On the Solvability for Parabolic Equations with one Space Variable

Martín López Morales
Department of Computer Science
Monterrey Institute of Technology.Campus México City. Mexico.
Calle del Puente 222.Ejidos de Huipulco.Tlalpan.14380
México D.F.. Mexico

Abstract :- In the present work we consider the higher order linear parabolic equation in a rectangle with initial and boundary conditions.We establish new a priori estimates for the solutions to this problem in general Hölder anisotropic norms, under the assumption that the coefficients and the independent term are continuous functions in the rectangle , they satisfy the general Hölder condition in the rectangle of exponent $a(l)$, with respect to the space variable only and they satisfy the general Hölder condition on the boundary of exponent $b(l)$, with respect to all variables . In this connection, however, we also obtain an estimate for the modulus of continuity with respect to the time of the higher derivatives with respect to x of the corresponding solutions.
On the basis of our new a priori estimates for the solution to this problem , we establish the corresponding theorem on the solvability in general Hölder anisotropic spaces.
We apply our results in the linear theory to establish the local solvability with respect to the time , in general Hölder anisotropic spaces, for the nonlinear parabolic equation , with the same type of initial and boundary conditions.

Key words: estimates, solvability, equations, parabolicity, solution, problem.

1.Introduction

In the present work we consider the higher order linear parabolic equation

$$Lu \equiv u_t + (-1)^m \sum_{r=0}^{2m} a_r(t,x) D_x^r \, u \; = \; f(t,x) \,, \qquad (1)$$

in the rectangle $\overline{R}_T = [0,T] \times [-\rho,\rho]$ with the initial condition

$$u \;|_{t=0} \; = \varphi_0(x), \qquad (2)$$

and the boundary conditions on $S_T = \{(t,x) \; ; \; 0 \le t \le T, \;\; x = \pm \rho\}$

$$b_p^{\pm}(D_x u) \;\big|_{x=\pm\rho} \;\equiv \sum_{q=0}^{m_p^{\pm}} b_{qp}^{\pm} D_x^q u \;\big|_{x=\pm\rho} \; = \varphi_p^{\pm}(t), \qquad (3)$$

where $0 \le t \le T$ $\quad p = 1,...,m$; $0 \leqslant m_p^{\pm} \leqslant 2m-1$, $b_{qp}^{\pm}$ – real numbers satisfying the complementary condition with respect to the operator Lu, $p = 1,...,m$; $q = 0,1,...,m_p^{\pm}$ (See [1]).
Here x is a point in $[-\rho,\rho]$, $t \in [0,T]$, $u_t = \dfrac{\partial u}{\partial t}$, $D_x^r \, u = \dfrac{\partial^r u}{\partial x^r}$, $r = 0,...,m$, $D_x^0 u \equiv u.$,
We establish new a priori estimates for the solutions to the problem (1), (2),(3) in general Hölder anisotropic norms, under the assumption that the coefficients $a_r(t,x)$ and the independent term $f(t,x)$are continuous functions in the rectangle $\overline{R}_T = [0,T] \times [-\rho,\rho]$, they

satisfy the general Hölder condition in the rectangle $\overline{R}_T = [0,T] \times [-\rho,\rho]$ of exponent $\beta(l), l > 0$ with respect to the space variable only and they satisfy the general Hölder condition on the boundary S_T of exponent $\alpha(l),\ l > 0$ with respect to the variables (t,x) (SeeTheorem 1).In this connection, however, we also obtain an estimate for the modulus of continuity with respect to the time t of the derivatives $D_x^r u,\ r = 0,...2m,$ but not u_t (SeeTheorem1).
Note that in the works [1]-[13] and in many others, the a priori estimates are obtained under the fulfillment of a (general) Hölder condition with respect to the totality of variables (t,x) on the coefficients of equation (1).
On the basis of new a priori estimates for the solutions to the problem (1), (2),(3) we establish the corresponding theorem on the solvability for this problem in general Hölder anisotropic spaces (SeeTheorem 2).We assume that the coefficients of Equation (1) satisfy the uniform parabolicity condition: for any $(t,x) \in \overline{R}_T = [0,T] \times [-\rho,\rho]$

$$(-1)^{m+1}\ a_{2m}(t,x) > \lambda\ \ ,\ .\ \lambda = const. > 0 \quad , \qquad (4)$$

We apply our results in the linear theory (for the problem (1),(2),(3)) to establish the local solvability with respect to the time t, in general Hölder anisotropic spaces, for the nonlinear parabolic equation

$$u_t = A(t,x,u,D_x u,...,D_x^{2m} u), \qquad (5)$$

in $\overline{R}_T = [0,T] \times [-\rho,\rho]$ with the initial condition (2) and the boundary conditions (3) (SeeTheorem 3).
In the present work, the equation (5) is linearized directly. No conditions are imposed here on the nature of the growth of the nonlinearity of the function $A(t,x,p^0,p^1...,p^{2m})$, where p^r-scalar, $0 \le r \le 2m,$which is defined for $(t,x) \in \overline{R}_T = [0,T] \times [-\rho,\rho]$ and any $p^r, .0 \le r \le 2m.$
The main assumption concerning to the function $A(t,x,p^0,p^1...,p^{2m})$is the paraboliicity condition: for any $(t,x) \in \overline{R}_T$ and $p^0,p^1...,p^{2m}$

$$(-1)^{m+1}\{A_{p^{2m}}(t,x,p^0,p^1...,p^{2m})\} > 0 \qquad (6)$$

We require less smoothness conditions from the functions $A(t,x,p^0,p^1...,p^{2m})$and $\varphi(x)$than in the works [8] , [10]and [11] (See theorem 3).
In almost all the work we suppose that in the equation (1),the function$f = f_1 + f_2$; the functions $a_r(t,x),\ 0 \le r \le 2m$ and $f_1$satisfy the general Hölder condition in $\overline{R}_T = [0,T] \times [-\rho,\rho]$ of exponent $\beta(l)\ , l > 0$ with respect to the space variable only and they satisfy the (complementary) condition general Hölder condition on the boundary S_T of exponent $\beta(l),\ l > 0$ with respect to the variables (t,x), f_2 satisfies the general Hölder condition in $\overline{R}_T = [0,T] \times [\rho,-\rho]$ of exponent $\alpha(l), l > 0$ with respect to the space variable only and it satisfies the (co0mplementary)general Hölder condition on the boundary S_T of exponent $\alpha(l),\ l > 0$ with respect to the variable (t,x) (See $[14]$)
All the coefficients and the independent terms are continuous in the rectangle $\overline{R}_T = [0,T] \times [-\rho,\rho]$ .Some close results have been established in [15] , [16], [17], [18], [19],[20] and [22].

