cycle. Comparing with other algorithms, this algorithm spends little time and computation for frequency of the signal. The proposed algorithm is simulated in Matlab software using a testing study example with satisfactory results.**

***Index Terms*—frequency estimation, fundamental harmonic, nonsinusoidal signal, numerical differentiation, Lagrange interpolation**

1 Introduction

Frequency estimation has been one of important task in intelligent instrumentation and metering. Many of well-proven techniques such as zero crossing technique [1]-[2], level crossing technique [3], least squares error technique[4]-[6], Newton method [7], Kalman filter [8]–[12], Fourier transform [13]-[19], wavelet transform [18] have been used for this purpose, and some estimation results in accuracy give us a helpful guide. However, larger errors in frequency measurement of signal are often brought in, and much more time and computation must need so as not to be applied in real-time measurement and control.

The algorithm proposed in this paper is developed to estimate the fundament frequency of non-sinusoidal signals with a frequency varying from 6Hz to 1MHz. This algorithm is based on numerical differentiation and central Lagrange interpolation with multi-points. Comparing with other algorithms, this algorithm spends little time and computation over a wide range at a high accuracy.

In section 1, the pioneering works in frequency estimation are described. In section 2, the proposed algorithm is presented. In section 3, the steps for the algorithm implementation are discussed. In section 4, a study case is simulated with Matlab software to illustrate the results of the proposed algorithm.

2 The Proposed Algorithm

2.1 Numerical Differentiation

Given a function of voltage signal:

$$v(t) = 0 \qquad (1)$$

At discrete points such as (t_i , v_i) and (t_j , v_j), i =0,1,2,…, $M-1$, j =0,1,2,…, $M-1$, the Taylor series expansion is expressed as:

$$v(t_i) = v(t_j) + \Delta t \left.\frac{dv}{dt}\right|_{t=t_j} + \frac{(\Delta t)^2}{2!}\left.\frac{d^2v}{dt^2}\right|_{t=t_j} + \frac{(\Delta t)^3}{3!}\left.\frac{d^3v}{dt^3}\right|_{t=t_j} + \ldots + \frac{(\Delta t)^M}{M!}\left.\frac{d^Mv}{dt^M}\right|_{t=t_j} + \ldots \qquad (2)$$

where $\Delta t = t_i - t_j$.

Without consideration of M order and higher order derivatives, we have central difference formulas such that:

$$v(t_{i+k}) = v(t_i) + (t_{i+k} - t_i)v'(t_i) + \frac{(t_{i+k} - t_i)^2}{2!}v''(t_i) + \frac{(t_{i+k} - t_i)^3}{3!}v^{(3)}(t_i) + \ldots + \frac{(t_{i+k} - t_i)^{M-1}}{(M-1)!}v^{(M-1)}(t_i) \qquad (3)$$

$\ldots$

$$v(t_{i+2}) = v(t_i) + (t_{i+2} - t_i)v'(t_i) + \frac{(t_{i+2} - t_i)^2}{2!}v''(t_i) + \frac{(t_{i+2} - t_i)^3}{3!}v^{(3)}(t_i) + \ldots + \frac{(t_{i+2} - t_i)^{M-1}}{(M-1)!}v^{(M-1)}(t_i) \qquad (4)$$

$$v(t_{i+1}) = v(t_i) + (t_{i+1} - t_i)v'(t_i) + \frac{(t_{i+1} - t_i)^2}{2!}v''(t_i) + \frac{(t_{i+1} - t_i)^3}{3!}v^{(3)}(t_i) + \ldots + \frac{(t_{i+1} - t_i)^{M-1}}{(M-1)!}v^{(M-1)}(t_i) \qquad (5)$$

$$v(t_{i-1}) = v(t_i) + (t_{i-1} - t_i)v'(t_i) + \frac{(t_{i-1} - t_i)^2}{2!}v''(t_i) + \frac{(t_{i-1} - t_i)^3}{3!}v^{(3)}(t_i) + \ldots + \frac{(t_{i-1} - t_i)^{M-1}}{(M-1)!}v^{(M-1)}(t_i) \qquad (6)$$

$$v(t_{i-2}) = v(t_i) + (t_{i-2} - t_i)v'(t_i) + \frac{(t_{i-2} - t_i)^2}{2!}v''(t_i)$$

$$+\frac{(t_{i-2}-t_i)^3}{3!}v^{(3)}(t_i)+\ldots+\frac{(t_{i-2}-t_i)^{M-1}}{(M-1)!}v^{(M-1)}(t_i) \quad (7)$$

