

DIRECT AND INDIRECT BLIND EQUALIZERS OF NOISY FIR CHANNELS : DETERMINISTIC METHODS

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MAROC

Abstract :

In the context of mobile communications, the aim of several researchers in recent years is the reduction of Intersymbol interference (ISI).

Blind equalizers reduce Intersymbol Interference using second-order statistics without the need for training sequences. Most current methods require channel estimation as a first step to estimate the equalizer. However, direct methods bypass channel estimation and equalize it directly.

In this paper, we present two different methods to equalize a SIMO FIR channel in indirect context using channel estimation and in direct context. The used methods are Zero Forcing (ZF) and Minimum Mean-Square Error (MMSE). A comparison of the two classes of algorithms is the goal of this paper.

Performance of the proposed methods are presented via simulations.

Index Terms— Blind channel identification, Blind channel equalization., ISI, ZF, MMSE, direct methods, indirect methods.

1 INTRODUCTION:

Indirect Blind equalization consists of two steps: Blind estimation of the channel impulse response (also known as blind identification) and construction of an equalizer based on the estimated impulse response. Early methods for blind identification were based on higher order statistics of the received signal. An identification method that uses only second order statistics was proposed to surmount this problem [1].

Indirect Blind equalization consists of two steps: Blind estimation of the channel impulse response (also known as blind identification) and construction of an equalizer based on the estimated impulse response. However, Direct methods bypass the channel estimation step and directly estimate a linear filter that can remove the Intersymbol interference (ISI) [2].

In this paper, we present two methods for finding linear equalizers from the data using two different algorithms Zero-Forcing (ZF) and Minimum Mean Square Error (MMSE) developed in direct and indirect approaches[3].

This paper is organised as follows. In section II, we present the model of an oversampled communication system, in section III, indirect blind channel equalization algorithms are presented and in section IV their direct versions. Simulation results are shown in section V and, we conclude our work in section VI

2 PROBLEM FORMULATION :

2.1 The system model:

The channel is considered Linear and Time Invariant (LTI) during a time window sufficient to allow its estimation. The continuous time received signal is (Fig.1)

$$y(t) = \int_{-\infty}^{+\infty} s(\tau)h(t - \tau)d\tau + v(t) \quad (1)$$

where $h(t)$:The composite channel, $s(l)$:Information symbols, T :symbol duration and $v(t)$:additive noise. In discretetime:

$$y(n) = \sum_{l=-\infty}^{+\infty} s(l)h(n - lN) + v(n) = x(n) + v(n) \quad (2)$$

The Single Input Single Output (SISO) relationship of (2) accepts an equivalent Single Input Multiple Output

(SIMO) description as given by (Fig.2):

$$y_i(n) = \sum_{l=-\infty}^{+\infty} s(l)h_i(n - l) + v_i(n) = x_i(n) + v_i(n) \quad (3)$$

Where: $y_i(n) = y(nN + i)$; $h_i(n) = h(nN + i)$

$x_i(n) = x(nN + i)$ and $v_i(n) = v(nN + i)$

For $i=0\dots N-1$.

At this stage, we adopt the following assumptions:

A1) The subchannels $h_i(n)$ do not have any common zero to allow the channel identifiability which will be needed in indirect approach.

A2) The input sequence $s(n)$ is white with unit variance: $\sigma_s^2 = 1$, i.e. $E[s(k)s^*(l)] = \delta(k - l)$ (4)

A3) The additive noise is white with variance σ_v^2 :
i.e. $E[v(k)v^*(l)] = \sigma_v^2\delta(k - l)$ (5)

In each symbol interval, a vector

$$y(n) = [y_0(n)\dots y_{N-1}(n)]^T \quad (6)$$

of length N is received. The channel impulse response can also be represented in vector form as

$$h(n) = [h_0(n)\dots h_{N-1}(n)]^T \quad (7)$$

And the noise as

$$v(n) = [v_0(n)\dots v_{N-1}(n)]^T \quad (8)$$

2.2 Fractionally Spaced Equalization (FSE):

In the receiver, ZF or MMSE equalizers can be used to extract the transmitted symbol. The i^{th} subchannel $h_i(n)$ is equalized by the filter $g(n)$, as shown in Fig. 3. The equalizer impulse response is :

$$g(n) = [g_0(n) \ g_1(n) \ \dots \ g_{N-1}(n)] \quad (9)$$

where $n=0,\dots,L_g-1$ and L_g is the length of the longest branch of the equalizer

A d -delay equalizer vector of length L_gN is constructed as: $g_d = [g(0) \ \dots \ g(L_g-1)]$ (10)

and the symbol estimate is obtained from

$$\hat{s}(n - d) = g_d y_{n,L_g} \quad (11)$$

$$y_{n,L_g} = [y(n) \ y(n+1) \ \dots \ y(n + L_g - 1)]^T \quad (12)$$

We also adopt the following assumptions.