2.Basic Notations. Auxiliary Propositions.

We shall say that the function $u(t,x)$ defined in the rectangle $\overline{R}_T = [0,T] \times [-\rho,\rho]$ satisfies the general Hölder condition of exponent $\beta(\ell),\ \ell > 0$ in $\overline{R}_T = [0,T] \times [-\rho,\rho]$ with respect to the space variables if there exists a constant $C > 0$ such that

$$|u(t,x) - u(t,y)| \le C|x-y|^{\beta(|x-y|)}, \quad (t,x),(t,y) \in \overline{R}_T.$$

The function $\beta(\ell)$ is defined and continuous in $0 < \ell < \infty$.Moreover it has the following properties:

Ia. $\beta(\ell) \to \sigma \in [0,1[$ if $\ell \to 0^+$ or $\ell \to +\infty$.

Ib. $\beta'(\ell)\ell\ln\ell \to 0$ if $\ell \to 0^+$ or $\ell \to +\infty$.

Ic. if $\sigma = 0$ then $\beta(\ell)\ln\ell \to -\infty$ for $\ell \to 0^+$ and $(\beta(\ell) + \beta'(\ell)\ell\ln\ell) > 0$ for $\ell \in]0,\ell_0[$, where ℓ_0 is sufficiently small number (we suppose that the derivative $\beta'(\ell)$ exists and it is a continuous function in $R_0 =]0,\ell_0[\cup]1/\ell_0,+\infty[)$.

Note that the condition Ic. introduces a new set of functions (the functions $u(t,x)$ that satisfy the general Hölder condition with respect to x only).In this new set of functions we will obtain the corresponding existence and uniqueness theorems for the solutions to the problems (1),(2),(3) and (5), (2),(3).

It follows from the conditions Ia, Ib, Ic that

IIa $\ell^{\beta(\ell)}$ is a monotonically increasing function for $\ell \in R_0$

IIb

$$\frac{(k\ell)^{\beta(k\ell)-\sigma}}{\ell^{\beta(\ell)-\sigma}} \to 1 \quad if \quad \ell \to 0^+ or\ \ell \to +\infty.$$

uniformly respect to k, $0 < a \le k \le b < \infty$.

We denote by Γ^i $i = 1,2,$the set of functions $\beta(\ell)$ for which

$$\Gamma^1\ell^{\beta(\ell)} \equiv \Gamma_{\beta(\ell)} \equiv \int_0^\ell t^{\beta(t)-1}dt < \infty \qquad ; \qquad \Gamma^2\ell^{\beta(\ell)} \equiv \Gamma_{\Gamma^1\ell^{\beta(\ell)}} < \infty$$

For the functions $\beta(\ell) \in \Gamma^1$ ($\beta(\ell) \in \Gamma^2$) we introduce the functions

$$B^1_{\beta(\ell)} \equiv B_{\beta(\ell)} = \frac{1}{\ln\ell}\ln\frac{\Gamma_{\beta(\ell)}}{\Gamma_{\beta(1)}}, \quad B^2_{\beta(\ell)} = B_{B_{\beta(\ell)}}.$$

The function $B^j_{\beta(\ell)}$ is a function of the type $\beta(\ell), j = 1,2$.

See the examples of functions of the type $\beta(\ell)$ in [21]

For the functions $u(t,x)$ defined and Hölder continuous (in the general sense) of exponent $\beta(\ell)$, $\ell > 0$ in the rectangle $\overline{R}_T = [0,T] \times [-\rho,\rho]$ with respect to the space variable, we introduce the following norms for $0 \leqslant t \leqslant T$

.

$$|u|^{[t]}_{0,0} = \sup_{0\le\ \tau \le t}|u|^{\tau}_{0,0}, \quad |u|^{[t]}_{0,\beta(\ell)} = \sup_{0\le\ \tau \le bt}|u|^{\tau}_{0,\beta(\ell)}, \qquad (7)$$

where

$$|u|^{t}_{0,0} = \sup_{x\in[-\rho,\rho]}|u(t,x)|, \quad |u|^{t}_{0,\beta(\ell)} = |u|^{t}_{0,0} + H^{t}_{\beta(\ell)}(u), \qquad (8)$$

$$H^{t}_{\beta(\ell)}(u) = \sup_{\substack{x,y\in[-\rho,\rho]\\ x\neq y}} \frac{|u(t,x)-u(t,y)|}{|x-y|^{\beta(|x-y|)}} \qquad (9)$$

For the functions $u(t,x)$ that have continuous derivatives with respect to x up to the order m $(m = 1,2,...)$ inclusively in the rectangle $\overline{R}_T = [0,T] \times [-\rho,\rho]$ and satisfying the general Hölder condition of exponent $\beta(\ell)$, $\ell > 0$, with respect to space variables in the rectangle $\overline{R}_T = [0,T] \times [-\rho,\rho]$ we define the norms

$$|u|^{\tau}_{m,\beta(\ell)} = \sum_{r=0}^{m}|D^r_x u|^{\tau}_{0,\beta(\ell)}\ ;\ |u|^{[t]}_{m,\beta(\ell)} = \sup_{0\le\tau\le\ t}|u|^{\tau}_{m,\beta(\ell)}\ ,\ m = 0,1,2,...,D^0_x u \equiv u \qquad (10)$$

We will denote by $C^{[t]}_{m,\beta(\ell)}(\overline{R}_t)$, $\quad m = 0,1,2,...$ the Banach space of functions $u(\tau,x)$ that are continuous in $\overline{R}_t = [0,t] \times [-\rho,\rho]$ together with all derivatives respect to x up to the order m, $m = 0,1,2,...$ inclusively and have a finite norms (10).

For the functions $u(t,x)$ that have continuous derivatives with respect to x up to the order m($m = 0,1,2,...)$ inclusively in the layer $\overline{\overline{\Pi}}_t = [0,t] \times E_1, 0 \le t \le T$, $E_1 =]-\infty, \infty[$ (or in the half layer $\overline{\overline{\Pi}}_t^+ = \{(\tau,x)\ ;\ 0 \le \tau \le t,\ x \ge 0\}, 0 \le t \le T$) and satisfying the general Hölder condition of exponent $\beta(\ell)$, $\ell > 0$, with respect to space variables in the layer $\overline{\overline{\Pi}}_t$ (or in the half layer $\overline{\overline{\Pi}}_t^+$) we define similarly the norms (7) ... (10) and the corresponding spaces $C^{[t]}_{m,\beta(\ell)}(\Pi_t)$ (or the spaces $C^{[t]}_{m,\beta(\ell)}(\overline{\overline{\Pi}}_t^+)$.

