Bargaining, Nontâtonnement, and Decreasing Density of Wealth Distribution

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Abstract: In studying the evolution of wealth distribution one needs some economically meaningful mechanism of wealth transfers. Traditional models of nontâtonnement with prices, e.g., the Edgeworth process, give us a good starting point. In this paper we show a model of discrete-time Edgeworth process that tends to yield a Paretian decreasing density of wealth distribution. Under the assumption that every agent has identical homothetic preferences, the process is characterized by the iteration of a bargaining, which can be reduced to a bargaining among "wealth class representatives". While there are "turnovers" of the agents, the representatives' bargaining at any instant of time renews the allocation of assets to another. We will analyze the final allocation of this process under some idealization, and see that the "class sum wealths" are equalized, so that the wealth distribution is hyperbolically decreasing, if the number of the agents is large and if the turnovers of the agents are frequent in the process. Some simulation results are also given, together with a remark on the significance of this self-organizing pattern to the Law of Demand. (*JEL Classification:* D310, D500)

Keywords: Self-organizing system, Wealth distribution, Edgeworth process, Bargaining, Law of demand

1 Introduction

Empirical wealth distributions are skewed to the right. This is well known since Pareto [10]. It was only recently, however, that we saw the general reasoning of this phenomenon. The progress was made by the so-called "econophysicists", who brought the tools of statistical physics into economics. This is a very promising approach, and would have wider applicability than to the problem at hand. But concerning to the problem of wealth distribution, their modeling of wealth transfers seems to have much to be done. For example, some assume that the wealths are directly transferred from poors to riches, not a very meaningful assumption in our market economy (where the equal value exchange, or *quid pro quo* is the premise). It should also be recognized that the most of recent econophysicists' efforts are poured into the establishment of Zipf's law, a specific version of Pareto's law, but the empirical validity of the former seems to be unsettled yet.

This paper is not a critique, but rather hopes to lay down the sound basis for the development of the new area, considering the problem in a more traditional framework in economics. In this paper we show a class of nontâtonnement processes (a discrete-time Edgeworth process) that tend to yield a decreasing density of wealth distribution. As one of traditional type of nontâtonnement models, it is a deterministic dynamic of the exchange of multiple "assets", which complies with the conservation of total assets and *quid pro quo.* Under the assumption that every agent has identical homothetic preferences, our process is characterized by the iteration of a bargaining, which can be reduced to a bargaining among "wealth class representatives". The representatives' bargaining is of the type of Nash [8]. While there are "turnovers" of the agents (the members of a class change because the prices change the wealth of each agent), the representatives' bargaining at any instant of time renews the allocation of assets to another, under the disequilibrium prices. We will analyze the final allocation of this nontâtonnement process, which is a Walrasian, no-trade equilibrium, and show that, under some idealization, the associated wealth distribution is hyperbolically decreasing, if the number of the agents is large, and if the turnovers of the agents are frequent during the process (in a specific sense in this paper). Basically, this follows from the fact that the "class sum wealths" are equalized in such a situation, leading to the reciprocal relationship between the individual level of wealth and the number of the agents belonging to each class.

It is important to note beforehand, however, that the process yields decreasing density of wealth distribution if, and only if, the prices change "wildly" enough during the process (i.e., if their convergence is oscillatory, for example). Here is the place where the tools of statistical physics would most usefully be applied. However, we will see this tendency only through some computer simulations in this paper. Formal analysis of the nonequilibrium state is left for the future studies, hopefully with the help of the new tools.

The paper is organized as follows. In Section 2 we state the model. In Section 3 we analyze the equilibrium of the nontâtonnement when the agents are numerous and the turnovers of the agents are frequent. Section 4 gives some simulation results. In Section 5 we give some concluding comments, especially on the possibility of generalizing the type of representatives' bargaining, and on the significance of the decreasing-ness to the emergence of the law of demand.

2 The Model

There are I agents $i = 1, \cdots, I$ and L assets $l = 1, \cdots, L$. Each i trades his *assets* $\bar{x}^i[t] \in X = \mathbb{R}^L_{++}$ under the *prices* $p[t] \in P = \mathbb{R}^L_{++}$ at each period $t = 0, 1, 2, \cdots$. We denote $(\bar{x}^1[t], \cdots, \bar{x}^I[t]) =: (\bar{x}^i[t])$ and call it the *allocation* at t. We make the following assumptions (U), (D), (B), and (A) to our discrete-time Edgeworth process (See Negishi [9] or Hahn [6] for the general discussion of Edgeworth process).

