Central Bank's Opportunism and the Fisher Effect: a threshold cointegration investigation.

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Abstract

This paper examines the long-run relationship between short term nominal interest rates and inflation using American data. As there is mixed evidence on the long run behaviour of real interest rates, we test for a unit root in the framework of a complete cointegration analysis and an Error Correcting Model (ECM) with switching regimes. As a first step, we conduct cointegration tests, while innovating by allowing a break in the cointegrating vector as well as a mean shift in the long-run equation, following Gregory and Hansen (1996) methodology. As a second step, we undertake Threshold AutoRegressive (TAR) tests for the residuals of the cointegration relationship as well as a test of non-linearity allowing a smooth transition from one regime to another. An application to the US data shows strong evidence for a threshold behaviour in the long run relationship. Non-linear mean reversion properties for the Treasury Bill market are consistent with asymmetric changes to inflation shocks for the Central Bank reaction function. This would imply that the forward-looking Fed runs a credible anti-inflationary policy by reacting differently to positive and to negative inflation surprises.

Key words: Threshold cointegration, structural breaks, Fisher effect, monetary policy regimes.

JEL classification: E4, C12, C22

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1 Introduction

Despite intensive empirical studies and a large literature, it seems that no consensus emerged about the statistical properties of the real rate of interest and more generally about the validation of the Fisher effect, which is basically a relationship that postulates a nominal interest rate in any period equal to the sum of the real interest rate and the expected rate of inflation.

This is a worrying fact for two main reasons: first of all, the real interest rate is a crucial determinant of investment, savings and indeed virtually for all intertemporal decisions. This also means that a potential nonstationarity of the real interest rate would have important consequences concerning monetary policies effects and also for economic and financial models interpretation. Secondly, it seems that a lot of economic articles using some theoretical models, such as the Consumption-based intertemporal capital Asset Pricing Model (CAPM), or some econometric methodologies, such as the Generalized Moments Methods (GMM), routinely assume that the real interest rate is a stationary process. This should be very confusing as empirical works indicate that this is not so or at best holds only over short periods. The statistical characterization of the real interest rate has been therefore investigated by many macroeconomists, with unfortunately only contradictory findings.

The main goal of this article is thus to apply recent econometric methods for the quarterly US Treasury Bill secondary market[1], so as to help us to resolve the Fisher effect "puzzle" for the period 1951-2000 and more precisely to understand why the instruments on this monetary market failed to provide a sufficient hedge against inflation.

[1] where the nominal interest rates are the average of daily closing bid for the T-bills

Before determining the modelization and the estimation of the Fisher effect for short term interest rates, several points have to be emphasized.

Firstly, most of the empirical findings call for a stochastic trend specification in both the nominal interest rates and the inflation rates series, leading us to rely on unit root and cointegration procedures for investigation of the relationship between these two variables.

Secondly, while long term interest rates are market determined, the short term interest rates are more likely to be driven by short run policy considerations; this would thus imply to take into consideration all the features of the monetary policies of the Federal Reserve Bank (hereafter Fed) of the United States, especially during the 70s and the 80s.

Thirdly, taking into consideration our long spanned data (1951-2000), structural breaks are very likely to occur during the period considered, according to Perron (1989),Zivot and Andrews (1992), Garcia and Perron (1996), Perron (1997). Another justification for introducing structural breaks in the cointegration model comes from the fact that, according to some measurements of inflation expectations, the inflation rate was under-expected during the oil crisis and over-expected during the deflation period. OECD forecasts with the help of econometric models confirm over-estimation of inflation rates due to systematic errors in the agents' anticipation during this period. Bismut (1988) for instance argue that expectations differ a lot with realization in the disinflation period of the early 80s. Thus, one could observe from the data that a decade of low real interest rates in the 70s gave way to a decade of high real rates in the 80s. These stylized facts are usually interpreted as a consequence of the change of monetary policy in 1979 and in 1982 following Paul Volcker's accession as the chairman of the Fed.

Moreover, as Fisher (1930) pointed in his study, the Fisher hypothesis is less supposed to be valid in the short run than in the long run. The interpretation of the Fisher effect as a mean reverting difference between the nominal interest rates and the inflation rates requires therefore some economic conditions which may not be fulfilled in the short run dynamics. For instance, a conventional interpretation of the causality in the Fisher relationship would be that excess monetary growth causes inflation and thus, combined with a stationary real interest rate, this will be reflected in the nominal interest rate. In the short run, this could however show up in movements of the real interest rates in the presence of sticky goods prices, so that a long run relationship investigation would be more likely to take into account some persistence of the real interest rate for the mean reversion process.

Finally, in a world with no market rigidities, no transaction costs, homogeneous behaviour for the agents, fully integrated markets and perfect accuracy for the inflation expectations, we would expect a perfect match for the Fisher effect in the relationship between interest rates and inflation (i. e. a one-to-one movements between the two variables). Since there is no evidence that these conditions were fulfilled, we have to deal with potential heterogeneities by introducing some non linearities in the model.

Furthermore, as it has been mentioned by Coakley and Fuertes (2002), the growing interest in inflation targeting discussed in Svensson (1997), Soderlind (1997) and Tobin (1998 Yale University CFDP 1187) and the opportunistic behaviour of the central banks are some of the reasons for exploring asymmetries in the key variables studied here. According to the proponents of the opportunistic approach (Orphanides & Wilcox (1996)), when inflation is moderate but still above the long-run objective, the Fed should not take deliberate anti-inflation action, but rather should wait for external circumstances (such as favorable supply shocks and unforeseen recessions) to deliver the desired reduction in inflation.

More precisely, according to the concept of inflation targeting and the forward looking estimations of Clarida et al. (1999) for the Taylor rule (cf Taylor (1999)), interest rate feedback rules imply that nominal interest rates should respond to increases in inflation with a more than one-to-one increase during the Volcker-Greenspan era or a less than one-to-one increase during the Burns-Miller period (cf Dolado Dolores & Ruge-Garcia (2002)), calling for a positive and a negative Fisher relation respectively, as suggested by Goto & Torous (2002). More precisely, since the arrival of Paul Volcker at the head of the Fed, monetary authorities are quick to raise nominal interest rates in response to inflationary pressures, which leads to a return of the real interest rates to their equilibrium value. On the other hand, in a falling inflation environment, the authorities may not be as quick to reduce the level of nominal interest rates, especially during the disinflation period. Hence, as verified empirically by Bec, Ben Salem & Collard (2002) and Kim, Osborn & Sensier (2002), there is strong evidence for a multiple-regime behaviour to inflation shocks in Central Bank reaction function, which imply that monetary authorities run a credible yet opportunistic anti-inflationary policy, reacting more strongly to positive[2] than to negative inflation surprises.

