A chaotic oscillator using the Van der Pol dynamic immersed into a Jerk system

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Abstract

In the present work a new chaotic oscillator is presented by immersing the dynamics of the Van der Pol oscillator into a Jerk system (a third order differential equation). The map of Poincaré was used to prove chaotic behavior.

Key-wods

Chaotic oscillator, Van der Pol oscillator, Jerk systems.

I. INTRODUCTION

In the paper presented by J. C. Sprott in 2000 [1], Sprott shows many Jerk systems that exhibit chaotic behavior. A great part of these chaotic systems can be implemented using analog electronics ([1]). Also, a huge amount of chaotic oscillators have appeared recently (see, for instance, the references given in [1]). In this paper, we present a new chaotic oscillator obtained by inmersing the dynamics of a slightly modified Van der Pol oscillator into a Jerk system. The chaotic test was done using the map of Poincaré (Banerjce, page 61, [3]).

II. JERK SYSTEMS AND CHAOTIC OSCILLATORS

A Jerk system is a third order differential equation (a nonlinear one) of the form [1]:

$$\dddot{x} = f(\ddot{x}, \dot{x}, x) \qquad (1)$$

where $x, \dot{x}, \ddot{x},$ and $\dddot{x}$ represent position, velocity, acceleration and Jerk (the time derivative of the acceleration), respectively. For instance, the following Jerk system:

$$\dddot{x} + \ddot{x} - x\dot{x} + ax + b = 0 \qquad (2)$$

presents chaotic motion with $a=0.9$ and $b=0.4$ [1]. Its strange attractor is shown in Fig. 1.

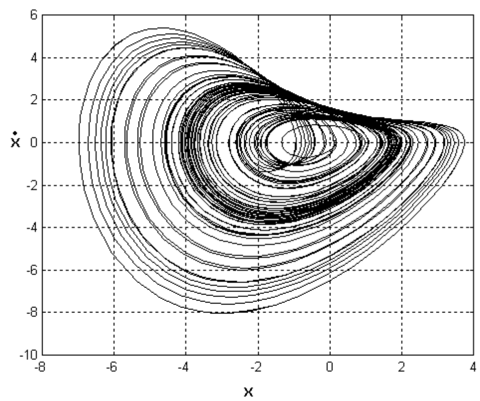

Fig. 1 Phase portrait using $x(0) = \dot{x}(0) = \ddot{x}(0) = 0.1$.

The Van der Pol dynamics is given by [2]:

$$\ddot{x} + \varepsilon(x^2 - 1)\dot{x} + x = 0 \qquad (3)$$

which is a second order differential equation. Its phase portrait is pictured in Fig. 2 where it is shown the presence of a limit cycle (a stable one).

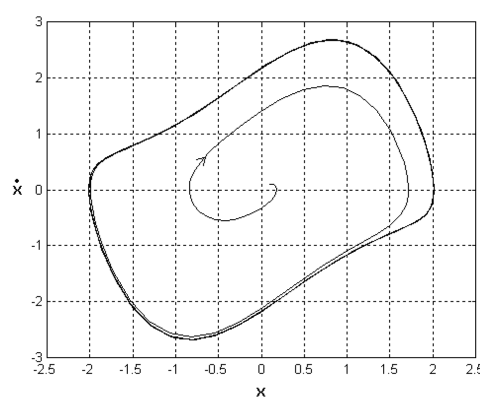

Fig. 2 Phase portrait using $x(0) = \dot{x}(0) = 0.1$ and $\varepsilon=1$.

Motivated by the fact that a Jerk system can produce chaos, first, we slightly modifiy the dynamics of Van der Pol as follows:

$$a\ddot{x}^2 sign(\ddot{x}) + \varepsilon(x^2 - 1)\dot{x} + x = 0 \qquad (4)$$

where a is a constant; after that, we immerse (4) into a Jerk system:

$$\begin{aligned} \dddot{x} &= f(\ddot{x}, \dot{x}, x) \\ &= -[a\ddot{x}^2 sign(\ddot{x}) + \varepsilon(x^2 - 1)\dot{x} + x] \end{aligned} \qquad (5)$$

Its phase portrait (of (5)), with $a=0.01$ and $\varepsilon=7$, is shown in Figure 3 and Figure 4, where $x(0) = \dot{x}(0) = \ddot{x}(0) = 0.1$.