(U) Every i has identical homothetic preference relation, which is represented by a homogeneous of degree one *utility* $u^i \colon X \to \mathbb{R}_+$.

(D) At each t, each i determines his *demand* $y^i[t]$, as the solution to the problem:

$$\max_{y^i} u^i(y^i) \quad \text{s.t.} \quad p[t] \cdot y^i = p[t] \cdot \bar{x}^i[t]. \tag{1}$$

Our process is a nontâtonnement, i.e., trade takes place even if $\sum_i y^i[t] \neq \sum_i \bar{x}^i[t]$.

Let $w^i[t] := p[t] \cdot \bar{x}^i[t]$ be the *wealth* of i at t. Since the *normalized wealth* $\tilde{w}^i[t] := w^i[t]/\max_j w^j[t] \in (0, 1]$, we divide this unit interval into K segments of length $1/K$, and call $C^k[t] := \{i \colon (k-1)/K < \tilde{w}^i[t] \leq k/K\}$ the *class* k at t, $k = 1, \cdots, K$. We call $w^k[t] := \sum_{C^k[t]} w^i[t]$ the *class sum wealth* of k at t. For convenience, we assume $I = K^2$ to study the histogram of $(\#C^k[t]/K)$ and its limit of $K \to \infty$, the density function.

(B) At each t, a bargaining determines *agreement* $(\hat{x}^i[t])$ as the solution to the problem:

$$\max_{(x^i)} \quad \prod_k \Big(\prod_{C^k[t]} u^i(x^i)^{\frac{w^i[t]}{w^k[t]}} \Big) \tag{2}$$

$$\text{s.t.} \quad \sum_i x^i = \sum_i \bar{x}^i[t], \tag{3}$$

$$p[t] \cdot x^i = p[t] \cdot \bar{x}^i[t] \quad \forall i, \tag{4}$$

$$u^i(\bar{x}^i[t]) \leq u^i(x^i) \quad \forall i. \tag{5}$$

We call the type of Eq. (3) the *conservation* (CSV) and Eq. (4) the *quid pro quo* (QPQ). Ineq. (5) is called the *individual rationality* (IR).

Literally, (B) is the maximization of the product of agents' utilities powered by their *wealth shares in class* $(w^i[t]/w^k[t],\ i \in C^k[t])$. It is well known that the preferences of agents are aggregated under (U) in that there is an aggregate preference relation that generates their aggregate demand (Antonelli [1]; see Shafer and Sonnenschein [11] for the issue of aggregation). Also, Eisenberg's [3] theorem tells us that the aggregate preference relation of the agents in $C^k[t]$ is represented by the homogeneous of degree one function

$$U^k(x^k) := \max_{(x^i)_k} \prod_{C^k[t]} u^i(x^i)^{\frac{w^i[t]}{w^k[t]}} \quad \text{s.t.} \quad \sum_{C^k[t]} x^i = x^k, \tag{6}$$

if the wealth shares are fixed (this is the case for our one-shot bargaining). It is then quite natural to have the following lemma.

Lemma. Let $\bar{x}^k[t] := \sum_{C^k[t]} \bar{x}^i[t]$, $\hat{x}^k[t] := \sum_{C^k[t]} \hat{x}^i[t]$. Then $(\hat{x}^k[t])$ solves the *representatives' bargaining*:

$$\text{(RB)} \qquad \max_{(x^k)} \quad \prod_k \tilde{U}^k(x^k) \tag{7}$$

$$\text{s.t.} \quad \sum_k x^k = \sum_i \bar{x}^k[t], \tag{8}$$

$$p[t] \cdot x^k = p[t] \cdot \bar{x}^k[t] \quad \forall k, \tag{9}$$

$$\tilde{U}^k(\bar{x}^k[t]) \leq \tilde{U}^k(x^k) \quad \forall k, \tag{10}$$

where

$$\tilde{U}^k(x^k) := \max_{(x^i)_k} \prod_{C^k[t]} u^i(x^i)^{\frac{w^i[t]}{w^k[t]}} \quad \text{s.t.} \quad \sum_{C^k[t]} x^i = x^k, \tag{11}$$

$$p[t] \cdot x^i = p[t] \cdot \bar{x}^i[t], \ u^i(\bar{x}^i[t]) \leq u^i(x^i) \quad \forall i \in C^k[t]. \tag{12}$$

