A chaotic oscillator using the Van der Pol dynamic immersed into a Jerk system

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Abstract
In the present work a new chaotic oscillator is presented by immersing the dynamics of the Van der Pol oscillator into a Jerk system (a third order differential equation). The map of Poincaré was used to prove chaotic behavior.

Key-words
Chaotic oscillator, Van der Pol oscillator, Jerk systems.

I. INTRODUCTION
In the paper presented by J. C. Sprott in 2000 [1], Sprott shows many Jerk systems that exhibit chaotic behavior. A great part of these chaotic systems can be implemented using analog electronics ([1]). Also, a huge amount of chaotic oscillators have appeared recently (see, for instance, the references given in [1]). In this paper, we present a new chaotic oscillator obtained by immersing the dynamics of a slightly modified Van der Pol oscillator into a Jerk system. The chaotic test was done using the map of Poincaré (Banerjce, page 61, [3]).

II. JERK SYSTEMS AND CHAOTIC OSCILLATORS
A Jerk system is a third order differential equation (a nonlinear one) of the form [1]:

\[ \dddot{x} = f(\ddot{x}, \dot{x}, x) \] (1)

where \( x, \dot{x}, \ddot{x}, \) and \( \dddot{x} \) represent position, velocity, acceleration and Jerk (the time derivative of the acceleration), respectively.

For instance, the following Jerk system:

\[ \dddot{x} + \dddot{x} - \dot{x} + ax + b = 0 \] (2)

presents chaotic motion with \( a=0.9 \) and \( b=0.4 \) [1]. Its strange attractor is shown in Fig. 1.

![Fig. 1 Phase portrait using \( x(0) = \dot{x}(0) = \dddot{x}(0) = 0.1 \).](image)

The Van der Pol dynamics is given by [2]:

\[ \dddot{x} + \varepsilon(x^2 - 1)\dot{x} + x = 0 \] (3)

which is a second order differential equation. Its phase portrait is pictured in Fig. 2 where it is shown the presence of a limit cycle (a stable one).

![Fig. 2 Phase portrait using \( x(0) = \dot{x}(0) = 0.1 \) and \( \varepsilon=1 \).](image)
Motivated by the fact that a Jerk system can produce chaos, first, we slightly modify the dynamics of Van der Pol as follows:

$$ax^2 \text{sign}(\ddot{x}) + \epsilon(x^2 - 1)\dot{x} + x = 0$$

(4)

where $a$ is a constant; after that, we immerse (4) into a Jerk system:

$$\dddot{x} = f(\dot{x}, \ddot{x}, x)$$

$$= -[ax^2 \text{sign}(\ddot{x}) + \epsilon(x^2 - 1)\dot{x} + x]$$

(5)

Its phase portrait (of (5)), with $a=0.01$ and $\epsilon=7$, is shown in Figure 3 and Figure 4, where $x(0) = \dot{x}(0) = \ddot{x}(0) = 0.1$.

In this way, and roughly speaking, the Poincaré map is a mapping of the points in which the trajectory intersects the Poincaré section. If the system is periodic, the Poincaré map will contain a few points; however, if the trajectory is chaotic, then, the Poincaré map will contain a huge amount of points distributed over the Poincaré section producing something similar to a strange attractor [3]. In Fig. 6 the map of Poincaré is shown placing the Poincaré section in the plane $(x - \dot{x})$ in $\ddot{x} = 0$ of our system.

From Fig. 6 follows that the proposed systems is a chaotic Jerk system.

**References**