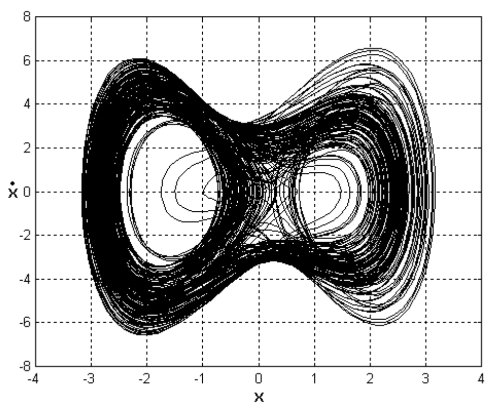

Fig. 3 Phase portrait.

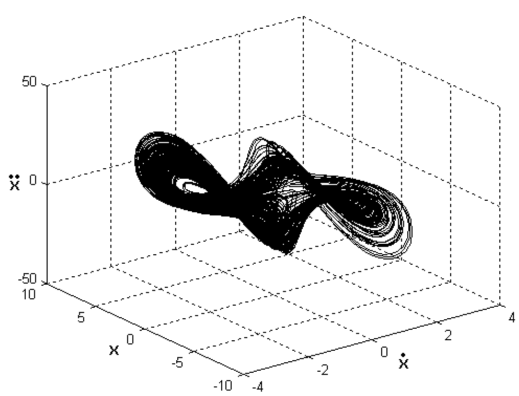

Fig. 4 Three-dimensional phase portrait.

Other values of a and ε that produce chaos are $(a,\varepsilon) = \{(0.03,7),(0.01,6)\}$. Hereafter we will use $a=0.01$ and $\varepsilon=7$.

III. CHAOTIC TEST

The chaotic test we use to prove chaotic motion for system (5) is the map of Poincaré [3]. This map is obtained when we place, in appropriate form, a surface, called the "Poincaré section", into the state space [3] (see Fig. 5).

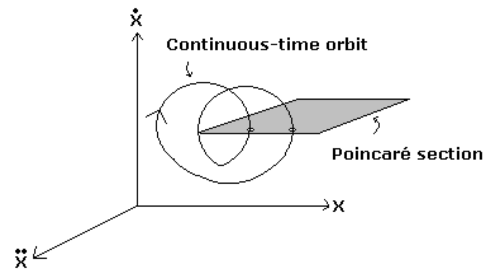

Fig. 5 Poincaré map.

In this way, and roughly speaking, the Poincaré map is a mapping of the points in which the trajectory intersects the Poincaré section. If the system is periodic, the Poincaré map will contain a few points; however, if the trajectory is chaotic, then, the Poincaré map will contain a huge amount of points distributed over the Poincaré section producing something similar to a strange attractor [3]. In Fig. 6 the map of Poincaré is shown placing the Poincaré section in the plane ($x - \dot{x}$) in $\ddot{x} = 0$ of our system.

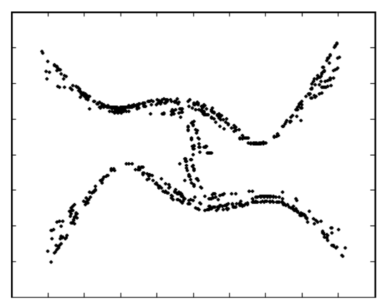

Fig. 6 Poincaré map.

From Fig. 6 follows that the proposed systems is a chaotic Jerk system.

References

[1] J.C. Sprott, "Simple Chaotic Systems and Circuits" *Am. J. Phys.*, Vol. 68, No. 8, August 2000, pp 758-763.
[2] L.O. Chua, C.A. Desoer, and E.S. Kuh, *Linear and Nonlinear Circuits*, McGrawHill, N.Y., 1987.
[3] S. Banerjce, G. C. Verghese, "Nonlinear Phenomena in Power Electronics: Attractors, Bifurcations, Chaos, and Nonlinear Control,", IEEE Press, N.Y., 2001.