Proof. We can regard (B) as a two-step problem as follows: 1) the choice of (x^k) such that $\sum_k x^k = \sum_k \bar{x}^k[t]$ and $p[t] \cdot x^k = p[t] \cdot \bar{x}^k[t]$ for all k; 2) the distribution of x^k such that $\max_{(x^i)_k} \prod_{C^k[t]} u^i(x^i)^{w^i[t]/w^k[t]}$ subject to $\sum_{C^k[t]} x^i = x^k$, $p[t] \cdot x^i = p[t] \cdot \bar{x}^i[t]$ and $u^i(\bar{x}^i[t]) \leq u^i(x^i)$ for all i. But this is nothing but the problem (RB). Note that, since $(x^k) = (\bar{x}^k)$ gives $\tilde{U}^k(\bar{x}^k[t]) = \tilde{U}^k(x^k[t]) \,\forall k$ and since the bargaining set is convex, there is a Pareto improving (x^k) such that $\tilde{U}^k(\bar{x}^k[t]) \leq \tilde{U}^k(x^k) \,\forall k$ in case $(\bar{x}^k[t])$ and its distributions are suboptimal. □

Note that the definition of $\tilde{U}^k$ has two additional constraints (QPQ and IR) compared to that of U^k. They are the restrictions of possible agreements in the bargaining. They could be dropped under (U), but we do not try it in this paper. By the addition of them $\tilde{U}^k$ depends on $p[t]$ and $(\bar{x}^i[t])$, and in this sense, it may not be the representation of "pure" preferences. Nevertheless, it does serve for our purpose. Note also that the representatives' bargaining is of the type of Nash [8].

Finally, we add the following assumption on the adjustments in the process.

(A) The allocation $(\bar{x}^i[t])$ and the prices $p[t]$ of the assets change as follows:

$$\bar{x}^i[t+1] = \bar{x}^i[t] + \gamma_1(\hat{x}^i[t] - \bar{x}^i[t]) \quad \forall i, \tag{13}$$

$$p[t+1] = p[t] + \gamma_2[t](\sum_i y^i[t] - \sum_i \bar{x}^i[t]), \tag{14}$$

where $\gamma_1 \in (0,1]$ is a constant, and $\gamma_2[t] > 0$ is an appropriate real number which secures that $p[t+1] \in P$.

Note that $(\bar{x}^i[t+1])$ satisfies $\sum_i \bar{x}^i[t+1] = \sum_i \bar{x}^i[t]$ (CSV) and $p[t] \cdot \bar{x}^i[t+1] = p[t] \cdot \bar{x}^i[t]$ $\forall i$ (QPQ), two common constraints in the nontâtonnement with prices (see Hahn [6]). However, it is generally the case that

$$p[t+1] \cdot \bar{x}^i[t+1] \neq p[t] \cdot \bar{x}^i[t], \tag{15}$$

i.e., $w^i[t+1] \neq w^i[t]$. This is the basic mechanism of *wealth transfers*, which cause the *turnovers* of the agents. Also note that, in the aggregate, it is generally the case that $w^k[t+1] \neq w^k[t]$, and even $\tilde{U}^k(\bar{x}^k[t+1]) < \tilde{U}^k(\bar{x}^k[t])$ is possible.

3 Equilibrium of the Process

Let us consider the convergence of the process. We can choose the simple sum of utilities

$$\sum_i u^i(\bar{x}^i[t]) \tag{16}$$

as the monotonically increasing Liapunov function, and prove the convergence of the allocation $(\bar{x}^i[t])$.[1] If the prices $p[t]$ also converge, then we can say that the process is globally stable. (See Negishi [9] or Hahn [6] for the global stability of Edgeworth process.) The equilibrium allocation then is a *Walrasian equilibrium with no trade*, in that every individual demand $y^i[t]$ specified by (D) equals $\bar{x}^i[t]$ (in the limit of $t \to \infty$). We will consider such an equilibrium state in this section.

What we want to know in this paper is the "shape" of this final allocation $(\bar{x}^i) := (\bar{x}^1, \cdots, \bar{x}^I)$ (we will omit $[t]$ when we speak of the final state), i.e., how the *total assets* $\bar{x} := \sum_i \bar{x}^i$ are distributed among the agents except that it is Walrasian. Is it condensed or sparse, or, in terms of the final wealths $w^i = p \cdot \bar{x}^i$, how does the density function of the wealth look like?