We define the parabolic distance between each two points $P = (\theta,x) \in \overline{R}_T,\ Q = (\tau,y) \in \overline{R}_T$, by the magnitude

$$d(P,Q) = [\ |\theta-\tau|^{\frac{1}{m}} + |x-y|^2\ ]^{1/2}. \qquad (11)$$

For the functions $u(t,x)$ that have continuous derivatives with respect to x up to the order m $(m = 0,1,2,...)$ inclusively in the rectangle $R_t =]0,t] \times]-\rho,\rho[$ and satisfying the general Hölder condition of exponent $\beta(\ell)$, $\ell > 0$, with respect to the variables (t,x) in the rectangle $R_t =]0,t] \times]-\rho,\rho[$ with we define the norms for $0 \leqslant t \leqslant T$

$0 \leqslant t \leqslant T$ with respect to the variables (t,x), we introduce the following norms:

$$|u|^{R_t}_{0,0} = \sup_{(\tau,x)\in R_t} |u(\tau,x)|,\ H^{R_T}_{\beta(\ell)}(u) = \sup_{\substack{P,Q\in R_t\\ P\neq Q}} \frac{u(P)-u(Q)}{[d(P,Q)]^{\beta(d(P,Q))}} \qquad (12)$$

$$|u|^{R_t}_{0,\beta(\ell)} = |u|^{R_t}_{0,0} + H^{R_t}_{\beta(\ell)}(u), \qquad (13)$$

$$|u|^{R_t}_{m,\beta(\ell)} = \sum_{r=0}^{m} |D^r_x u|^{R_t}_{0,\beta(\ell)} \qquad m = 0,1,2,..., \qquad D^0_x u \equiv u \qquad (14)$$

We will denote by $C_{m,\beta(\ell)}(R_t)$, $m = 0,1,2,...,0 \le t \le T$ the Banach space of functions $u(\tau,x)$ that are continuous in $R_t = [\ 0,t\ ] \times]-\rho,\rho[$ together with all derivatives respect to x up to the order m, $m = 0,1,2,...$inclusively and have a finite norm (14) With respect to the coefficients of the equation (1) we assume that $a_r(t,x) \in C^{[T]}_{0,\beta(\ell)}(\overline{R}_T)$, $0 \le r \le 2m$ and

$$\sum_{|k|=0}^{2m} |a_k|^{[T]}_{0,\beta(l)} = B < \infty, \qquad \sum_{|k|=0}^{2m} |a_k|^{S_T}_{0,\beta(l)} = C < \infty, \qquad (15)$$

For equation (5) we consider in addition to the parabolicity condition (6) that there exists a domain

$H_M = \{(t,x) \in \overline{R_T};\ |u| \le M, |p^r| \le M,\ 1 \le r \le 2m,\ M = const.\}$ in which the function $A(t,x,p^0,p^1,...,p^{2m})$, and its derivatives with respect to $p^0,\ p^1,...,\ p^{2m}$, up to the second order inclusively are continuous, satisfy the Lipschitz condition with respect to $p^0,\ p^1,...,\ p^{2m}$, the general Hölder condition of exponent $\beta(\ell)$ with respect to x and with the constant B_M.They also satisfy the general Hólder condition of exponent $\frac{\beta(\ell)}{2m}$ with respect to t on the boundary $x =^{+}_{-} \rho$, $0 \leqslant t \leqslant T$ with the constant B_M.Moreover

$$A(t,x,0,0,0,) \in C^{[t]}_{0,\alpha(\ell)}(\overline{R_T}) \quad , \qquad |A(t,x,0,0,0)|^{[T]}_{0,\alpha(\ell)} \le B_0 \qquad (16)$$

All the mentioned derivatives are bounded in H_M by the constant B_M.

3.Bounds for solutions to the general boundary problem for linear parabolic equations.

Now we shall consider the equation (1) in the rectangle $\overline{R}_T$ with the initial zero condition

$$u\ |_{t=0} = o \qquad (17)$$

and the boundary conditions (3)

We define

$$\|\varphi_p\|^{[0,t]}_{\beta_p(l)} = |\varphi_p|^{[0,\,t]}_{0,\frac{\beta(l)+2m-m_p}{2m}}, \qquad for\ 1 \leqslant m_p,$$
$$\|\varphi_p\|^{[0,t]}_{\beta_p(l)} = |D_t\varphi_p|^{[0,t]}_{0,\frac{\beta(l)}{2m}} + |\varphi_p|^{[0,t]}_{0,\frac{\beta(l)}{2m}},\ for\ m_p = 0. \qquad (18)$$

Let $\overset{+}{\beta_p^-}(l) = \frac{\beta(l) + 2m - \overset{+}{m_p^-}}{2m}$, $p = 1,...,m$ .The functions $\overset{+}{\beta_p^-}(l) = \frac{\beta(l) + 2m - \overset{+}{m_p^-}}{2m}$,$p = 1,...,m$, are functions of the type $\beta(l)$, where $\beta(\ell) \to \sigma \in [0,1[$ if $\ell \to 0^+$or $\ell \to +\infty$. Note that

$\beta_p(l) \to \overset{+}{\beta_p^-}(l) \to \frac{\sigma + 2m - \overset{+}{m_p^-}}{2m} \in]0,1[$ if $\ell \to 0^+$or $\ell \to +\infty$.

In all the work we suppose that

$$\left\| \overset{+}{\varphi_p^-} \right\|_{\overset{+}{\beta_p^-}(l)}^{[0,T]} < \infty, \qquad p = 1,...,m \tag{19}$$

Theorem1. Let $u(t,x) \in C^{[T]}_{2m,B^2_{\beta(\ell)}}(\overline{R}_T)$ be a solution to the problem (1), (17), (3) in the rectangle $\overline{R}_T$. Assume that $f_1 \in C^{[T]}_{0,\beta(\ell)}(\overline{R}_T)$,

$f_2 \in C^{[T]}_{0,\alpha(\ell)}(\overline{R}_T)$, $\beta(\ell)$, $\alpha(\ell) \in \Gamma^2$, $\beta(\ell) \to \sigma_1$, $\alpha(\ell) \to \sigma_2$ if $\ell \to 0^+$ or $\ell \to +\infty$, $0 \le \sigma_1 < \sigma_2 < 1$,

$|f_1|^{S_T}_{0,\beta(l)} < \infty$, $|f_2|^{S_T}_{0,\beta(l)} < \infty$.