A4) The data length and channel (equalizer) order satisfy

$$P - L_g \geq L_g + L_h + 1 \quad (13)$$

Which is easily met in practice by collecting sufficient data. [4]

A5) The triplet (N, L_g, L_h) (number of channels, equalizer order, channel order) must obey [4]

$$N(L_g + 1) \geq L_g + L_h + 1$$

$$\text{i.e. } L_g \geq L_{g\min} = \frac{\hat{e} L_h}{\hat{e} N - 1} - \frac{1}{\hat{e}} \frac{\hat{u}}{\hat{u}} \quad (14)$$

3 INDIRECT BLIND EQUALIZERS

3.1 Indirect Zero Forcing Equalizer:

In the absence of noise, it is obvious to choose

$$\hat{s}(n - d) = s(n)$$

$$\sum_{i=0}^{N-1-L_h-1} \hat{a}_i \sum_{l=0}^{L_h-1} h_i(l) g_i^d(m-1) = \delta(m-d) \quad (15)$$

This type of equalizer is known as Zero Forcing (ZF) equalizer where d refers to the delay $d \in [0, L_h + L_g]$

This definition allows the ZF condition to be written in matrix form as: $\mathbf{H}^T \mathbf{g}_d^T = \mathbf{e}_{d+1}$ [2]

Where

$$\mathbf{e}_{d+1} = (\underbrace{0 \dots 0}_{d \text{ zeros}} 1 0 \dots 0) \quad (16)$$

is a $(L_g + L_h + 1) \times 1$ vector, so:

$$\mathbf{g}_d = ((\mathbf{H}^T)^{\#} \mathbf{e}_{d+1})^T \quad (17)$$

Where $\#$ indicates pseudoinverse.

3.2 Indirect MMSE Equalizer:

The MMSE equalizer minimize the cost function

$$J_{\text{MMSE}}(\mathbf{g}_d) = E[(s(n) - \hat{s}(n-d))^2] \quad (18)$$

$$\text{so: } E \left[\sum_{k=0}^{L_g-1} \hat{a}_k^T \mathbf{g}_d^T(k) y(n-k) - s(n) \right]^2 = 0 \quad (19)$$

$$\text{And } \mathbf{g} = \left\{ \mathbf{H} E[\mathbf{S}\mathbf{S}^T] \mathbf{H}^T + \sigma_v^2 \mathbf{I} \right\}^{-1} \mathbf{H} E[\mathbf{S}\mathbf{S}^T] \\ = (\mathbf{H}\mathbf{H}^T + \sigma_v^2 \mathbf{I})^{-1} \mathbf{H} \quad (20)$$

The ZF equalizer do not perform optimally in the presence of noise[3] in comparison with MMSE equalizer.

4 DIRECT BLIND EQUALIZERS

4.1 Direct Zero Forcing Equalizer:

ZF equalizers are used normally to suppress ISI in free noise case:

$$\mathbf{R}_{xx} = \mathbf{R}_{yy} = \mathbf{H}\mathbf{R}_{ss}\mathbf{H}^H \quad (21)$$

Since the input $s(n)$ is i.i.d then :

$$\mathbf{R}_{ss} = \sigma_s^2 \mathbf{I} = \mathbf{I}; \text{so: } \mathbf{R}_{xx} = \mathbf{H}\mathbf{H}^H \quad (22)$$

$$\text{And: } \mathbf{R}_{xx}^T \mathbf{g}_d = \mathbf{H}^T \mathbf{e}_1 = \mathbf{H}^T(:,1) \quad (23)$$

Where $\mathbf{H}(:,1)$ denotes the first column of \mathbf{H}

We can easily find that the ZF equalizer yields [3]

$$\mathbf{g}_d = (\mathbf{R}_{xx}^T)^{\#} \mathbf{H}^T(:,1) \quad (24)$$

4.2 Direct MMSE Equalizer:

From(18),we obtain:

$$\hat{s}(n) = \sum_{i=0}^{N-1} \sum_{l=0}^{L_g-1} \hat{a}_i g_i^d(l) y_i(n-l) \quad (25)$$

In matrix form:

$$\mathbf{R}_{yy}^T \mathbf{g}_d = \mathbf{H}^T(:, d+1) \quad (26)$$

$$\text{and } \mathbf{H}^T(:, d+1) = \mathbf{R}_{xx,d}^T \mathbf{R}_{xx}^T \mathbf{H}^T(:,1) \quad (27)$$

$$\text{Thus: } \mathbf{g}_d = (\mathbf{R}_{yy}^T)^{\#} \mathbf{R}_{xx,d}^T \mathbf{R}_{xx}^T \mathbf{H}^T(:,1) \quad (28)$$