The consequences of this multiple-regime behaviour should be considered

[2] which will correspond to a decrease of the real interest rates, nominal interest rates being kept constant.

while studying the relationship between interest rates and inflation rates for the T-Bill secondary market.

Moreover, there are some empirical motivations for a non linear modelization of the series studied. Hamilton (1988), Sola and Driffil (1994) and Gray (1996) all find strong evidence for non-linear behaviour in U.S. nominal interest rates, using Markov switching models. Furthermore, Anderson (1997), Enders and Granger (1998) and Enders and Siklos (1999) found evidence of non-linearity in nominal yields using threshold autoregressions. Finally, Pippinger and Goering (1993) and Caner and Hansen (2001) have underlined the poor power of Augmented Dickey Fuller (ADF) tests in distinguishing between non-linearity and non-stationarity.

The main innovation of our methodology is therefore to coincide asymmetries investigation (as smooth transition extension of the Self-Exciting Threshold AutoRegressive (SETAR) tests) with a cointegration test robust to structural changes. We will perform linearity tests and unit root tests in the framework of a cointegration relationship between short term nominal interest rates and inflation rates in two steps.

As a first step we conduct cointegration tests by allowing a break in the cointegrating vector as well as a mean shift for the constant in the long-run equation, following Gregory and Hansen (1996) methodology.

As a second step, following Balke and Fomby (1997), we undertake Self-Exciting Threshold AutoRegressive (SETAR) tests for the residuals of the long run relationship, as well as a test of non-linearity allowing a smooth transition from one regime to another. For the unit root test, the null will be the first order integration hypothesis while the alternative hypothesis will be the stationary

Logistic Smooth Transition AutoRegressive (LSTAR) model[3] so that the shifts between the two regimes are driven by a logistic transition function. Empirical findings show here strong evidence for a threshold cointegration relationship for quarterly nominal interest rates and inflation, especially when the model allows the transitions between regimes to occur smoothly.

The article is structured in the following way. The next section will describe cointegration testing procedures and SETAR models for unit root and linearity tests to deal with the Fisher effect as a switching regimes cointegration relationship in which we allow the constant and/or the cointegrating vector to shift in the long-run equation. Linearity will also be tested in the context of a LSTAR model. The third section will display the main econometric results obtained from these tests applied to the US data. The last section concludes.

2 Threshold cointegration analysis robust to structural breaks

2.1 Presentation of the Fisher Hypothesis

The Fisher effect represents a relation of determination between the nominal interest rates and the expected inflation rates, the former reflecting at each time the latter. Provided that $1 + i_t = (1 + r_t^e)(1 + \pi_t^e)$, a nominal interest rate of i_t will thus guarantee an ex ante real rate of r_t^e as soon as the anticipated price change expected by the agents is π_t^e. For small values of interest rates and inflation rates, the Fisher equation is commonly simplified as:

$$i_t = r_t^e + \pi_t^e \tag{1}$$

[3] with a possibility of a unit root in one of the regimes, so that the variable will be gloablly covariance-stationary.

This implies that if the inflation expectations are perfectly accurate, the nominal interest rate will follow the inflation evolution.

Since Fama (1975) and Fama and Gibbons (1982), the Fisher hypothesis has been the framework for testing market efficiency and stationarity of real returns, as Fisher (1930) claimed a one-to-one relationship between inflation and interest rates in a world of perfect foresight. This would mean that real interest rates are not related to the expected rate of inflation and determined entirely by the real factors in an economy, such as the productivity of capital and the time preference. This is an important prediction of the Fisher hypothesis for, if real rates are related to the expected rate of inflation, changes in the real rate will not lead to full adjustment in nominal rates in response to expected inflation.

The literature clearly indicates that the nominal interest rate is nonstationary (Fama and Gibbons (1982) and Mankiw and Miron (1986)). However, it has proven difficult to provide definitive evidence concerning the ex ante real interest rate, as it is inherently unobservable. Rose (1988) tested for cointegration using the techniques suggested by Engle and Granger (1987). At the annual frequency, none of the tests indicated cointegration at even the ten percent significance level. Mishkin (1992) raised an interesting problem about the Fisher effect's lack of robustness depending on the period considered. Mishkin therefore conducts a reexamination of the Fisher effect in the postwar United States and finds that the evidence does not support a short-run relationship in which a change in expected inflation is associated with a change in interest rates. More recently, Garcia and Perron (1996) reanalyzed data over the period 1961-1986 using Markov Switching (MS) methods and found support for a stable real rate of interest, subject to infrequent changes in the constant. Then these authors concluded that the ex ante real rate of interest was effectively stable, but subject to occasional mean shifts over 1961-1985. Three regimes were found over this time period. However, according to the graphs for the ex post real interest rate series calculated in the same way over the longer period 1951-1999, Phillips (1998) pointed out that a larger number of mean shifts is needed to accommodate this approach and the results seem to be much less satisfying.

To summarize, the empirical evidences reviewed just before give a mixed picture about the statistical properties of the real rate of interest, and it is probably fair to say that the generating mechanism for the real rate is not perfectly understood.

Assumption is made that all the economic agents have rational expectations.

The forecast error ε_t represents the difference between the inflation rates expected *ex ante* by the agents in the economy and the inflation rates really observed *ex post*:

$$\varepsilon_t = \pi_t^e - \pi_t = E\left(\pi_t/\Omega_t\right) - \pi_t \tag{2}$$

Under rational expectations assumption, ε_t will be unforecastable given Ω_t. In most of the empirical works, the expectation errors have been assumed to be covariance-stationary in level but we will loose this assumption here by considering them to be a martingale difference sequence with respect to the history of the time series up to time $t-1$ so that they will be defined as a process orthogonal to the current information set Ω_t available to the agents at time t.

2.2 Non linearities in the Fisher relationship

Considering the underlying model involving expected inflation, the Fisher hypothesis asserts that the coefficient b should be be unity (or very near unity) in a relation of the form :

$$i_t = a + b\pi_t^e + w_t \tag{3}$$

and that the residuals w_t should be stationary.