$\ldots$

$$v(t_{i-k})=v(t_i)+(t_{i-k}-t_i)v^{'}(t_i)+\frac{(t_{i-k}-t_i)^2}{2!}v^{"}(t_i)$$

$$+\frac{(t_{i-k}-t_i)^3}{3!}v^{(3)}(t_i)+\ldots+\frac{(t_{i-k}-t_i)^{M-1}}{(M-1)!}v^{(M-1)}(t_i) \quad (8)$$

where M is interpolation number, k =floor($M/2$). floor(A) rounds the elements of A to the nearest integers less than or equal to A.

Given (t_0, v_0), (t_1, v_1),..., (t_M, v_M) with regular spaced h, we have the following relationship:

t_0 , $t_1=t_0+h$, $t_2=t_0+2h$, $t_3=t_0+3h$,..., $t_M=t_0+Mh$

So that equation (3)-(8) become:

$$v(t_{i+k})=v(t_i)+khv^{'}(t_i)+\frac{(kh)^2}{2!}v^{"}(t_i)$$

$$+\frac{(kh)^3}{3!}v^{(3)}(t_i)+\ldots+\frac{(kh)^{M-1}}{(M-1)!}v^{(M-1)}(t_i) \quad (9)$$

$\ldots$

$$v(t_{i+2})=v(t_i)+2hv^{'}(t_i)+\frac{(2h)^2}{2!}v^{"}(t_i)$$

$$+\frac{(2h)^3}{3!}v^{(3)}(t_i)+\ldots+\frac{(2h)^{M-1}}{(M-1)!}v^{(M-1)}(t_i) \quad (10)$$

$$v(t_{i+1})=v(t_i)+hv^{'}(t_i)+\frac{h^2}{2!}v^{"}(t_i)$$

$$+\frac{h^3}{3!}v^{(3)}(t_i)+\ldots+\frac{h^{M-1}}{(M-1)!}v^{(M-1)}(t_i) \quad (11)$$

$$v(t_{i-1})=v(t_i)-hv^{'}(t_i)+\frac{h^2}{2!}v^{"}(t_i)$$

$$-\frac{h^3}{3!}v^{(3)}(t_i)+\ldots+(-1)^{M-1}\frac{h^{M-1}}{(M-1)!}v^{(M-1)}(t_i) \quad (12)$$

$$v(t_{i-2})=v(t_i)-2hv^{'}(t_i)+\frac{(-2h)^2}{2!}v^{"}(t_i)$$

$$+\frac{(-2h)^3}{3!}v^{(3)}(t_i)+\ldots+\frac{(-2h)^{M-1}}{(M-1)!}v^{(M-1)}(t_i) \quad (13)$$

$\ldots$

$$v(t_{i-k})=v(t_i)-khv^{'}(t_i)+\frac{(-kh)^2}{2!}v^{"}(t_i)$$

$$+\frac{(-kh)^3}{3!}v^{(3)}(t_i)+\ldots+\frac{(-kh)^{M-1}}{(M-1)!}v^{(M-1)}(t_i) \quad (14)$$