Of course *any* Walrasian equilibrium with no trade can be the final allocation, if we start from there with its supporting price vector. We thus start from a disequilibrium state. Also, in order for the "natural" tendency of wealth movement to reveal itself, if any, a sufficient amount of wealth transfers under QPQ (realized by Ineq. (15)) is needed. In our discrete-time model, an oscillatorily converging price may entail such transfers.

Now, as we noted earlier, the turnovers of the agents generally cause $w^k[t+1] \neq w^k[t]$ and $\tilde{U}^k(\bar{x}^k[t+1]) \neq \tilde{U}^k(\bar{x}^k[t])$ for any class k. Hence, throughout the entire iterations of (RB), only the values of (CSV) are definite, and those of the (QPQ) and (IR) are under incessant fluctuations. This is a difficult point, but, in reference with the results of simulations, we want to hypothesize that the effects of those fluctuating constraints are wiped out, if the turnovers are such big.

Claim. Suppose that *frequent turnovers of the agents* drive the entire process to

$$\max_{(x^k)} \quad \prod_k \tilde{U}^k(x^k) \tag{17}$$

$$\text{s.t.} \quad \sum_k x^k = \sum_i \bar{x}^k[t]. \tag{18}$$

Then the equilibrium density of wealth distribution is hyperbolically decreasing if the agents are numerous.

Proof. Note that $(\hat{x}^i) = (\bar{x}^i)$ and $(\bar{x}^i)$ is colinear at equilibrium. To formulate the Lagrangian, we use the logarithm of the maximand. It is

$$\sum_k \ln \tilde{U}^k(x^k) - \mu \cdot (\sum_k x^k - \sum_k \bar{x}^k), \tag{19}$$

and the first-order conditions are

$$\frac{\nabla \tilde{U}^k(\hat{x}^k)}{\tilde{U}^k(\hat{x}^k)} = \hat{\mu} \quad \forall k. \tag{20}$$

Now, as $I = K^2 \to \infty$, we have $K(w^i/w^k) \to 1/(\#C^k/K)$ (where $\#C^k/K$ is finite), $\nabla u^i(\hat{x}^i)/u^i(\hat{x}^i) = \nabla u^j(\hat{x}^j)/u^j(\hat{x}^j)$ for any $i, j \in C^k$, and

$$K\frac{D\tilde{U}^k(\hat{x}^k)}{\tilde{U}^k(\hat{x}^k)} = K\sum_{C^k} \frac{w^i}{w^k} \frac{Du^i(\hat{x}^i)}{u^i(\hat{x}^i)} D\hat{x}^i(\hat{x}^k) \tag{21}$$

$$\to \frac{1}{\#C^k/K} \frac{Du^i(\hat{x}^i)}{u^i(\hat{x}^i)} \quad i \in C^k, \tag{22}$$

where $D\hat{x}^i(\hat{x}^k)$ is the Jacobian of the distribution rule in $\tilde{U}^k$, and by $\sum_{C^k} \hat{x}^i(x^k) \equiv x^k$, their sum over C^k is identity matrix. $Du^i(\hat{x}^i)$ is identical for every i, and $u^i(\hat{x})$ is proportional to w^i at equilibrium. The identicalness of $K\nabla\tilde{U}^k(\hat{x}^k)/\tilde{U}^k(\hat{x}^k)$ then implies that the class sum wealths are identical among the classes, and the density of wealth distribution is hyperbolically decreasing. □

Actually the QPQ of (RB) has no effect at equilibrium because the equilibrium allocation is in the *core* (in the sense of Debreu and Scarf [2]). Thus we may also add IR of the form

$$0 \leq \tilde{U}^k(x^k) \quad \forall k \tag{23}$$

in the above hypothesis, without changing the result.

[1] By IR, $u^i(\bar{x}^i[t+1]) \geq u^i(\bar{x}^i[t])$ $\forall i$. The inequality holds for at least one i during the disequilibrium state, so that $\sum u^i(\bar{x}^i[t+1]) > \sum u^i(\bar{x}^i[t])$ during the process. By the boundedness of $\bar{x}^i[t]$ the utility sum reaches its maximum, and the maximizer is unique by virtue of the concavity of u^i. Note that the final outcome depends on the choice of $\gamma_2[t]$ during the process.