The real interest rates could then be expressed in the following way with $b' = b - 1$:

$$r_t^e = a + b'\pi_t^e + w_t \tag{4}$$

From the last equation, a potential non stationarity of the ex ante real interest rates r_t^e will result only from structural breaks in the deep parameter a or a coefficient b' significantly different from 0 (i. e. ruling out the Fisher hypothesis).

However, the inflation expectations and consequently the ex ante real interest rate could not be directly measured. So we have to rely on nominal interest rates measured at the beginning of the period m and future inflation measured at the end of the period m, so as to test for a Fisher effect, with the idea that the results will lead to the same interpretations as long as the assumption of rationality for expectations is held[4].

It will then be possible to write from equation 3 the ex post real interest rate as:

[4] Fortunately, as Mishkin (1992) pointed out, it is very easy to show that a test of the correlation of interest rates with future inflation is also a test for the correlation of interest rates and expected inflation.

$$r_t = a + b\pi_t^e - \pi_t + w_t = a + \varepsilon_t + b'\pi_t^e + w_t \tag{5}$$

which have the same statistical properties as 4 notwithstanding the expectations errors ε_t. In this case, the volatility of the ex post real interest rates could therefore come from the same sources as the ones in 4, in addition to potential volatility from the expectation errors process.

Under the assumption that a (nearly) full Fisher effect holds for the underlying model (which means that a coefficient b would be very close or equal to unity), we have than the following regression equation between ex post inflation rates π_t and the nominal interest rates i_t :

$$i_t = \alpha + \beta\pi_t + z_t \tag{6}$$

An estimation value close to unity is then expected for the parameter β while the residuals z_t encompass the fluctuations w_t as well as the expectation errors ε_t, as in (5).

So it is necessary to replace the Fisher effect in the framework of a cointegration equation (as in (6)), using Error Correcting Models (ECM) so as to discriminate the short-run Fisher effects with the long run ones.

Consequently, according to Mishkin (1992), it is interesting to emphasize about the less obvious link between a cointegration relation for π_t and i_t (which is a long-run Fisher Effect) and a unit root test on r_t[5].

Moreover, these are also tests for unit root in the ex ante real interest rate under the hypothesis of stationary expectation errors. In this case, a cointegration link between π_t and i_t will imply that i_t and π_t^e will be also cointegrated.

[5] Obviously this link could only be possible in the case of a full Fisher Effect.

So, a unit root test applied on r_t will let us conclude about a cointegrating relation between i_t and π_t^e, i. e. a Fisher effect. As a consequence, testing for a unit root in $i_t - \pi_t$ against stationary alternatives is thus equivalent to test for a unit root in r_t^e against a stationary ex ante real rate. Looking at the cointegration tests in this light indicates that the full long-run Fisher effect can be interpreted as the hypothesis that the ex ante real rate is stationary.

Since Gregory and Hansen (1996), it is possible to consider cointegration relationships in which the parameters are no longer time invariant. This means that the long-run relation holds over some period of time and shifts to a new long run equilibrium.

Perron (1989), Zivot and Andrews (1992) , Gregory and Hansen (1996) and Perron (1997) all argue that standard unit root tests are biased towards the null of non-stationarity (and the null of no cointegration in the case of residual-based cointegration tests) in the presence of unanticipated structural breaks or regime changes.

Basically, two types of model will be relevant for our analysis, according to the alternative of treating the structural change as changes in the intercept and/or changes in the slope (i.e. the cointegration vector)..

In the level shift model, the equilibrium equation shifts in a parallel fashion as only the intercept changes.

Level shift model (model S):

$$i_t = \alpha_1 + (\alpha_2 - \alpha_1)DU_t + \beta\pi_t + Z_t \tag{7}$$

with $DU_t = \begin{cases} 0 & if\, t < T_b \\ 1 & if\, t \geq T_b \end{cases}$

In the regime shift model, we allow a change in the coefficient of the long run equilibrium in addition to a level shift.

Regime shift model (model C/S):

$$i_t = \alpha_1 + (\alpha_2 - \alpha_1)DU_t + \beta_1 \pi t + (\beta_2 - \beta_1)\pi_t DU_t + Z_t \tag{8}$$

2.3 Asymmetric mean reversion process and smooth transition between the regimes

In the standard ECM, the short run adjustment towards long run equilibrium is supposed to be always present and time-invariant. The parameters which measure the mean reverting intensity towards equilibrium are considered as fixed. Nevertheless, according to Balke and Fomby (1997), the movements towards equilibrium value do not always appear or at least do not have the same intensity. So as to take into account the possible non-linearities in the adjustment dynamics towards equilibrium, we introduce two regimes in the dynamics of the error term Z_t.

To detect any asymmetries, we undertake a simple Self Exciting Threshold AutoRegressive (SETAR) test for sign asymmetries in which the dynamics and the threshold properties of the variable studied depends on the level of this variable.

Following Balke and Fomby (1997), we estimate the cointegration relationship (8) in which the residuals Z_t follow a SETAR process: $\Delta Z_t = \rho_1 Z_{t-1}(1 - I_t) + \rho_2 Z_{t-1} I_t + \sum_{j=1}^{p-1} \Phi_j \Delta Z_{t-j} + u_t$ where I_t is the Heaviside function : $I_t = \left\{ \begin{array}{ll} 1 & \text{if } Z_{t-l} < \tau \\ 0 & \text{if } Z_{t-l} \geq \tau \end{array} \right\}$.

In testing whether the SETAR model is statistically significant relative to a linear $AR(p)$ one faces the problem that the threshold parameter is not identified under the null hypothesis. However, Hansen (1996) shows that given a set of possible threshold values $\lambda \in \Lambda = [\lambda_1, \lambda_2]$ along with the least squares threshold estimate $\hat{\lambda}$, one can perform a sequence of Wald tests over the values in this set. Evidence for the null hypothesis of linearity can be assessed using the test statistic:

$$W_T = W_T\left(\hat{\lambda}\right) = \sup_{\lambda \in \Lambda} W_T\left(\lambda\right)$$

where $W_T\left(\hat{\lambda}\right)$ is the Wald test statistic of the null hypothesis. The asymptotic null distribution of W_T is non-standard. Appropriate critical values can be found by bootstrapping the data. Caner and Hansen (2001) perform a Monte-Carlo experiment to explore the size and power properties of the Bootstrapped W_T test. The evidence suggests that the test is free from size distortions and that the power of the test increases with the magnitude of the threshold effect.