From equation (9)-(14), the s order differentiation of $v(t)$ can mathematically be concluded as:

$$v^{(s)}(t)=\sum_{\substack{u_1=0\\u_1\neq 0}}^{M-1}\sum_{\substack{u_2=0\\u_2\neq 0,u_1}}^{M-1}\sum_{\substack{u_3=0\\u_3\neq 0,u_1,u_2}}^{M-1}\ldots\sum_{\substack{u_s=0\\u_s\neq 0,u_1,u_2,u_3,\ldots,u_s}}^{M-1}\frac{\prod_{\substack{j=0\\j\neq 0,u_1,u_2,u_3,\ldots,u_s}}^{M-1}(t-t_j)}{\prod_{\substack{j=0\\j\neq 0}}^{M-1}(t_0-t_j)}v_0$$

$$+\sum_{\substack{u_1=0\\u_1\neq 1}}^{M-1}\sum_{\substack{u_2=0\\u_2\neq 1,u_1}}^{M-1}\sum_{\substack{u_3=0\\u_3\neq 1,u_1,u_2}}^{M-1}\ldots\sum_{\substack{u_s=0\\u_s\neq 1,u_1,u_2,u_3,\ldots,u_s}}^{M-1}\frac{\prod_{\substack{j=0\\j\neq 1,u_1,u_2,u_3,\ldots,u_s}}^{M-1}(t-t_j)}{\prod_{\substack{j=0\\j\neq 1}}^{M-1}(t_1-t_j)}v_1$$

$$+\sum_{\substack{u_1=0\\u_1\neq 2}}^{M-1}\sum_{\substack{u_2=0\\u_2\neq 2,u_1}}^{M-1}\sum_{\substack{u_3=0\\u_3\neq 2,u_1,u_2}}^{M-1}\ldots\sum_{\substack{u_s=0\\u_s\neq 2,u_1,u_2,u_3,\ldots,u_s}}^{M-1}\frac{\prod_{\substack{j=0\\j\neq 2,u_1,u_2,u_3,\ldots,u_s}}^{M-1}(t-t_j)}{\prod_{\substack{j=0\\j\neq 2}}^{M-1}(t_2-t_j)}v_2$$

$$+\sum_{\substack{u_1=0\\u_1\neq 3}}^{M-1}\sum_{\substack{u_2=0\\u_2\neq 3,u_1}}^{M-1}\sum_{\substack{u_3=0\\u_3\neq 3,u_1,u_2}}^{M-1}\ldots\sum_{\substack{u_s=0\\u_s\neq 3,u_1,u_2,u_3,\ldots,u_s}}^{M-1}\frac{\prod_{\substack{j=0\\j\neq 3,u_1,u_2,u_3,\ldots,u_s}}^{M-1}(t-t_j)}{\prod_{\substack{j=0\\j\neq 3}}^{M-1}(t_3-t_j)}v_3$$

$$+\ldots$$

$$+\sum_{\substack{u_1=0\\u_1\neq m}}^{M-1}\sum_{\substack{u_2=0\\u_2\neq m,u_1}}^{M-1}\sum_{\substack{u_3=0\\u_3\neq m,u_1,u_2}}^{M-1}\ldots\sum_{\substack{u_s=0\\u_s\neq m,u_1,u_2,u_3,\ldots,u_s}}^{M-1}\frac{\prod_{\substack{j=0\\j\neq m,u_1,u_2,u_3,\ldots,u_s}}^{M-1}(t-t_j)}{\prod_{\substack{j=0\\j\neq m}}^{M-1}(t_m-t_j)}v_m$$

$$+\ldots+$$

$$\sum_{\substack{u_1=0\\u_1\neq M-1}}^{M-1}\sum_{\substack{u_2=0\\u_2\neq M-1,u_1}}^{M-1}\sum_{\substack{u_3=0\\u_3\neq M-1,u_1,u_2}}^{M-1}\ldots\sum_{\substack{u_s=0\\u_s\neq M-1,u_1,u_2,u_3,\ldots,u_s}}^{M-1}\frac{\prod_{\substack{j=0\\j\neq M-1,u_1,u_2,u_3,\ldots,u_s}}^{M-1}(t-t_j)}{\prod_{\substack{j=0\\j\neq M-1}}^{M-1}(t_{M-1}-t_j)}v_{M-1} \quad (15)$$

where s is the differentiation order number for $v(t)$, $u_1, u_2, u_3, \ldots, u_s$ is differentiation index.