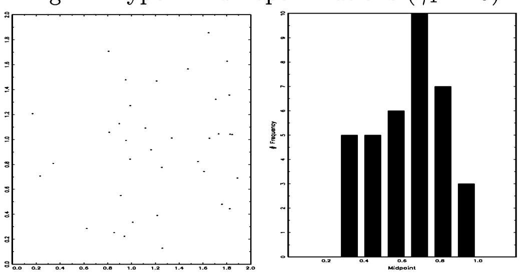
4 Simulations

The simulations were performed on PC with GAUSS 5.0 (Aptech). The built-in function `sqpSolve` was used to compute the solution of (B). This function uses the Sequential Quadratic Programming method for the computation.

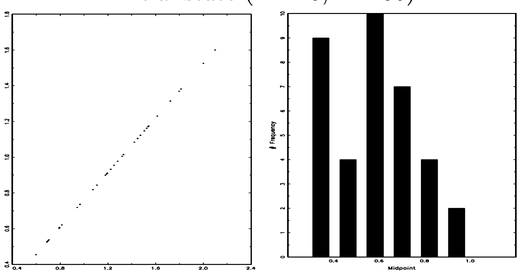
In order to visualize the movement of asset allocation the number of assets was fixed to two ($L = 2$). The utility function used was Cobb–Douglas, $x_1^{2/5} x_2^{3/5}$. During the runs the parameter γ_1 for the allocation adjustment was fixed to .5 or 1. The parameter $\gamma_2[t]$ for the price adjustment was calculated at each iteration, such that it would be 1) initially set to one, and 2) decreased by multiplying .9 until both prices became positive and made at least one pair of seller and buyer. This seemed to produce a sufficient fluctuation of prices, which were also converging.

We performed two types of simulation as follows. In the Type 1, the number of agents was 36 ($K = 6$, $I = 36$) and the "full" optimization (where the internal maximum iteration parameter `_sqp_MaxIters` of `sqpSolve` was default) was done on each step of the process. The parameter γ_1 was set to .5, to let the process be more susceptible to the fluctuation of prices. The setting of type 1 simulation is best conforming to that of our model, and its typical result is shown in Figure 1. The 2D-plot of equilibrium allocation and the histogram of the associated wealth distribution are shown.

Fig. 1: Type 1. Full optimizations ($\gamma_1 = .5$)



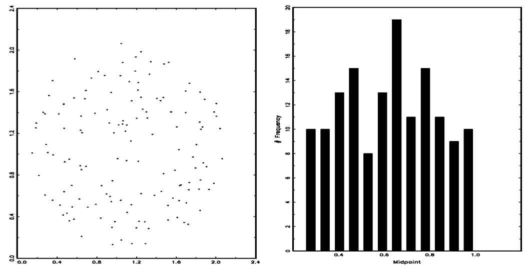
Initial state ($K = 6$, $I = 36$)



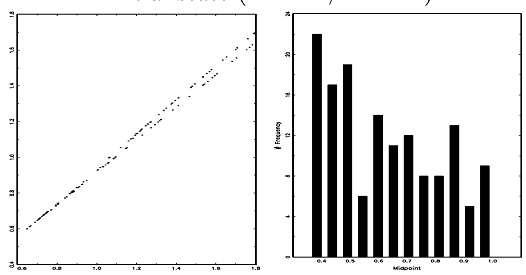
After 2,000 iterations

To perform simulations with a larger number of agents, we set, in the simulation of Type 2, the internal maximum iterations `_sqp_MaxIters` of `sqpSolve` to 1. That is, we truncated the computation of true maximum on each step. This may have the effect that is similar to setting $\gamma_1 < 1$, but the facial value of γ_1 was set to 1. Figure 2 is a typical result. The number of agents was 144.

Fig. 2: Type 2. Truncated optimizations ($\gamma_1 = 1$)



Initial state ($K = 12$, $I = 144$)



After 2,000 iterations

Many patterns were tested for each type of simulations and we obtained similar results. Although the scale of the simulations is very small, they all seem to show the characteristic of the process considered.

5 Concluding Comments

We have considered the emergence of a decreasing density of wealth distribution in the framework of Edgeworth process. Our special assumption was on the "bargaining powers" of the agents. They were set as if the agents were performing a coalitional bargaining among the wealth classes.