Such a nonlinear extension is also able to incorporate the smooth transition mechanism in an ECM to allow for asymmetric adjustment. Intuitively, market frictions often suggest that the degree of error correction is function of the size of the deviation from the equilibrium.

It would thus be interesting to introduce smoothness in the transition function by using a two regime Vector Smooth Transition Auto Regressive (VSTAR) model where we define the error correction term as Z_t, the deviation from the long run equilibrium relationship defined as $\beta' X_t$ where X_t includes a $(K \times 1)$ vector Y of k $I(1)$ variables (and $K-k$ deterministic as well as dummy variables in the case of the Gregory & Hansen methodology) and β a $(K \times 1)$ vector. We have then the Vectorial Smooth Transition Error-Correction Model [VSTECM]:

$$\Delta Y_t = (\Phi_{1,0} + \alpha_1 Z_{t-1} + \sum_{j=1}^{p-1} \Phi_{1,j} \Delta Y_{t-j}) G(Z_{t-1}, c, \gamma) \quad (9)$$

$$+ (\Phi_{2,0} + \alpha_2 Z_{t-1} + \sum_{j=1}^{p-1} \Phi_{2,j} \Delta Y_{t-j}) [1 - G(Z_{t-1}, c, \gamma)] + \varepsilon_t \quad (10)$$

where $\Phi_i = [\phi_{i,0}, \alpha_i, \phi_{i,1}, ..., \phi_{i,p-1}]$ is a $(P + 1 \times k)$ vector of parameters.

Here, we choose the transition function $G(Z_{t-1}, c, \gamma)$ to be the first order logistic function $[1 + \exp(-\gamma_i (z_{t-1} - c_i))]^{-1}$ for $\gamma > 0$ and $c > 0$, so as to detect asymmetric behaviour for small and large equilibrium errors. This results in gradually changing strength of adjustment for larger (both positive and negative) deviations from equilibrium.

In the test for linearity, according to Luukkonen and Terasvirta (1988), we replace the transition function G by a suitable first order Taylor approximation. In the reparametrised equation, the identification parameter is no longer present so that the linearity can be tested by means of a Lagrange Multiplier (LM) statistic with a standard asymptotic χ^2 distribution under the null hypothesis of linearity: $LM \xrightarrow{H_0} \chi^2 (p + 1)$.

3 Empirical results

The data used here for interest rates are the monthly average of daily closing bid of the 3 months Treasury Bill Rate and span from 1951.1 to 1999.12. The inflation rates are computed from monthly values of the urban CPI[6] (see figure 1).

[6] The choice of 1951 for the beginning of the data could be explained by the fact that tests for periods prior to 1951 would be meaningless. During World War II and up to the Treasury-Federal Reserve Accord of 1951, interest rates on Treasury Bill were pegged by the government with the result that Treasury Bill rates did not adjust to predictable changes in inflation rates.

If we run ADF tests (see Dickey and Fuller (1981) and Said and Dickey (1984)) with or without the modifications of the efficient DF-GLS test of Elliott et al. (1996) (ERS) as well as the Kwiatkowski D. and Shin (1992) (KPSS) for confirmatory analysis, there is strong evidence for a random walk with no drift as the driving process of each series. All the test statistics are not significant for the null hypothesis of unit root and significant for the null hypothesis of stationarity at the 5% level (see table 1).

We will distinguish two cases: the first one corresponds to the case in which we will apply Threshold AutoRegressive (TAR) tests to the residuals of the cointegration relation robust to structural breaks. In the second case, we will see if the model still displays mean reversion via the real interest rates when the assumption of a full Fisher effect is considered, while still allowing structural shifts.

3.1 First step: the long run relationship between inflation and interest rates

3.1.1 Cointegration tests

We will follow both the Engle and Granger (1987) (EG) and the Johansen procedures to test for a unit root in the residuals of the long run relationship between inflation and nominal interest rates.

The standard method of EG to test the null hypothesis of no cointegration is a residual-based one. The candidate cointegrating relation is estimated by OLS and a unit root test is applied to the regression errors Z_t in 6.

In the framework of a cointegration relationship, we have the following long run equation:

$i_t = 3.17 + 0.56\pi_t + \hat{Z}_t$

Unfortunately, the results are contradictory, as the Johansen procedure is supporting the hypothesis of cointegration while the residual-based one is not (cf appendix).

If we expect any structural breaks to occur in the sample studied, then it is preferable to rely on the residual based cointegration tests of Gregory and Hansen (1996), who propose a test for a cointegration relation with a structural change against an $I(1)$ alternative. This will help us to specify correctly any sudden and exogenous change in the process, such as the different post oil-crisis monetary policy conducted after the nomination of Volcker as the chairman of the Fed in 1979.

We propose to test the Fisher effect for both models, level shift model (model S) and the regime shift model (model C/S), described in the previous section. In all cases, the time break is treated as unknown and is estimated with a data dependent method which corresponds to the minimum of the t-stats computed on a trimmed sample (excluding outliers values). Here, the results lead us to introduce a structural break in July 1979 ($T_b = 1979.7$).

According to the empirical findings, it is now possible to reject the null hypothesis of no cointegration when allowing a level shift in the cointegration relationship while the regime shift does not give better results.

3.1.2 SETAR tests on the residuals of the cointegration relationship

We then apply the TAR unit root tests on the residuals Z_t so as to have two different adjustment procedures towards the equilibrium relationship, according to which regime belongs the error term Z_t in the following VECM:

$\Delta Y_t = \mu_i + \gamma_i Z_{t-1} + A_i(L)\Delta Y_t + v_t^i$ with $Y_t = \begin{bmatrix} i_t \\ \pi_t \end{bmatrix}$

SETAR1

In the case of a SETAR model, we have the two following regimes[7] for the error term Z_t:

$$\Delta Z_t = \left\{ \begin{array}{c} \underset{(0.031)}{-0.017} - \underset{(0.029)}{0.074}.Z_{t-1} + \underset{(0.052)}{0.17}.\Delta Z_{t-2} + ... - \underset{(0.053)}{0.23}.\Delta Z_{t-9} \\ \text{when } Z_{t-8} < 2.05 \\ \underset{(0.28)}{0.166} - \underset{(0.08)}{0.1}.Z_{t-1} + \underset{(0.13)}{0.465}.\Delta Z_{t-1} + ... - \underset{(0.09)}{0.215}.\Delta Z_{t-12} \\ \text{when } Z_{t-8} \geq 2.05 \end{array} \right\} + \hat{u}_t$$

The Wald statistic is maximum for a delay of 8, estimated by the program for delays between 1 to 12.