For example, the 2nd-order differentiation of $v(t)$ with respect to t has the following formulation:

$$v^{"}(t)=\sum_{\substack{u=0\\u\neq 0}}^{M-1}\sum_{\substack{v=0\\v\neq 0,u}}^{M-1}\frac{\prod_{\substack{j=0\\j\neq 0,u,v}}^{M-1}(t-t_j)}{\prod_{\substack{j=0\\j\neq 0}}^{M-1}(t_0-t_j)}v_0+\sum_{\substack{u=0\\u\neq 1}}^{M-1}\sum_{\substack{v=0\\v\neq 1,u}}^{M-1}\frac{\prod_{\substack{j=0\\j\neq 1,u,v}}^{M-1}(t-t_j)}{\prod_{\substack{j=0\\j\neq 1}}^{M-1}(t_1-t_j)}v_1$$

$$+\ldots$$

$$+\sum_{\substack{u=0\\u\neq m}}^{M-1}\sum_{\substack{v=0\\v\neq m,u}}^{M-1}\frac{\prod_{\substack{j=0\\j\neq m,u,v}}^{M-1}(t-t_j)}{\prod_{\substack{j=0\\j\neq m}}^{M-1}(t_m-t_j)}v_m+\ldots$$

$$+\sum_{\substack{u=0\\u\neq M-1}}^{M-1}\sum_{\substack{v=0\\v\neq M-1,u}}^{M-1}\frac{\prod_{\substack{j=0\\j\neq M-1,u,v}}^{M-1}(t-t_j)}{\prod_{\substack{j=0\\j\neq M-1}}^{M-1}(t_{M-1}-t_j)}v_{M-1} \quad (16)$$

The value of $v''(p)$ at point p is expressed as:

$$
\begin{aligned}
v''(p) &= (-1)^0 \frac{2\prod_{\substack{j=0\\ j\neq 0}}^{k-1} t_p - t_j)\prod_{\substack{j=k+1\\ j\neq M-1}}^{M-1}(t_p - t_j)}{(M-1)!\,h^{M-1}}[\,v_{M-1} + v_0\,] \\
&+ (-1)^1 \frac{2\prod_{\substack{j=0\\ j\neq 1}}^{k-1}(t_p - t_j)\prod_{\substack{j=k+1\\ j\neq M-2}}^{M-1}(t_p - t_j)}{(M-2)!\,h^{M-1}}[\,v_{M-2} + v_1\,] + \ldots \\
&+ (-1)^m \frac{2\prod_{\substack{j=0\\ j\neq m}}^{k-1}(t_p - t_j)\prod_{\substack{j=k+1\\ j\neq M-(m+1)}}^{M-1}(t_p - t_j)}{m![M-(m+1)]!\,h^{M-1}}[\,v_{M-(m+1)} + v_m\,] \\
&+ \ldots + \\
&(-1)^{k-1} \frac{2\prod_{\substack{j=0\\ j\neq k-1}}^{k-1}(t_p - t_j)\prod_{\substack{j=k+1\\ j\neq M-[(k-1)+1]}}^{M-1}(t_p - t_j)}{(k-1)!\{M-[(k-1)+1]\}!\,h^{M-1}}[\,v_{M-[(k-1)+1]} + v_{k-1} \\
&]\text{-}[\frac{2\prod_{\substack{j=0\\ j\neq 0}}^{k-1}(t_p - t_j)\prod_{\substack{j=k+1\\ j\neq M-1}}^{M-1}(t_p - t_j)}{k!k!\,h^{M-1}} \\
&+ \frac{2\prod_{\substack{j=0\\ j\neq 1}}^{k-1}(t_p - t_j)\prod_{\substack{j=k+1\\ j\neq M-2}}^{M-1}(t_p - t_j)}{k!k!\,h^{M-1}} + \ldots \\
&+ \frac{2\prod_{\substack{j=0\\ j\neq m}}^{k-1}(t_p - t_j)\prod_{\substack{j=k+1\\ j\neq M-(m+1)}}^{M-1}(t_p - t_j)}{k!k!\,h^{M-1}} + \ldots \\
&+ \frac{2\prod_{\substack{j=0\\ j\neq k-1}}^{k-1}(t_p - t_j)\prod_{\substack{j=k+1\\ j\neq M-[(k-1)+1]}}^{M-1}(t_p - t_j)}{k!k!\,h^{M-1}}\,]\,v_k \qquad (17)
\end{aligned}
$$