Furthermore the conditions (4) , (15)and (19)hold. Then there exists a constant K, depending only on $n,m,\lambda,B,C,T,\alpha(\ell),\beta(\ell)$, $B_{\beta(\ell)}$ and on the constants of the complementary condition in S_T and in $t = 0$.such that for $0 \le t \le T$ the next estimate holds

$$|u|^{\overline{R}_t}_{2m,B^2_{\beta(\ell)}} \le K \, [\left| f_1 \right|^{[t]}_{0,\beta(\ell)} + \left| f_1 \right|^{S_t}_{0,\beta(\ell)} + t^{\frac{\sigma_2-\sigma_1}{2m}} \left(|f_2|^{[t]}_{0,\alpha(\ell)} + |f_2|^{S_t}_{0,\alpha(\ell)}\right) +$$

$$+ \sum_{p=1}^{m} \left(\| \varphi_p^+ \|^{[0,t]}_{0,\, \beta_p^-(l)} + \| \varphi_p^- \|^{[0,t]}_{0,\, \beta_p^+(l)} \right)]. \tag{20}$$

Proof. Let $x_o \in [-\rho,\rho]$ and $d \in]0,1[$. We introduce the function $\eta(x) = \mu(|x - x_0|)$, where $\mu(l)$ is a decreasing infinitely differentiable function on $l > 0$, $\mu(l) = 1$ for $0 \le l \le d$; $\mu(l) = 0$ for $l \ge 2d$ and satisfying the inequality

$$\sum_{s=1}^{2m+1} d^s|\mu^{(s)}(l)| \le K \tag{21}$$

The function $\omega(t,x) = \eta(x)u(t,x)$ satisfies the equation

$$L^1_0\omega \equiv \omega_t + (-1)^m a_{2m}(t,x_0)D^{2m}_x\omega = [a_{2m}(t,x) - a_{2m}(t,x_0)]D^{2m}_x\omega + \hat{f}_1(t,x) + \eta(x)f_2(t,x) \tag{22}$$

in the rectangle $\overline{R}_T$, where

$$\hat{f}_1(t,x) = \left[(-1)^{m+1} \sum_{r=0}^{2m-1} a_r(t,x)D^r_x u + f_1(t,x) \right] \eta(x) + (-1)^m a_{2m}(t,x) \left[\sum_{r=0}^{2m-1} C^{2m}_r \, \eta^{(2m-r)} D^r_x u \right],$$

C^{2m}_r –binomial coefficients

The function $\omega(t,x) = \eta(x)u(t,x)$ $\left(\omega \in C^{[T]}_{2m,B^2_{\beta(\ell)}}(\overline{R}_T)\right)$satisfies the initial zero condition (17) and the boundary conditions (for $x_0 = -\rho$)

$$\overset{+}{b_p^-}(D_x\omega) \; |_{x=\overset{+}{-}\rho} \equiv \sum_{q=0}^{\overset{+}{m_p^-}} \overset{+}{b_{qp}^-} D^q_x\omega \; |_{x=\overset{+}{-}\rho} = \overset{+}{\varphi_p^-}(t) \; ; \quad p = 1,...,m \; ; \; 0 \leqslant \overset{+}{m_p^-} \leqslant 2m - 1 \tag{23}$$

We next find a bound for the solution $\omega(t,x) = \eta(x)u(t,x)$ to the problem (22) ,(17), (23) in $[-\rho,-\rho + 2d]$.

The mapping $y = x - (-p)$ $(y = x + \rho$), $x \in [-\rho,\rho]$ transforms the equation (22) into the equation

$$L^1_0\overline{\omega} \equiv \overline{\omega}_t + (-1)^m\overline{a}_{2m}(t,0)D^{2m}_y\overline{\omega} = [\overline{a}_{2m}(t,y) - \overline{a}_{2m}(t,0)]D^{2m}_y\overline{\omega} + \overline{f_1}(t,y) + \overline{\eta}(y)\overline{f}_2(t,y) \tag{24}$$

where

$\overline{\varpi}(t,y) = \overline{\eta}(y)\overline{u}(t,y) \equiv \eta(y-\rho)u(t,y-\rho), \quad y_0 = x_0 + \rho = 0 \ , \overline{a}_{2m}(t,y_0) = a_{2m}(t,0\ ),$
$\overline{f}_1(t,y) = \hat{f}_1(t,y-\rho); \ \overline{f}_2(t,y) = f_2(t,y-\rho);$
The function $\overline{\varpi}(t,y)$ satisfies the equation (24) in the rectangle $\overline{R}^+_T = [0,T] \times [0,2\rho]$, the initial zero condition

$$\varpi \ |_{t=0} \ = o \qquad (25)$$

and the boundary conditions

$$b^-_p(D_x\overline{\varpi}) \ |_{y=0} \ \equiv \sum_{q=0}^{m^-_p} b^-_{qp}D^q_x\overline{\varpi} \ |_{y=0} \ = \varphi^-_p \ (t) \ , \quad p = 1,...,m \qquad ;0 \ \leqslant m^-_p \leqslant 2m-1 \qquad (26)$$

We next find a bound for the solution $\overline{\varpi}(t,y) = \overline{\eta}(y)\overline{u}(t,y)$ to the problem (24) ,(25), (26) in the rectangle $\widetilde{R}_{t,2d} = [0,t] \times [0,2d+\rho], 0 \le t \le T.$
The function $\overline{\varpi}(t,y) \in C^{[T]}_{2m,B^2_{\beta(\ell)}}(\Pi^+_T)$ satisfies the equation (24) in the half layer $\Pi^+_T = \{(t,y) \ ; \ 0 \leqslant t \leqslant T, \ y \geqslant 0\}$ (observe that the functions $\overline{\eta}(y)$ and $\overline{\varpi}(t,y)$ vanishes for $|y| \ge 2d, \quad 2d < \rho.$),the initial zero condition (25) and the boundary conditions (26).
We can represent the solution to the problem (25),(26),(27) in the form

$$\overline{\varpi}(t,y) \ = u^1 + u^2 \qquad (27)$$

$$u^1(t,z) = \int_0^t \int_{E_1} G_{x_0}(t,\tau;z-\xi) \ \sum_{j=1}^{2} \ \overline{F}^*_j(\tau,\xi) \ d\xi d\tau \qquad (28)$$

where
$G_{x_0}(t,\tau;y-\xi)$ is the fundamental solution to the operator
L^1_0 , $\overline{F}_1(\tau,\xi) = [\overline{a}_{2m}(t,\xi) - \overline{a}_{2m}(t,0)]D^{2m}_y\overline{\varpi} + \overline{f_1}(t,\xi)$, $\overline{F}_2(\tau,\xi) = \overline{\eta}(\xi)\overline{f}_2(t,\xi)$ and

$$\overline{F}^*_j(\tau,\xi) = \left\{ \begin{array}{ll} \overline{F}_j(\tau,\xi) \quad , & \xi \ge 0 \\ \overline{F}_j(\tau,-\xi) \quad , & \xi < 0 \end{array} \right\}, \quad (j = 1,2)$$