According to the tests results (see appendix), we find evidence for both stationarity and asymmetry in the residuals of the long run cointegration relationship between inflation and interest rates. We are then able to reject the null hypothesis of no cointegration in a non-linear context, leading us not to reject the Fisher hypothesis in the long run as soon as we correctly specify the non-linearities features of the underlying model. So it is possible to retain a non linear Fisher effect.

3.2 Second step: the full Fisher effect

According to the results of unit root tests versus an alternative SETAR applied to the residuals of the long run relationship between inflation rates and interest rates, there is a strong evidence for non linearity and stationarity concerning the residuals of the long run relationship between nominal interest rates and inflation rates. As the final stage, the hypothesis of a pure non linear Fisher effect will be assumed, implying that the residuals of the cointegration relationship will be the real interest rates series with structural breaks.

As before in the Gregory and Hansen (1996) and Perron (1989) spirit, we check for structural breaks by regressing the real interest rates on a constant

[7] Note that $Z_t \geq x$ is equivalent to $i_t - 0.56\pi_t \geq x + 3.17$.

and dummy variables:

$$i_t = \alpha_1 + (\alpha_2 - \alpha_1)DU_t + \alpha_3 D\left(T_b\right)_t + \pi_t + Z_t \tag{11}$$

In this particular case where the cointegrating vector is $[1, -1]$,the cointegration procedure becomes a unit root test with structural breaks whose critical values can be found in Perron (1989) and in Perron (1997). We have then the following long run regression:

$r_t = 0.46 + 2.43 DU_t - 2.99.D\left(1979.4\right)_t + \hat{Z}_t$

with $DU_t = \begin{cases} 0 & if\, t < 1979.4 \\ 1 & if\, t \geq 1979.4 \end{cases}$ and $D\left(1979.4\right)_t = \begin{cases} 0 & if\, t < 1979.4 \\ 1 & if\, t = 1979.4 \end{cases}$

Due to the particular strong assumption of a full Fisher effect, there seems to remain some persistence of the long run relationship residuals $\hat{Z}_t$ of (11). The non stationarity of $\hat{Z}_t$ has been tested and discussed in (Million (0275)) and there is evidence that the residuals are stationary for the entire sample.

3.2.1 SETAR test on the real interest rates

In the context of a SETAR model, we have the following:

SETAR2

Regime 1

$$\Delta Z_t = \underset{(0.604)}{-0.63} - \underset{(0.168)}{0.44}\, Z_{t-1} - 0.095.\Delta Z_{t-2} + ... + 0.21.\Delta Z_{t-11} + \hat{u}_t$$

if $r_{t-4} < -3$

Regime 2

$$\Delta Z_t = \underset{(0.0512)}{0.015} - \underset{(0.036)}{0.09}\, Z_{t-1} + 0.12\Delta Z_{t-1} + 0.15.\Delta Z_{t-2} + ... - 0.07.\Delta Z_{t-12} + \hat{u}_t$$

if $r_{t-4} \geq -3$

The Wald statistic is maximum for a delay of 4, estimated by the program for delays between 1 to 12. Moreover, unit root tests run for each of the regime suggest that the regime 1 is the most stationary between the two. However, most of the observations this time belong to the high regime (which represent in total 90% of the observations), contrary to the previous TAR model in which the first regime has the largest number of observations. This could come the lack of relevance of the estimation of the threshold (-3) in comparison of the previous TAR model applied for the residuals of the cointegration relationship, suggesting that the strong constraint of a sudden change in the regime has to be loosened.

3.2.2 Smooth transition between the regimes

The hypothesis of a sudden transition function will be relaxed by estimating a STAR model for which SETAR2 model is a special case. We choose to modelize the residuals in the ECM with the help of a Logistic Smooth Transition AutoRegressive (LSTAR) process[8].

We have then two regimes in the STAR model for the residuals of the real interest rate series (see table of results at the end of the article).

The significance test statistic of the first order autoregressive coefficient of ΔZ_t equals to 23.4 in the low regime.

Here our test statistic for linearity is equal to 49.3 which corresponds to a p-value less than 0.4%, thus allowing us to reject the linearity hypothesis for the real interest rates. The transition speed between the two regimes is equal to 1.23.

We have then the following two regime ECM (with the p-value for the sig-

[8] Usually, two interpretations of the STAR model are possible. On the one hand, the STAR model can be thought of as a regime-switching model that allows for two regimes, associated with the extreme values of the transition function (i.e. 0 and 1) where the transition from one regime to the other is smooth. On the other hand, the STAR model can be said to allow for a 'continuum' of regimes, each associated with a different value of $G(Z_{t-1}, c, \gamma)$ between 0 and 1. In this paper we will use the 'two regime' interpretation.

nificancy of the respective parameters below in parenthesis):

$$\Delta i_t = \left\{ \begin{array}{c} \underset{(4.4\%)}{-0.025}.Z_{t-1} + \underset{(0\%)}{0.57}.\Delta i_{t-1} + ...\underset{(0\%)}{0.2}.\Delta i_{t-6} - \underset{(0\%)}{0.07}.\Delta \pi_{t-2} - ...\underset{(1.1\%)}{-0.04}.\Delta \pi_{t-4} \\ if \quad Z_{t-1} < 2,2 \\ \underset{(22.1\%)}{-0.064}.Z_{t-1} - \underset{(0\%)}{0.44}.\Delta i_{t-3} + ... - \underset{(0\%)}{0.05}.\Delta i_{t-6} \\ if \quad not \end{array} \right\} + \hat{u}_t$$

Here, the model clearly displays a regime of mean reversion in the low regime where the relative coefficient for mean reversion is negative (-0.025) and significant while in the high regime the coefficient is not significant at the 5% level. This would mean that the series in this high regime display weaker mean-reversion effects, then being less likely to return to the long run equilibrium value[9].