where k =floor($M/2$). floor(A) rounds the elements of A to the nearest integers less than or equal to A.

The 4th-order differentiation of $v(t)$ with respect to t is obtained:

$$
\begin{aligned}
v^{(4)}(t) &= \sum_{\substack{u=0\\ u\neq 0}}^{M-1}\sum_{\substack{v=0\\ v\neq 0,u}}^{M-1}\sum_{\substack{w=0\\ w\neq 0,u,v}}^{M-1}\sum_{\substack{s=0\\ s\neq 0,u,v,w}}^{M-1} \frac{\prod_{\substack{j=0\\ j\neq 0,u,v,w,s}}^{M-1}(t-t_j)}{\prod_{\substack{j=0\\ j\neq 0}}^{M-1}(t_0 - t_j)} v_0 \\
&+ \sum_{\substack{u=0\\ u\neq 1}}^{M-1}\sum_{\substack{v=0\\ v\neq 1,u}}^{M-1}\sum_{\substack{w=0\\ w\neq 1,u,v}}^{M-1}\sum_{\substack{s=0\\ s\neq 1,u,v,w}}^{M-1} \frac{\prod_{\substack{j=0\\ j\neq 1,u,v,w,s}}^{M-1}(t-t_j)}{\prod_{\substack{j=0\\ j\neq 1}}^{M-1}(t_1 - t_j)} v_1 + \ldots \\
&+ \sum_{\substack{u=0\\ u\neq m}}^{M-1}\sum_{\substack{v=0\\ v\neq m,u}}^{M-1}\sum_{\substack{w=0\\ w\neq m,u,v}}^{M-1}\sum_{\substack{s=0\\ s\neq m,u,v,w}}^{M-1} \frac{\prod_{\substack{j=0\\ j\neq m,u,v,w,s}}^{M-1}(t-t_j)}{\prod_{\substack{j=0\\ j\neq m}}^{M-1}(t_m - t_j)} v_m + \ldots \\
&+ \sum_{\substack{u=0\\ u\neq M-1}}^{M-1}\sum_{\substack{v=0\\ v\neq M-1,u}}^{M-1}\sum_{\substack{w=0\\ w\neq M-1,u,v}}^{M-1}\sum_{\substack{s=0\\ s\neq M-1,u,v,w}}^{M-1} \frac{\prod_{\substack{j=0\\ j\neq M-1,u,v,w,s}}^{M-1}(t-t_j)}{\prod_{\substack{j=0\\ j\neq M-1}}^{M-1}(t_{M-1} - t_j)} v_{M-1}
\end{aligned}
$$

(18)

With to M =7, the 1st order derivative of $v(t)$ at point p is expressed as follow:

$$
\begin{aligned}
v'(p) &= \{[\,v(p+3)\text{ - }v(p-3)\,]\text{-}9\,[\,v(p+2)\text{ - }v(p-2)\,] \\
&+45[\,v(p+1)\text{ - }v(p-1)\,]\}/60\,h \qquad (19)
\end{aligned}
$$

The 2nd order derivative of $v(t)$ at point p is expressed as follow:

$$
\begin{aligned}
v''(p) &= \{2[\,v(p+3)+v(p-3)\,]\text{-}27[\,v(p+2)+v(p-2)\,] \\
&+270[\,v(p+1)+v(p-1)\,]\text{-}490\,v(p)\,\}/\ 180\,h^2 \qquad (20)
\end{aligned}
$$