The function $u^2 = \overline{\varpi}(t,y) - \ u^1$ is a solution to the problem

$$L^1_0(u^2) = 0 \qquad (29)$$

$$u^2 \ |_{t=0} \ = 0 \qquad (30)$$

$$b_p(D_y)u^2 \ |_{y=0} \ = \psi_p(t) \qquad (31)$$

Where

$$\psi_p(t) \equiv \left[\overline{\eta}(0)\varphi^-_p \ (t) \ - b_p(D_x)u^1(t,0)\right], \quad p = 1,...,m$$

The function $u^1(t,z) \in C^{[T]}_{2m,B^2_{\beta(\ell)}}(\overline{\overline{\Pi}}_T)\}$ is a solution to the Cauchy problem for the equation

$$L^1_0u^1 \equiv u^1_t + (-1)^m\overline{a}_{2m}(t,0)D^{2m}_yu^1 = [\overline{a}_{2m}(t,z) - \overline{a}_{2m}(t,0)]D^{2m}_zu^1 + $$
$$+\overline{f_1}(t,z) + \overline{\eta}(z)\overline{f}_2(t,z) \qquad (32)$$

with the initial zero condition (17) in the layer $\overline{\overline{\Pi}}_T = [0,T] \times E_1 \ , E_1 = \left]-\infty \ , \ \infty\right[.$
Reasoning as in the proof of theorem 1 in [19] , choosing $0 < d \ = d_1 < 1$ small enough (see [19]), such that $d^{B^2_{\beta(d)}} < \frac{1}{2K}d$ and $d < \frac{\rho}{2}$ we obtain that the solution $u^1(t,z)$ to the Cauchy problem (32),(17) satisfies the next inequality for $0 \le t \le T.(\overline{\varpi}(t,y) \in C^{[T]}_{2m,B^2_{\beta(\ell)}}(\Pi^+_T))$

$$|u^1|^{[t]}_{2m,B^2_{\beta(\ell)}} \le K \ [d_1^{-(2m+1)}|\overline{u}|^{[t],2d_1}_{2m-1,B^2_{\beta(\ell)}} + d_1^{-(2m-1)}\left(\left|\overline{f}_1\right|^{[t],2d_1}_{0,B_{\beta(\ell)}} + t^{\frac{\sigma_2-\sigma_1}{2m}}|\overline{f}_2|^{[t],2d_1}_{0,B_{\alpha(\ell)}}\right)]. \qquad (33)$$

where the symbol $[t], 2d_1$ means that the norms in the right hand side of (33) are consider in the rectangle $[0,t] \times [0,2d_1]$.
Now we shall consider the solution $u^2(t,x)$ to the problem (29) - (31).At first we next find a

bound for the norm $\|\psi_p(t)\|^{[0,t]}_{B^2\beta_p(l)}$, $p = 1,...,m$; $0 \le t \le T$. Reasoning as in the proof of theorem 3 in [19], we find the next estimate for $0 \le t \le T$

$$\|\psi_p\|^{[0,t]}_{B_{\beta p(l)}} < K\{[d_1^{-(2m+1)}|\overline{u}|^{[t],2d_1}_{2m-1,B^2_{\beta(l)}} + t^{\frac{\sigma_2-\sigma_1}{2m}}\left(d_1^{-(2m-1)}|\overline{f}_2|^{[t],2d_1}_{0,B_{\alpha(l)}} + \left|\tilde{f}_2\right|^{[t]}_{0,\left(\frac{B_{\alpha(l)}}{2M}\right)}\right) +$$

$$+ d_1^{-(2m-1)}\left|\overline{f}_1\right|^{[t],2d_1}_{0,B_{\beta(l)}} + \left|\tilde{f}_1\right|^{[t]}_{0,\left(\frac{B_{\beta(l)}}{2M}\right)}$$

$$+ \sum_{r=0}^{2m-1}|D_y^r\overline{u}(t,0)|^{[0,t]}_{0,\frac{B^2_{\beta(l)}}{2m}}] + \|\varphi_p^-\|^{[0,t]}_{B_{\beta p(l)}}\}, \quad p = 1,...,m \qquad (34)$$

where $\tilde{f}_j(t) = \overline{f}_j(t,0)$, $j = 1,2$

Now we consider the equation

$$L_0(v) \equiv v_t + (-1)^m D_y^{2m} v = 0, \qquad (35)$$

The functions

$$v_p(t,y) = \int_0^t G_p(t-\tau\ ;y)\psi_p(\tau)d\tau, \qquad p = 1,...,m \qquad (36)$$

where

$G_p(t-\tau\ ;y)$ - component of the Green function to the problem (35), (30),(31).

We can represent the solution $v(t,y) \in C_{2m,B^2_{\beta(l)}}(\overline{\Pi}^+_T)$, $\overline{\Pi}^+_T = \{(t,x)\ ;\ 0 \leqslant t \leqslant T,\ x \geqslant 0\}$ to the problem (35),(30),(31) in the form (See [8],[10])

$$v(t,y) = \sum_{p=1}^{m} v_p(t,y) \qquad (37)$$

and the next inequality holds

$$|v_p|^{[t]}_{2m\ ,\ B^2_{\beta(l)}} \leqslant K\ \|\psi_p\|^{[0,t]}_{B_{\beta p(l)}}\ , \qquad p = 1,...,m \qquad (38)$$

K is a constant depending on $n,m,\lambda,B,T,C,\beta(l),B^2_{\beta(l)}$ and on the constants of the complementary condition in $S_T = \{(t,x)\ ;\ 0 \leqslant t \leqslant T, x = \pm\ \rho\}$ and in $t = 0$.