As the mean reversion is less likely to occur in the high regime, this would mean that as soon as the long run residuals (which are here basically the real interest rates with structural dummies) are above the threshold value of 2.2, they are less likely to return to their equilibrium than in the low regime. This is consistent with the apparent persistence in real interest rates and especially why real interest rates were kept so high in the 80s in the United States so that, facing stagflation situations, the Fed put the priority on the decline of the inflation.

4 Interpretations and conclusion.

Throughout this article, we have examined the long-run relationship between nominal interest rates and inflation with an application to the US data

[9] We could note that the second equation of the VECM (where the first difference of the inflation rate $\Delta\pi_t$ is regressed on lagged variables) consistently displays also only one regime with mean reversion while in the other the error correcting variable is not statistically significant.

and showed strong evidence for a threshold behaviour[10] in the residuals of this cointegration relationship. The introduction of structural breaks in the long-run relationship helped us to remove some structural instability from the model prior to the analysis of asymmetries in the short run dynamics.

Through SETAR tests, we underlined the existence of size asymmetries. We explain size asymmetries as differential adjustments to small and large equilibrium errors. This results in gradually changing strength of adjustment for larger (both positive and negative) deviations from equilibrium. Moreover, we found strong evidence for a smooth transition between the regimes of high and low interest rates (which could referred to a regime of increasing nominal rates or decreasing inflation rates respectively, *ceteris paribus*). This kind of nonlinear behaviour may usually result from non-synchronous interventions, heterogeneous agents and some intervention costs, as in the T-Bill market.

Furthermore, according to Orphanides () this could be interpreted as a feedback effect of an opportunistic behaviour of the Central Bank for the short term monetary market. Proponents of the opportunistic approach hold that when inflation is moderate but still above the long-run objective, the Fed should not take deliberate anti-inflation action, but rather should wait for external circumstances (such as favorable supply shocks and unforeseen recessions) to deliver the desired reduction in inflation. All this means that the policy maker (still pursuing an objective of price stability) will change his behaviour depending

[10] As for any statistical test, we are only able to reject or not the null hypothesis of the test in consideration, without having any indication concerning the alternative candidates to be held. However, as there is strong evidence against a linear model, we presume that the LSTAR model performs better than the linear model and also better than other non-linear models for several reasons. The ESTAR gave less statisfying results and does not encpmpass the SETAR models as a particular case, while the LSTAR model does. Furthermore, Marine Carrasco gives evidence that Threshold tests have better power than other non linear models (such as the Markov-Switching models) especially in the presence of spurious regimes.

on the level of inflation. Whenever the inflation rate decreases and falls on a band of tolerable inflation, the policy maker will be more reluctant to conduct an active policy (by decreasing nominal rates for instance), but merely engage in a policy of watchful waiting (which is consistent with a stance of inflation targeting). However, in a context of increasing inflation (which will correspond to our low regime where real interest rates are low and/or decreasing, *ceteris paribus*), the monetary authorities will change nominal interest rates so that inflation rates will go back to acceptable values.

These empirical findings confirm the different forward looking estimations of Clarida et al. (1999) for the Taylor rule (cf Taylor (1999)) in which interest rate feedback rules imply that nominal interest rates should respond to increases in inflation with a more than one-to-one increase during the Volcker-Greenspan era or a less than one-to-one increase during the Burns-Miller period. More precisely, since the arrival of Paul Volcker at the head of the Fed, monetary authorities are quick to raise nominal interest rates in response to inflationary pressures, which leads to a return of the real interest rates to their equilibrium value. On the other hand, in a falling inflation environment, the authorities may not be as quick to reduce the level of nominal interest rates, especially during the disinflation period. Hence, as verified empirically by Bec, Ben Salem & Collard (2002) and Kim, Osborn & Sensier (2002), there is strong evidence for a multiple-regime behaviour to inflation shocks in Central Bank reaction function, which imply that monetary authorities run a credible yet opportunistic anti-inflationary policy, reacting more strongly to positive[11] than to negative inflation surprises.

[11] which will correspond to a decrease of the real interest rates, nominal interest rates being kept constant.

This evidence should also resolve the puzzle of why the Fisher effect appears to be strong in some periods but not in others. Just as this analysis predicts, a long-run Fisher effect appears to be strong in the periods when interest rates and inflation exhibit stochastic trends: these two series will trend together and thus there will be a strong correlation between inflation and interest rates. On the other hand, as soon as those variables do not exhibit stochastic trends simultaneously, a strong correlation between interest rates and inflation will not appear if there is no short-run Fisher effect. Thus, the presence of a long-run but not a short-run Fisher effect predicts that a Fisher effect will not be detectable during periods when interest rates and inflation do not have trends. It is exactly in these periods that Mishkin (1992) was unable to detect any evidence for a Fisher effect.

Indeed, according to Mishkin, the findings here are more consistent with the views expressed in Fisher (1930) than with the standard characterization of the so-called Fisher effect in the last twenty years. The evidence in this paper thus supports a return to Irving Fisher's original characterization of the inflation interest rate relationship.

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6 Appendix

A Threshold AutoRegressive (TAR) model could be used to best define the ECM for the bivariate time series $Y_t = \begin{bmatrix} y_t \\ x_t \end{bmatrix}$, whose components y_t, x_t each contain a unit root.

If both components have a joint stochastic trend B_t ($y_t + \beta x_t = B_t$ where $B_t = B_{t-1} + \eta_t$), it is then possible to find a stationary linear combination of these two integrated variables:

$y_t + \alpha x_t = Z_t$, where $Z_t = \rho Z_{t-1} + u_t$ with $|\rho| < 1$.

So as to take into account possible non-linearities in the model, we introduce two different adjustment dynamics towards the equilibrium relationship, according to which regime the error term Z_t belongs. Given that the coefficients γ_i are defined according to the autoregressive coefficient ρ, the coefficients in the ECM also depends of the regime $i = \{1, 2\}$:

$$\Delta Y_t = \mu_i + \gamma_i . Z_{t-1} + \Phi_i(L) \Delta Y_t + v_t^i$$

with $\gamma_i = \begin{bmatrix} -(1-\rho_i)\beta / (\beta - \alpha) \\ -(1-\rho_i) / (\beta - \alpha) \end{bmatrix}$

A time-varying adjustment mechanism could then be specified since there will be mean-reversion as soon as γ_i will be negative and significant (and none if it is positive or non significant). Here the components of Y_t are linked by a long-run equilibrium relationship, whereas the adjustment towards this equilibrium is nonlinear and can be characterized as regime switching, with the regimes determined by the size and/or sign of the deviation from equilibrium.