In the same form, the 3rd, 4th, 5th and 6th order derivative of $v(t)$ at point p is written respectively:

$$
\begin{aligned}
v^{(3)}(p) &= \{\text{-}[\,v(p+3)\text{ - }v(p-3)\text{ +}8\,[\,v(p+2)\text{ - }v(p-2)\,] \\
&\text{-}13[\,v(p+1)\text{ - }v(p-1)\,]\}/8\,h^3 \qquad (21)
\end{aligned}
$$

$$
\begin{aligned}
v^{(4)}(p) &= \{\text{-}2[\,v(p+3)+v(p-3)\,]\text{+}24[\,v(p+2)+v(p-2)\,] \\
&\text{-}78[\,v(p+1)+v(p-1)\,]\text{+}112\,v(p)\,\}/\ 12\,h^4 \qquad (22)
\end{aligned}
$$

$$
\begin{aligned}
v^{(5)}(p) &= \{[\,v(p+3)\text{ - }v(p-3)\text{ -}4\,[\,v(p+2)\text{ - }v(p-2)\,] \\
&\text{+}5[\,v(p+1)\text{ - }v(p-1)\,]\}/2\,h^5 \qquad (23)
\end{aligned}
$$

$$
\begin{aligned}
v^{(6`)}(p) &= \{[\,v(p+3)+v(p-3)\,]\text{-}6[\,v(p+2)+v(p-2)\,] \\
&\text{+}15[\,v(p+1)+v(p-1)\,]\text{-}20\,v(p)\,\}/\ 12\,h^6 \qquad (24)
\end{aligned}
$$

With to M =15, the 2nd order differentiation of $v(t)$ at point p is expressed as follow:

$$
\begin{aligned}
v''(p) &= \frac{10368}{871782912h^2}[\,v(p+7)+v(p-7)\,] \\
&\text{-}\frac{14112}{62270208h^2}[\,v(p+6)+v(p-6)\,] \\
&+\frac{2032128}{958003200h^2}[\,v(p+5)+v(p-5)\,] \\
&\text{-}\frac{3175200}{239500800h^2}[\,v(p+4)+v(p-4)\,] \\
&+\frac{56448}{870912h^2}[\,v(p+3)+v(p-3)\,] \\
&\text{-}\frac{435456}{127008h^2}[\,v(p+2)+v(p-2)\,] \\
&+\frac{508032}{290304h^2}[\,v(p+1)+v(p-1)\,] \\
&\text{-}\frac{435456}{127008h^2} \qquad (25)
\end{aligned}
$$

2.2 Frequency Estimation

Without loss of generality, a non-sinusoidal signal with 3rd -order harmonics is taking into consideration:

$$
\begin{aligned}
v(n) &= V_1\sin(2\pi f t_s + \phi_1) + V_2\sin(2(2\pi f)t_s + \phi_2) \\
&+ V_3\sin(3(2\pi f)t_s + \phi_3) \qquad (26)
\end{aligned}
$$

The 1st-order differentiation of $v(n)$ is formulated:

$$v^{'}(n) = V_1\, 1(2\pi f)\ \sin(2\pi f t_s + \phi_1) + V_2\ 2(2\pi f)\ \cos(2(2\pi f)t_s + \phi_2) + V_3\ 3(2\pi f)\ \cos(3(2\pi f)t_s + \phi_3) \quad (27)$$

The 2nd-order differentiation of $v(n)$ is formulated:

$$v^{''}(n) = -V_1\, [1(2\pi f)]^2\ \sin(2\pi f t_s + \phi_1) - V_2\ [2(2\pi f)]^2\ \sin(2(2\pi f)t_s + \phi_2) - V_3\ [3(2\pi f)]^2\ \sin(3(2\pi f)t_s + \phi_3) \quad (28)$$