5 SIMULATIONS:

In this paper, we evaluated the performances of direct and indirect approaches for channel equalization through simulations. The channel order is $L_h=6$ and two antennas were used. Fig 4 depicts the NMSE of the equalizers versus the SNR which ranges from 15dB to 35dB for a number of data $P=500$. Fig 5 depicts the NMSE versus the number of data P which ranges from 500 to 2500 for $\text{SNR}=25\text{dB}$. Fig 6 and 7 depict a comparison between ZF and MMSE. For all simulations, a $T=200$ Monte Carlo runs was considered.

We can deduce that direct methods provide better results than indirect methods for the ZF and MMSE algorithms.

6 CONCLUSION:

In this paper, we have treated two classes of channel equalization: Direct and Indirect approaches. In Indirect algorithms, we have used Subspace algorithms of [4] to estimate the channel, we coped then to two different algorithms ZF and MMSE usually used in indirect approaches, we used their direct versions and compare their

performances through simulations. From simulations, Direct methods provide good results than the indirect ones, and the MMSE provide better results than ZF in the Direct and Indirect algorithms.

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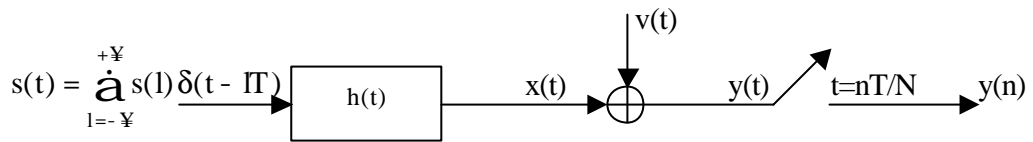


Fig. 1. Fractionally sampled communication system.

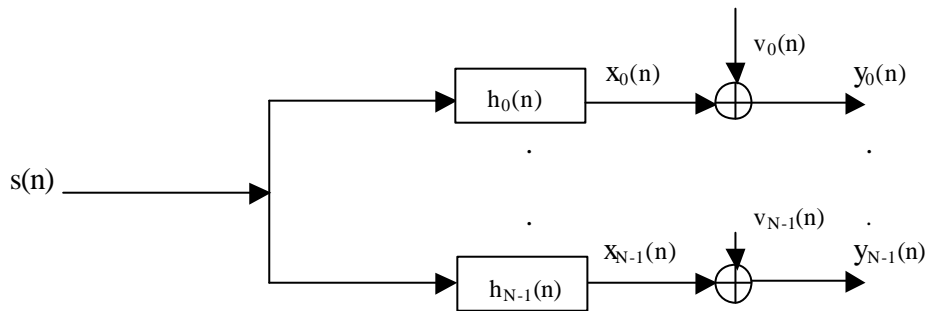


Fig.2 . Single Input Multi Output (SIMO) channel model.

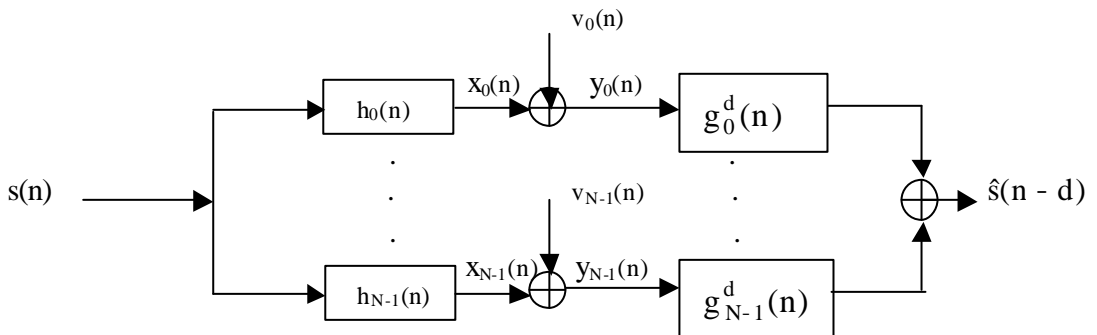


Fig.3 .Equalization of a SIMO channel.