Self Exciting Threshold AutoRegressive (SETAR) tests could be applied so as to detect any sign asymmetries the variable studied, whose dynamics will depend on the level of this variable.

In the context of the cointegration analysis with structural breaks, the residuals Z_t are computed from the estimates $\hat{\alpha}$ of the cointegration relationship defined as $\beta' X_t$ where X_t includes a $(K \times 1)$ vector Y of k $I(1)$ variables (and $K - k$ deterministic as well as dummy variables) and α a $(K \times 1)$ vector.(cf Gregory and Hansen (1996)) and are assumed to follow a SETAR process: $Z_t = \rho_1 Z_{t-1}(1 - I_t) + \rho_2 Z_{t-1} I_t + u_t$ where I_t is the Heaviside function : $I_t = \left\{ \begin{array}{ll} 1 & \text{if } Z_{t-l} < \tau \\ 0 & \text{if } Z_{t-l} \geq \tau \end{array} \right\}$.

Here the threshold τ will correspond to the attractor as soon as $\tau = 0$.

In testing whether the SETAR model is statistically significant relative to a linear $AR(p)$ one faces the problem that the threshold parameter is not identified under the null hypothesis. However, Hansen (1996) shows that given a set of possible threshold values $\lambda \in \Lambda = [\lambda_1, \lambda_2]$ along with the least squares threshold estimate $\hat{\lambda}$, one can perform a sequence of Wald tests over the values in this set. Evidence for the null hypothesis of linearity can be assessed using the test statistic:

$$W_T = W_T\left(\hat{\lambda}\right) = \sup_{\lambda \in \Lambda} W_T\left(\lambda\right)$$

where $W_T\left(\hat{\lambda}\right)$ is the Wald test statistic of the null hypothesis. Appropriate critical values can be found by bootstrapping the data since the asymptotic null distribution of W_T is non-standard. Caner and Hansen (2001) perform a Monte-Carlo experiment to explore the size and power properties of the Bootstrap W_T test. The evidence suggests that the test is free from size distortions and that the power of the test increases with the magnitude of the threshold effect.

Such a nonlinear extension is also able to incorporate the smooth transition mechanism in an ECM to allow for asymmetric adjustment.

Again, we define the error correction term as Z_t, the deviation from the long run equilibrium relationship Here, it is interesting to introduce smoothness in the transition function by using a two regime Vector Smooth Transition Auto Regressive (VSTAR) model. We have then the Smooth Transition Error-Correction Model [STECM]:

$$\begin{aligned}\Delta Y_t &= (\Phi_{1,0} + \alpha_1 Z_{t-1} + \sum_{j=1}^{p-1} \Phi_{1,j} \Delta Y_{t-j}) G(Z_{t-1}, c, \gamma) && (12)\\ &+(\Phi_{2,0} + \alpha_2 Z_{t-1} + \sum_{j=1}^{p-1} \Phi_{2,j} \Delta Y_{t-j})[1 - G(Z_{t-1}, c, \gamma)] + v_t && (13)\end{aligned}$$

where $\Phi_i = \left[\phi_{i,0}, \alpha_i, \phi_{i,1}, ..., \phi_{i,p-1}\right]$ is a $(P + 1 \times k)$ vector of parameters.

For instance, the transition function I_t is replaced by the first order logistic function $G(Z_{t-1}, c, \gamma) = \left[1 + \exp\left(-\gamma_i \left(z_{t-1} - c_i\right)\right)\right]^{-1}$ for $\gamma > 0$ and $c > 0$, so as to detect asymmetric behaviour for small and large equilibrium errors. This results in gradually changing strength of adjustment for larger (both positive and negative) deviations from equilibrium.

The transition function goes monotonically from zero to one as Z_{t-1} increases, being equal to 0.5 for $Z_{t-1} = c$. Consequently, the parameter c may be viewed as the threshold between two regimes. The parameter γ governs the smoothness of the transition between regimes. An advantage of the logistic function is that for $\gamma \to 0$, the function collapses to a constant (equal to $0,5$). Hence, the model becomes linear when $\gamma = 0$ and the LSTAR model nests a two-regime SETAR model as a special case.

In the test for linearity, according to Luukkonen and Terasvirta (1988), we replace the transition function G by a suitable first order Taylor approximation.

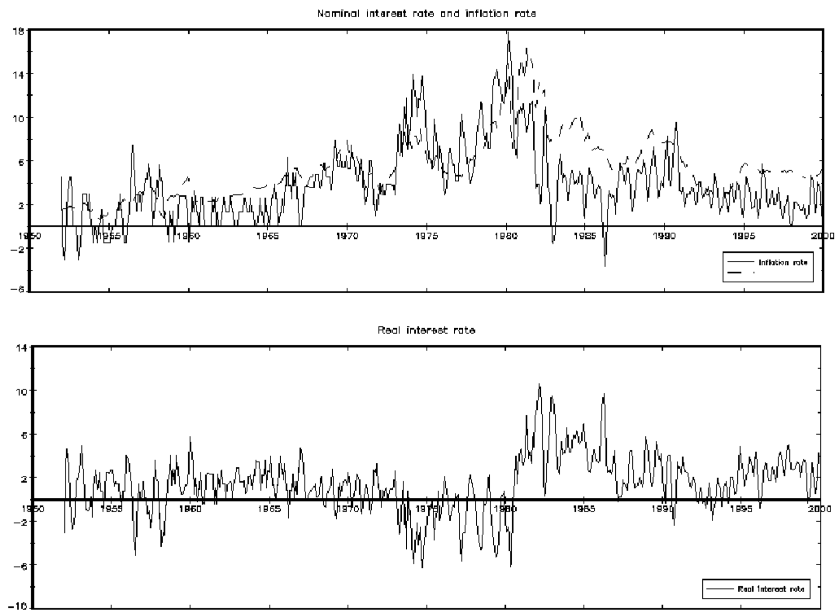

Figure 1: Real interest rates as the difference between nominal interest rates and inflation rates

Type of Test	i_t	π_t	r_t	5% *c.v.*
ADF	-2.52	-2.5	-2.7	-2.88
DF-GLS	-0.9	-1.3	-1.9	-1.96
KPSS	1.16*	2.10*	0.78*	0.463

Table 1: Unit root test results

In the reparametrised equation, the identification parameter is no longer present so that the linearity can be tested by means of a Lagrange Multiplier (LM) statistic with a standard asymptotic χ^2 distribution under the null hypothesis of linearity: $LM \xrightarrow{H_0} \chi^2(p+1)$ (see Van Dijk and Franses (2002) for a detailed review).