Actually, the formulation of bargaining in (B) mostly comes from a rather naïve thought: If the demand behaviors of the agents aggregate, why not their bargaining behaviors? If we push this thought a step further, however, a more appropriate formulation of bargaining would be

$$(\mathrm{B}') \qquad \max_{(x^i)} \prod_i u^i(x^i)^{\frac{w^i[t]}{w[t]}} \tag{24}$$

$$\text{s.t.} \quad \sum_i x^i = \sum_i \bar{x}^i[t], \tag{25}$$

$$p[t] \cdot x^i = p[t] \cdot \bar{x}^i[t] \quad \forall i, \tag{26}$$

$$u^i(\bar{x}^i[t]) \leq u^i(x^i) \quad \forall i, \tag{27}$$

where $w[t] := \sum_i w^i[t]$ is the *total wealth* of the agents at t. The reason is as follows. Let $(S^\lambda[t])$ be *any* partition of I agents at t. Then since

$$\prod_\lambda \Big(\prod_{S^\lambda[t]} u^i(x^i)^{\frac{w^i[t]}{w^\lambda[t]}} \Big)^{\frac{w^\lambda[t]}{w[t]}} = \prod_i u^i(x^i)^{\frac{w^i[t]}{w[t]}}, \tag{28}$$

(B$'$) is represented by

$$(\mathrm{RB}') \qquad \max_{(x^\lambda)} \prod_\lambda \tilde{U}^\lambda(x^\lambda)^{\frac{w^\lambda[t]}{w[t]}} \tag{29}$$

$$\text{s.t.} \quad \sum_\lambda x^\lambda = \sum_\lambda \bar{x}^\lambda[t], \tag{30}$$

$$p[t]\cdot x^\lambda = p[t]\cdot \bar{x}^\lambda[t] \quad \forall\lambda, \tag{31}$$

$$\tilde{U}^\lambda(\bar{x}^\lambda[t]) \le \tilde{U}^\lambda(x^\lambda) \quad \forall\lambda, \tag{32}$$

where $\tilde{U}^\lambda$ and other symbols are similarly defined as in (RB). That is, we can reduce (B$'$) to *any* form of representatives' bargaining (RB$'$). That this *arbitrary* reducibility requires the form of bargaing (B$'$) may also be clear (consider the Eisenberg's theorem).

If we use (B$'$) and (RB$'$) instead of (B) and (RB) in our model, however, the conclusion of our Claim, the identicalness of $(1/(\#C^k/K))(\nabla u^i(\hat{x}^i)/u^i(\hat{x}^i))$, is modified to that of

$$\frac{1}{\#C^k/K}\frac{w^k}{w}\frac{Du^i(\hat{x}^i)}{u^i(\hat{x}^i)}. \tag{33}$$

The reason we did not adopt (B$'$) was that we cannot deduce implications from the identicalness of the above expression (except that the equality of w^k/w is equivalent to the previous condition, which is almost tautological), for one thing, and the results of simulation using (B$'$) were ambiguous, for another thing. However, there is a strong reason that (B$'$) may also lead to a similar result: In our process, those who gain are those who buy or sell on the long side of the markets. For example, if one sells as much as he wants in the excess supply market, he can avoid the loss from falling price the best. Roughly speaking, (B$'$) favors for the wealthier agents. As some econophysicists' simple models suggest, this would lead to some kind of decreasing wealth distribution.

There is another, possibly more significant, theoretical importance in the problem of wealth distribution. We will shortly comment on this point here. Since the pioneering work of Hildenbrand [7], it is known that the law of demand holds in the consumption sector of economy if the wealth (income, or expenditure, in his treatment) distribution is decreasing. The heterogeneity of the agents' tastes is also known to yield the law of demand (Grandmont [4, 5]). In our view, however, the distribution of wealth is more easily observed than that of preferences, and, more importantly, would be better treated as an "endogenous" variable of our economic system. It would be worth mentioning that the hyperbolically decreasing density of wealth distribution conforms to the type of distribution that Grandmont ([4], Proposition 1) has generalized the Hildenbrand's [7], which secures the establishment of the law of demand.

Finally, we note that our analysis was solely limited to the equilibrium state of the system (with some idealization). In order to study the disequilibrium state of the process, it is certainly desirable to exploit the tools in statistical physics, by modeling the large economy from the outset. It may also be true that the more satisfactory simulations will reveal the nature of the process more precisely. We hope that our study could show a possible link between the traditional mainstream economics and those of the newborn active areas of researches.

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