We can represent the solution $u^2(t,y)$ to the problem (29) - (31) in the form (37) and applying the inequalities (38) and (34) ,we obtain

the next estimate (by going back to the variable t) for $0 \le t \le T$.

$$|u^2|^{[t]}_{2m\ ,\ B^2_{\beta(l)}} \le K\{[d_1^{-(2m+1)}|\overline{u}|^{[t],2d_1}_{2m-1,B^2_{\beta(l)}} + t^{\frac{\sigma_2-\sigma_1}{2m}}\left(d_1^{-(2m-1)}|\overline{f}_2|^{[t],2d_1}_{0,B_{\alpha(l)}} + \left|\tilde{f}_2\right|^{[t]}_{0,\left(\frac{B_{\alpha(l)}}{2M}\right)}\right) +$$

$$+d_1^{-(2m-1)}\left|\overline{f}_1\right|^{[t],2d_1}_{0,B_{\beta(l)}} + \left|\tilde{f}_1\right|^{[t]}_{0,\left(\frac{B_{\beta(l)}}{2M}\right)} + \sum_{r=0}^{2m-1}|D_y^r\overline{u}(t,0)|^{[0,t]}_{0,\frac{B^2_{\beta(l)}}{2m}}] + \sum_{p=1}^{m}\|\varphi_p^-\|^{[0,t]}_{B^-_{\beta p(l)}}\}, \qquad (39)$$

Combining the inequalities (33) and (39) we can get from (27) the estimate ($0 \le t \le T$)

$$|\varpi|^{[t]}_{2m\ ,\ B^2_{\beta(l)}} \le K\{[d_1^{-(2m+1)}|\overline{u}|^{[t],2d_1}_{2m-1,B^2_{\beta(l)}} + \mathrm{t}^{\frac{\sigma_2-\sigma_1}{2m}}\left(d_1^{-(2m-1)}|\overline{f}_2|^{[t],2d_1}_{0,B_{\alpha(l)}} + \left|\widetilde{f}_2\right|^{[t]}_{0,\left(\frac{B_{\alpha(l)}}{2M}\right)} \right) +$$

$$+d_1^{-(2m-1)}\left|\overline{f}_1\right|^{[t],2d}_{0,B_{\beta(l)}} + \left|\widetilde{f}_1\right|^{[t]}_{0,\left(\frac{B_{\beta(l)}}{2M}\right)} + \sum_{r=0}^{2m-1}|D_y^r\overline{u}(t,0)|^{[0,t]}_{0,\frac{B^2_{\beta(l)}}{2m}}] + \sum_{p=1}^{m}\|\varphi_p^-\|^{[0,t]}_{B^-_{\beta p(l)}}\} \qquad (40)$$

By other hand from the estimates of the moduli of continuity with respect to the time for the derivatives of the solution to the Cauchy problem (32), (17) in the layer $\overline{\overline{\Pi}}_T = [0,T] \times E_1$, $E_1 =]-\infty , \infty[$ (See theorem 3 in [19]) and from the estimates of the moduli of continuity with respect to the time for the derivatives $D_y^{2m-j}u^2(t,y),\ j = 0,1,...,2m-1$, $y \ge 0$, $0 \le t_2 < t_1 \le T$ of the solution $u^2(t,y)$ to the problem (29), (30) , (31) in the half layer $\overline{\Pi}_T^+ = \{(t,y)\ ;\ 0 \leqslant t \leqslant T,\ y \geqslant 0\}$ (See [10]) and from the estimates (34),(37), (39) we can get from (27) the estimate

$$\frac{\left|D_y^{2m-j}\overline{\varpi}(t_1,y)\ - D_y^{2m-j}\overline{\varpi}(t_2,y)\right|}{\left(|t_1-t_2|^{\frac{1}{2m}}\right)^{B^2_{\beta\left(|t_1-t_2|^{\frac{1}{2m}}\right)}+J}} \le K\{[d_1^{-(2m+1)}|\overline{u}|^{[t_1],2d_1}_{2m-1,B^2_{\beta(l)}} +$$

$$+\mathrm{t}_1^{\frac{\sigma_2-\sigma_1}{2m}}(d_1^{-(2m-1)}|\overline{f}_2|^{[t_1],2d_1}_{0,B_{\alpha(l)}} + \left|\widetilde{f}_2\right|^{[t_1]}_{0,\left(\frac{B_{\alpha(l)}}{2M}\right)})$$

$$+d_1^{-(2m-1)}\left|\overline{f}_1\right|^{[t_1],2d_1}_{0,B_{\beta(l)}} + \left|\widetilde{f}_1\right|^{[t_1]}_{0,\left(\frac{B_{\beta(l)}}{2M}\right)} +$$

$$+\sum_{r=0}^{2m-1}|D_y^r\overline{u}(t_1,0)|^{[0,t_1]}_{0,\frac{B^2_{\beta(l)}}{2m}}] + \sum_{p=1}^{m}\|\varphi_p^-\|^{[0,t_1]}_{B^-_{\beta p(l)}}\}, \qquad (41)$$

Choosing $o < d = d_0$ small enough (See [19]), such that $d^{B^2_{\beta(d)}} < \frac{1}{2K}d$ and $d = \frac{1}{2}\min\{1,\frac{\rho}{2}\}$ we find that (by going back to the space variable x)

$$|u|^{R_{t,d_0}}_{2m\ ,\ B^2_{\beta(l)}} \le |\omega|^{R_{t,d_0}}_{2m\ ,\ B^2_{\beta(l)}} \le K\{|u|^{\overline{R}_t}_{2m-1,B^2_{\beta(l)}} + \mathrm{t}^{\frac{\sigma_2-\sigma_1}{2m}}\left(|f_2|^{[t],2d_0}_{0,B_{\alpha(l)}} + \left|\ f_2\right|^{S_t}_{0,\frac{B_{\alpha(l)}}{2m}}\right) + \left|\ f_1\right|^{S_t}_{0,\frac{B_{\beta(l)}}{2m}} +$$

$$+\left|\ f_1\right|^{[t],2d_0}_{0,B_{\beta(l)}}. + \sum_{p=1}^{m}\|\varphi_p^-\|^{[0,t]}_{B^+_{\beta p(l)}}\} \qquad (42)$$

For $x_0 = \rho$ we obtain similarly the next estimate

$$|u|^{R_{t,d_0}}_{2m\ ,\ B^2_{\beta(l)}} \le K\{|u|^{\overline{R}_t}_{2m-1,B^2_{\beta(l)}} + \mathrm{t}^{\frac{\sigma_2-\sigma_1}{2m}}\left(|f_2|^{[t],2d_0}_{0,B_{\alpha(l)}} + \left|\ f_2\right|^{S_t}_{0,\frac{B_{\alpha(l)}}{2m}}\right) + \left|\ f_1\right|^{S_t}_{0,\frac{B_{\beta(l)}}{2m}} +$$

$$+|f_1|^{[t],2d_0}_{0,B_{\beta(l)}}. + \sum_{p=1}^{m}\|\varphi_p^+\|^{[0,t]}_{B^+_{\beta p(l)}}\} \qquad (43)$$

where the norms in the right hand side are given in the rectangle $[0,t] \times [\rho - 2d_0, \rho]$.