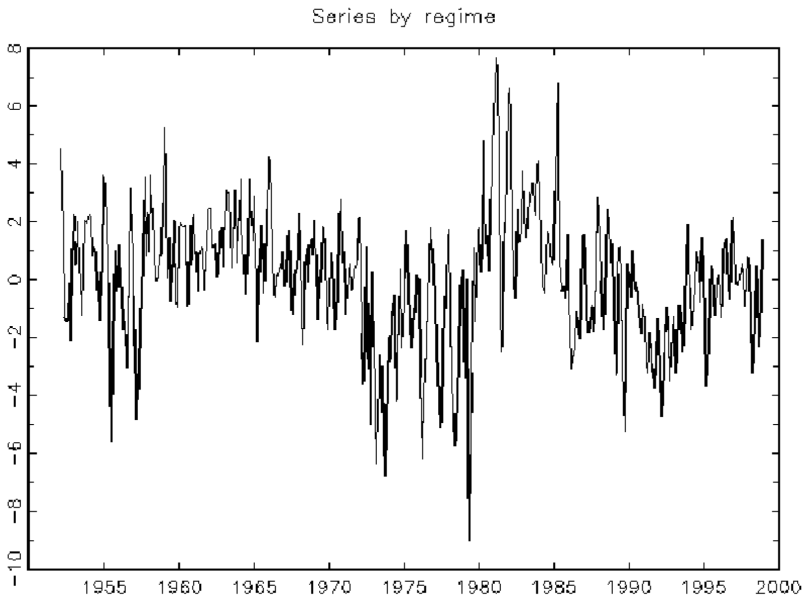

Figure 2: High and low regimes for the residuals of the cointegration relationship

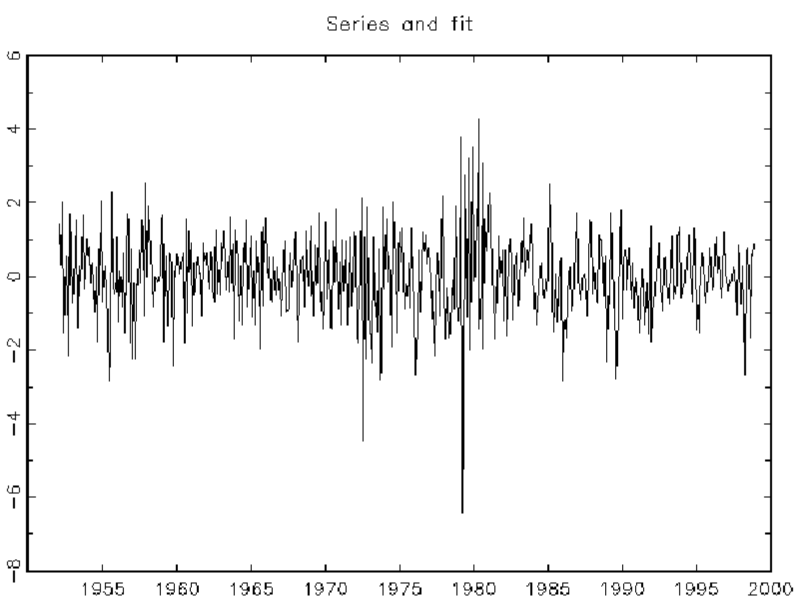

Figure 3: Residuals of the real rates on a constant and a dummy

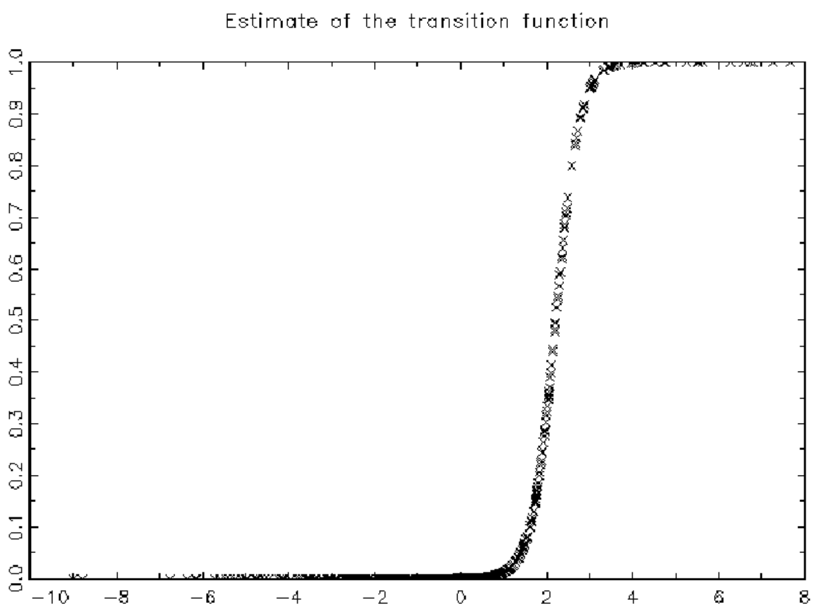

Figure 4: Logistic transition function versus the threshold variable (residuals of the real interest rates).

Type of test	Test Statistic	5% critical value
EG-ADF	-2.49	-3.36
Johansen (rk=0)	24.09*	20.0
Johansen (rk$\leqslant$1)	5.99	9.2
GH-C	-5.33*	-4.61
GH-C/S	-3.35	-4.95

Table 2: Cointegration test results

	SETAR1	SETAR2	LSTAR
number of lags	12	12	12
UR test p-value	0%	0.3%	0%
Delay order	8	4	1
Linearity test statistic	90.2	55.8	49.24
Linearity test p-value	0%	0.3%	0.4%
Threshold estimate	2.05	-3.0	2.2
β_{LR}	0.56	1	1

Table 3: SETAR tests results