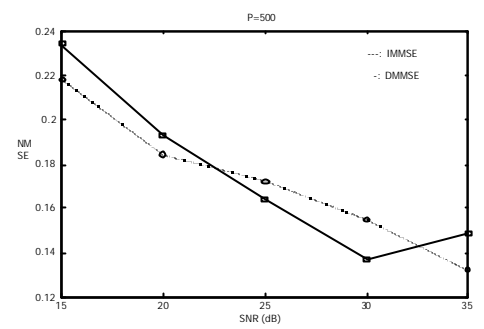
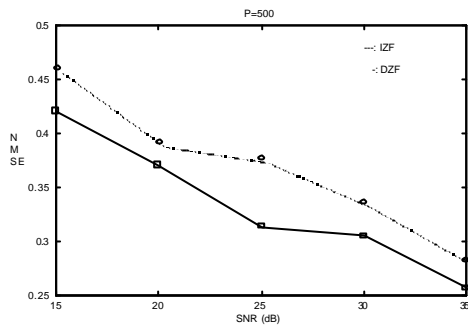


Fig.4. Comparison between IZF and DZF (4-a)- IMMSE and DMMSE (4-b) NMSE versus SNR (for P=500 and T=200 Monte Carlo runs).

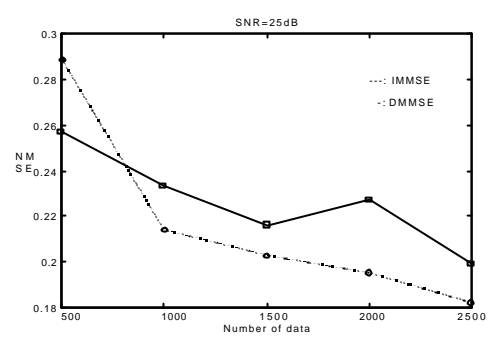
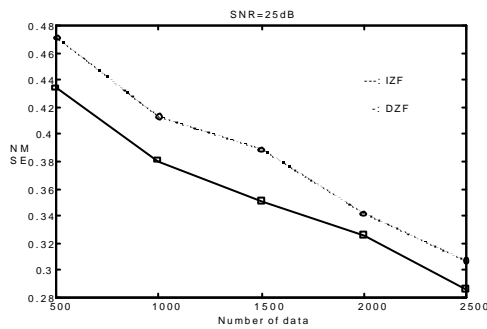


Fig.5. Comparison between IZF and DZF (5-a)- IMMSE and DMMSE (5-b) NMSE versus number of data (for SNR=25dB and T=200 Monte Carlo runs).

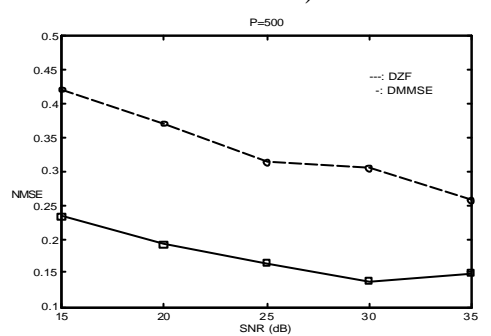
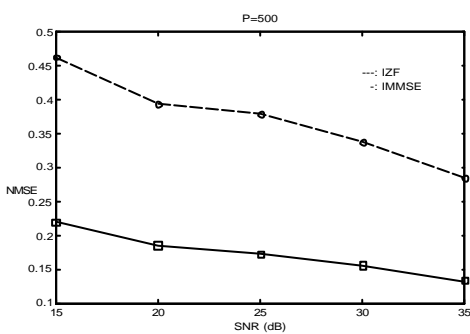


Fig.6. Comparison between IZF and IMMSE (6-a)- DZF and DMMSE (6-b) NMSE versus SNR (for P=500 and T=200 Monte Carlo runs).

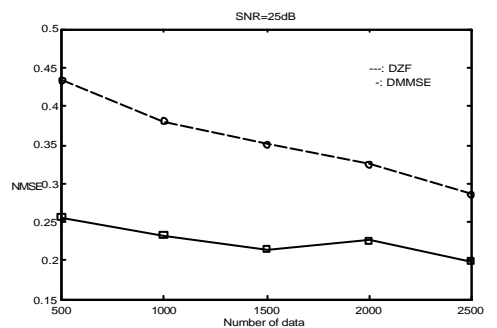
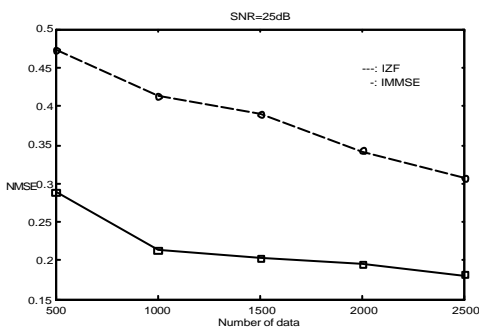


Fig.7. Comparison between IZF and IMMSE (7-a)- DZF and DMMSE (7-b) NMSE versus number of data (for SNR=25dB and T=200 Monte Carlo runs).