For $x_0 \in \mathbf{I}_\rho^{\frac{d_0}{2}} \equiv \left[-\rho + \frac{d_0}{2}, \rho - \frac{d_0}{2}\right]$ we derive similarly the next interior estimate in the rectangle $\mathbf{R}^\rho_{t,d_0} = [0,t] \times [-\rho + d_0, \rho - d_0]$, $0 \le t \le T$

$$|u|^{R^\rho_{t,d_0}}_{2m\ ,\ B^2_{\beta(l)}} \le K\{\ \left|\ f_1 b\right|^{[t],\mathbf{I}_\rho^{\frac{d_0}{2}}}_{0,B_{\beta(l)}} + \mathrm{t}^{\frac{\sigma_2-\sigma_1}{2m}}|f_2|^{[t],\mathbf{I}_\rho^{\frac{d_0}{2}}}_{0,B_{\alpha(l)}}b. + |u|^{[t],\mathbf{I}_\rho^{\frac{d_0}{2}}}_{2m-1,B^2_{\beta(l)}}\ \} \qquad (44)$$

where the symbol $[t],\mathbf{I}_\rho^{\frac{d_0}{2}}$ means that the norms in the right hand side of (44) are considered in

the rectangle $[0,t]\times\left[-\rho+\frac{d_0}{2},\rho-\frac{d_0}{2}\right]$.
Now we can get from (42)-(44) the following estimate in the rectangle $\overline{\mathbf{R}}_t, 0\leq t\leq T$:

$$|u|^{\overline{\mathbf{R}}_t}_{2m\,,\,B^2_{\beta(l)}}\leq K\{\left|f_1\right|^{S_t}_{0,\frac{B_{\beta(l)}}{2m}}+\left|f_1\right|^{[t]}_{0,B_{\beta(l)}}.+|u|^{\overline{R}_t}_{2m-1,B^2_{\beta(l)}}+\mathrm{t}^{\frac{\sigma_2-\sigma_1}{2m}}\left(|f_2|^{[t]}_{0,B_{\alpha(l)}}+\left|f_2\right|^{S_t}_{0,\frac{B_{\alpha(l)}}{2m}}\right)+$$

$$+\sum_{p=1}^{m}\left(\|\varphi_p^+\|^{[0,t]}_{0,\,\beta_p^+(l)}+\|\varphi_p^-\|^{[0,t]}_{0,\,\beta_p^-(l)}\right)\}$$

Applying the interpolation inequality (See [19]) with ε small enough and arguing as in the last part of the proof of theorem 1 in [16] to eliminate $|u|^{\overline{R}_t}_{0,0}$ we get the estimate (21)
Remark 1 We can reduce the boundary problem(1), (2) ,(18) with non-zero initial condition $u\mid_{t=0}=\varphi_0(x)$ to the boundary value problem (1), (17),(18) by means of the transformation $\overline{u}=u(t,x)-\varphi_0(x)$,where $\varphi_0(x)\in C_{2m,\beta(l)}([-\rho,\rho])$.

4.Existence and uniqueness theorems.

Theorem 2. Suppose that all conditions of theorem 1 are true.Assume furthermore that the following consistency conditions hold

$$\sum_{q=0}^{\overset{+}{m}_{\bar{p}}}\overset{+}{b}_{\bar{q}p}D_x^q\varphi_0\mid_{x=\overset{+}{-}\rho}=\overset{+}{\varphi}_{\bar{p}}(0),\text{if } \overset{+}{m}_{\bar{p}}\geqslant 1,\ p=1,...,m\ ;\ \overset{+}{b}_{\bar{q}p}\varphi_0(x)\mid_{x=\overset{+}{-}\rho}=\overset{+}{\varphi}_{\bar{p}}(0),\text{if } \overset{+}{m}_{\bar{p}}=0.$$

$$\overset{+}{b}_{\bar{0}p}\left[f_1(0,x)+f_2(0,x)-(-1)^m\sum_{r=0}^{m}a_r(0,x)D_x^r\varphi_0(x)\right]\mid_{x=\overset{+}{-}\rho}=\left(\overset{+}{\varphi}_{\bar{p}}\right)(0).$$

Then there exists a unique solution $u(t,x)\in C^{[T]}_{2m,B^2_{\beta(l)}}(\overline{R_T})$ to the boundary problem (1),(2),(3) with continuous derivatives u_t in $\overline{R_T}$.
We can get the proof of this theorem on the basis of the new a priori estimates established in this work and with the aid of the method of continuity in a parameter (see [9] and [24|]).
We proceed now to formulate the local existence theorem for solutions to the non- linear boundary value problems for the equation (5).
(with the initial zero condition (17) and the boundary conditions (3))
Let $\overset{+}{\beta}_{\bar{p}}(l)=\frac{\beta(l)+2m-\overset{+}{m}_{\bar{p}}}{2m}$.We will suppose that

$$\left\|\overset{+}{\varphi}_{\bar{p}}\right\|^{[0,T]}_{\overset{+}{\beta}_{\bar{p}}(l)}<\infty \tag{45}$$

Here we consider that the function
$A(t,x,u,D_xu,...,D_x^{2m}u)=L(u)+F(t,x,u,D_xu,...,D_x^{2m}u)+L(u)+A(t,x,0,0,...,0)$
where
$L(u)=A_{P^0}(t,x,0,...,0)u+A_{P^1}(t,x,0,...,0)D_xu+...+A_{P^{2m}}(t,x,0,...,0)D_x^{2m}u,$
$F(t,x,u,D_xu,...,D_x^{2m}u)=A(t,x,u,D_xu,...,D_x^{2m}u)-L(u)-A(t,x,0,0,...,0)$
Theorem 3. Suppose that all assumptions with respect to the function $A(t,x,p^0,p^1,...,p^{2m})$ hold. Moreover $0\leqslant\sigma_1<\sigma_2<1$ and the
following consistency conditions hold

$$\overset{+}{\varphi}_{\bar{p}}(0)=0\ ,\ \text{if}\quad \overset{+}{m}_{\bar{p}}\geqslant 1,\quad \overset{+}{\varphi}_{\bar{p}}(0)=0\ ,$$

$$\left(\overset{+}{\underset{-}{\varphi}}_p\right)'(0) = b_{0p}\ A(0.x,0,...,0)\ |_{x=\overset{+}{-}\rho}\ , \quad \text{if} \quad \overset{+}{\underset{-}{m}}_p = 0 \tag{46}$$

Then there exists $t_0 \in (0,T)$, determined by the above assumptions, such that the problem , (5), (17), (3) has in the rectangle
$\overline{R}_{t_0} = [0,t_0] \times [-\rho,\rho]$ a unique solution $u(t,x) \in C_{2m,B^2_{\beta(\ell)}}(\overline{R}_{t_0})$ with a continuous derivative u_t in $\overline{R}_{t_0} = [0,t_0] \times [-\rho,\rho]$.