From equation (27) and (28), we get the odd-order and even-order differentiation of $v(n)$ is expressed respectively:

$$v^{(o)}(n) = (-1)^{\frac{o+1}{2}}\ V_1\, [1(2\pi f)]^o\ \cos(2\pi f t_s + \phi_1) + (-1)^{\frac{o+1}{2}}\ V_2\ [2(2\pi f)]^o\ \cos(2(2\pi f)t_s + \phi_2) + (-1)^{\frac{o+1}{2}}\ V_3\ [3(2\pi f)]^o\ \cos(3(2\pi f)t_s + \phi_3) \quad (29)$$

$$v^{(e)}(n) = (-1)^{e/2}\ V_1\, [1(2\pi f)]^e\ \sin(2\pi f t_s + \phi_1) + (-1)^{e/2}\ V_2\ [2(2\pi f)]^e\ \sin(2(2\pi f)t_s + \phi_2) + (-1)^{e/2}\ V_3\ [3(2\pi f)]^e\ \sin(3(2\pi f)t_s + \phi_3) \quad (30)$$

where e is even-order differentiation index, o is odd-order differentiation index.

So, we get the 4th-order differentiation of $v(n)$:

$$v^{(4)}(n) = V_1\, [1(2\pi f)]^4\ \sin(2\pi f t_s + \phi_1) + V_2\ [2(2\pi f)]^4\ \sin(2(2\pi f)t_s + \phi_2) + V_3\ [3(2\pi f)]^4\ \sin(3(2\pi f)t_s + \phi_3) \quad (31)$$

where we define:

$$v_1(n) = V_1\ \sin(2\pi f t_s + \phi_1) \quad (32)$$

From equation (26) and (32), we obtain:

$$v_1(n) = \{36\,(2\pi f)^4\ v(n) + 13\,(2\pi f)^2\ v^{''}(n) + v^{(4)}(n)\ \}/\,(2\pi f)^4 \quad (33)$$

From equation (28), we get the 1st-order differentiation of $v_1(n)$:

$$v_1^{'}(n) = \{36\,(2\pi f)^4\ v^{'}(n) + 13\,(2\pi f)^2\ v^{(3)}(n) + v^{(5)}(n)\ \}/\,(2\pi f)^4 \quad (34)$$

From equation (32), the 1st and 2nd-order differentiation of $v_1(n)$ are written respectively:

$$v_1^{'}(n) = V_1\ 2\pi f\ \ \cos(2\pi f t_s + \phi_1) \quad (35)$$

$$v_1^{''}(n) = -V_1\ (2\pi f)^2\ \sin(2\pi f t_s + \phi_1) = -\,(2\pi f)^2\ v_1(n) \quad (36)$$

Using Equation (36), the fundamental frequency is estimated by:

$$f = \frac{1}{2\pi}\sqrt{\frac{-v_1^{''}(n)}{v_1(n)}} \quad (37)$$

There is greater error using equation (37) for estimating the fundamental frequency of non-sinusoidal signals due to the magnitude and phase of the voltage signal are not taken into consideration in the computation process of frequency estimation. In order to compensate this large error in computation and to estimate the frequency with high accuracy in a wide range of frequency, a coefficient, saying η, must be jointed into the right of Equation (37):

$$f_e = \eta\ \frac{1}{2\pi}\sqrt{\frac{-v_1^{''}(n)}{v_1(n)}} \quad (38)$$

where η is a coefficient depending not only upon the magnitude, the frequency and the phase of the sampled signal but also upon the sample frequency, f_s . At a great degree, η is determined by experiences, which influence the estimation accuracy of frequency.

Making use of equation (38), The estimated value of fundamental frequency is obtained at a higher accuracy in at most 1 cycle.

3 Implementation Process

The algorithm proposed in this paper is based on the assumption that the frequency, amplitude and phase angle of non-sinusoidal signals are all unknown. However, its the fundamental frequency can be estimated at a higher accuracy over a very wide range with the proposed algorithm. The steps for the implementation of the proposed algorithm may be written as follow:

Step1: Sample the non-sinusoidal signal with a fixed sample frequency: 512×50Hz=25600Hz.

Step2: Basing on central numerical differentiation of 7 points, compute the 2nd-order to 5th-order differentiation of the signal basing equation (16) -(25).