Proof. We will prove Theorem 3 by means of an iterative process in which one successively solves the equations

$$u^\nu_t = L(u^\nu) + F(t,x,u^{\nu-1},D_x u^{\nu-1},...,D^{2m}_x u^{\nu-1}) + A(t,x,0,...,0)\ ,\ \nu = 1,2,..., \tag{47}$$

with the zero initial condition (17) and with the boundary conditions

$$\overset{+}{\underset{-}{b}}_p(D_x u)\ |_{x=\overset{+}{-}\rho} = \overset{+}{\underset{-}{\varphi}}_p(t), \quad p = 1,...,m \tag{48}$$

Furthermore

$$u^0(t,x) = 0 \tag{49}$$

We will show that there exists such a sequence of functions $u^\nu(t,x), \nu = 1,2,...,$defined in some rectangle $[0,t_0] \times [-\rho,\rho]$ with t_0 small enough. Moreover they are bounded in $C_{2m,B^2_{\beta(\ell)}}(\overline{R}_{t_0})$.
We assume that there exists the functions $u^\mu(t,x),\ \mu \le \nu - 1$ and they satisfy the inequality

$$|u^\mu|^{\overline{R}_{t_0}}_{2m,B_{\beta(L)}} \le q = const. = \min(1,M) \tag{50}$$

where the numbers t_0 and η are still to be determined.
Now we can consider the equation (47) as a linear equation of type (1), where
$f_1(t,x) = F(t,x,u^{\nu-1},D_x u^{\nu-1},...,D^{2m}_x u^{\nu-1}), \quad f_2(t,x) = A(t,x,0,...,0)$.
Reasoning as in the proof of theorem 1 ,using lemma1 and theorem 2, we conclude that there exists $u^\nu(t,x) \in C_{2m,B^2_{\beta(\ell)}}(\overline{R}_{t_0})$ and it satisfies the inequality

$$|u^\nu|^{\overline{R}_{t_0}}_{2m,B^2_{\beta(\ell)}} \le K_1\left[\ |u^{\nu-1}|^{[t_0]}_{2m,B^2_{\beta(\ell)}} + |u^{\nu-1}|^{S_{t_0}}_{2m,B^2_{\beta(\ell)}} + t_0^\gamma\ \right] \tag{51}$$

We will use$K_1, K_2, K_{3,...,}$ to denote constants, depending on $n,m,\lambda,\alpha(\ell),\beta(\ell),\ B^2_{\beta(\ell)}, C, ; B_0, B_M$ but not depending on t_0. From the inequality (55) it follows the estimates

$$|u^\nu|^{\overline{R}_{t_0}}_{2m,B^2_{\beta(\ell)}} \le 2K_1\left[\ q^2 + t_0^r\ \right]$$

$$\omega_\nu \equiv |u^\nu - u^{\nu-1}|^{\overline{R}_{t_0}}_{2m,B^2_{\beta(l)}} \le K_2\left(|u^{\nu-1}|^{[t_0]}_{2m,B^2_{\beta(l)}} + |u^{\nu-2}|^{[t_0]}_{2m,B^2_{\beta(l)}}\right) \bullet |u^{\nu-1} - u^{\nu-2}|^{[t_0]}_{2m,B^2_{\beta(l)}} + \\ +\left(|u^{\nu-1}|^{S_{t_0}}_{2m,B^2_{\beta(l)}} + |u^{\nu-2}|^{S_{t_0}}_{2m,B^2_{\beta(l)}}\right) \bullet |u^{\nu-1} - u^{\nu-2}|^{S_{t_0}}_{2m,B^2_{\beta(l)}} \tag{52}$$

Putting $\quad 2K_1 q \le \frac{1}{2}, \quad 2K_1 t_0^\gamma = \frac{1}{2}q$ we find that $|u^\nu|^{\overline{R}_{t_0}}_{2m,B^2_{\beta(\ell)}} \le \ \frac{1}{2}q + \frac{1}{2}q = q.$

In the last inequalities we have selected

$$q = \min\left(\tfrac{1}{4K_1}, \tfrac{1}{8K_2}, 1, M\right) \quad , \ t_0 = \left(\tfrac{q}{4K_1}\right)^{\frac{1}{\gamma}} \tag{53}$$

From (50) and (53) it follows that $\ \omega_\nu \leqslant 4K_2 q\omega_{\nu-1}$, then

$\omega_\nu \le \frac{1}{2}\omega_{\nu-1}\ \ ,\quad \omega_\nu \le \frac{1}{2^{\nu-1}}\omega_1 = \frac{1}{2^{\nu-1}}|u^1|^{\overline{R}_{t_0}}_{2m,B^2_{\beta(l)}}$

We have determined the number t_0 by (53) , then the sequence $u^\nu(t,x)$ converges in the space $C_{2m,B^2_{\beta(l)}}(\overline{R}_{t_0})$.Consequently we can get a limit for $\nu \to \infty$ in the equation (47).

We proceed now to prove the uniqueness of the solution of the problem (5), (17), (3). Assume that there exists two solutions $u^1(t,x),\ u^2(t,x)$ of this problem in the corresponding rectangles $\overline{R}_{t_0}$ and consider the function $u = u^1(t,x) - u^2(t,x)$. This function satisfies the equation (in the smaller of the corresponding rectangles $\overline{R}_{t_0}$

$$u_t = Lu$$

$$Lu \equiv \int_0^1 A_u(t,x,\tau z^1 + (1-\tau)z^2)d\tau u + \left(\int_0^1 A_p(t,x,\tau z^1 + (1-\tau)z^2)d\tau, D_x u\right) + .$$
$$+ \left(\int_0^1 A_{p^{2m}}(t,x,\tau z^1 + (1-\tau)z^2)d\tau, D_x^{2m} u\right) \qquad (54)$$

Here the components of the vector function $z^r = (u^r, D_x u^r, ..., D_x^{2m} u^r), r = 1,2$ belong to the space $C_{0,B^2_{\beta(l)}}(\overline{R}_{t_0})$ and the coefficients of the operator L also belong to this space. Since $u(t,x)$ satisfies the zero initial condition (17) and the boundary conditions

$$\overset{+}{b_p^-}(D_x u)\ |_{x=\overset{+}{-}\rho} = 0, \quad p = 1,...,m \qquad (55)$$

it follows that $u(t,x) \equiv 0$ .This completes the proof of Theorem 3.

Remark 2. We can reduce the boundary problem (5) , (2), (3) with non-zero initial condition $u\ |_{t=0} = \varphi_0(x)$ to the boundary problem (5),(17),(3) with the zero initial condition $u\ |_{t=0} = 0$ by means of the transformation $\overline{u} = u(t,x) - \varphi_0(x)$, where .$\varphi(x) \in C_{2m,\beta(l)}(E_n)$.

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