Step3: calculate the estimated frequency f_{e1} using equation (37).

Step4: Compute the signal consequences basing on the following formulation:

$$v(n) = \sum_{k=1}^{K} V_{kc}\ \sin(2\pi k f n T / N + \phi_{kc}) \quad (39)$$

where $T = 1/\ f_{e1}$.

Calculate the estimated frequency f_{e2} using equation (37).

Step5: Compute the signal consequences basing on equation (38) with $T = 1/\ f_{e2}$. Calculate the estimated frequency f_{e3} using equation (37).

f_{e3} is the measurement value of the fundamental frequency, namely $f_e = f_{e3}$.

4 Simulation Results

The tests of numerical simulation for a non-sinusoidal signal with 3 order harmonics are carried out in Matlab codes. The tested signal is shown as the following equation (40). The amplitude of the fundamental harmonic change from 1V to 100V, and the phase of the fundamental harmonic, the amplitudes and phases of the 2nd and 3rd-order harmonic are fixed.

$$v(t) = 20\sqrt{2}\ \sin(2\pi f_1 t + \phi_1) + 0.8\sqrt{2}\ \sin(2\pi f_2 t + 61^\circ) + 1.6\sqrt{2}\ \sin(2\pi f_3 t + 12^\circ) \quad (40)$$

Basing on the implementation steps, a fixed sample frequency: 512×50Hz is used for frequency estimation. The results of estimation for fundamental frequency are shown in Table I. From the table, it is seen that the fundamental frequency is estimated at an accuracy of 0.001% over a range varying from 2Hz to 1MHz. Over wide frequency range, the relative errors are retained at 0.001% or smaller. However, the relative errors and he absolute are all very small when the frequency varies from 2Hz to 40kHz while the relative errors are small and the absolute errors are large when the frequency varies from 40kHz to 1MHz.

TABLE I
FREQUENCY MEASURED FROM 2HZ TO 1MHZ

No.	Real-value	Measurement value
1	1000000	999998.43353
2	982735.758	982738.83195
3	827657.357	827660.03799
4	637657.357	637674.00867
5	592384.274	592398.17806
6	535989.274	535993.77602
7	400000.578	400001.44807
8	348679.578	348680.61951
9	285372.579	285372.26743
10	137657.357	137657.11574
11	118564.387	118564.18878
12	108357.375	108357.21213
13	98356.747	98357.16082
14	86375.356	86375.22484
15	73745.742	73745.66320
16	63568.275	63568.18502
17	56856.356	56856.25765
18	43576.256	43576.19551
19	29837.246	29837.23577
20	12385.356	12385.35507
21	9356.588	9356.58760
22	6256.274	6256.27538
23	3853.578	3853.57179
24	1358.257	1358.25706
25	956.246	956.24604
26	635.746	635.74602
27	357.472	357.47201
28	128.583	128.58300
29	88.563	88.56300
30	38.385	38.38500
31	18.385	18.38500
32	6.856	6.85600

5 Conclusion

Basing on numerical differentiation and central Lagrange interpolation with multi-points, an algorithm is presented for the fundamental frequency estimation in this paper. One advantage of the proposed algorithm is that the fundamental frequency of non-sinusoidal signals with multi-components is estimated at a high accuracy of 0.001% over a very wide range varying from 2Hz to 1MHz in at most 1 cycle.

Basing on the proposed algorithm, the parameters of the sampled signals such as amplitudes and the phase angles of the fundamental harmonic and other harmonic need not to be known. At a great degree, the frequency range with a higher accuracy is dependent on the sample frequency and the coefficients for the computation, which comes from the experiences.

Over wide frequency range, the relative errors are retained at 0.001% or smaller. However, the relative errors and he absolute are all very small when the frequency varies from 2Hz to 40kHz while the relative errors are small and the absolute errors are large when the frequency varies from 40kHz to 1MHz.

Comparing with other algorithms, this algorithm spends little time and computation over a wide range at a high accuracy duo to the use of numerical differentiation and central Lagrange interpolation with multi-points.

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