Testing for Changing Persistence in U.S. Treasury On/Off Spreads Under Weighted-Symmetric Estimation

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Abstract: We extend the recursive break test procedure of Leybourne et al. by using weighted-symmetric estimation to detect a single change in time series persistence. An application to U.S. Treasury bond on/off spreads finds a significant change in persistence from $I(0)$ to $I(1)$ in the late 1990s.

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1 Introduction

Commentators have noted that U.S. budget surpluses in the late 1990s led to a staged contraction in the supply of Treasury bonds with a series of debt management policy changes since 1998. The Treasury's debt buyback program, launched in March 2000, repurchased longer term bonds from the markets expecting that the surpluses would continue, but the U.S. fiscal position began reverting in response to the macroeconomic slowdown of 2000-2001 and the impact of September 11, 2001. Against this background, financial market uncertainty was growing since the Asian crises of 1997-1998, compounded by Russia's default and the hedge fund LTCM's near-collapse in autumn 1998.

Global market turmoil prompted research on the dynamics of the Treasury bond market.[1] In principle, Treasury bonds whose remaining time to maturity and other characteristics are similar should trade at approximately the same price. In practice, however, illiquid bond yields tend to be higher than their more liquid counterparts. Thus, researchers have interpreted this yield differential as a time-varying liquidity premium, which is expected to be mean-reverting. Its significance has been documented across the term structure, with the debt buyback found to be an important explanatory variable.

[1] See Boni and Leach (2002), Fleming (2000, 2002), Furfine and Remolona (2002), Krishnamurthy (2003) and Longstaff (2003).

This raises the question of whether the above events and related uncertainty led to a change in persistence of the Treasury liquidity premium from a stationary, $I(0)$, to a nonstationary, $I(1)$, process. Determining the location and direction of such changes is a key issue for policy makers and market forecasters alike (Kim (2000) and Newbold et al. (2001)).

We address this question using the U.S. Treasury's 1-year bill and 5-year note daily on/off spreads, defined as the yield differential between the on-the-run (most recent, or active) and the first off-the-run (older) bond issue for each maturity. The null hypothesis is that the data is $I(1)$ throughout, and the alternative is a change from $I(0)$ to $I(1)$. The recursive procedure of Leybourne et al. (2003b)—henceforth LKSN—is extended by adopting weighted-symmetric estimation of the unit root coefficient.[2] LKSN develop GLS-based recursive and sequential DF unit root tests for detecting a single possible change in persistence under the alternative.[3] These tests allow for an unknown breakpoint and, in their general form, unknown direction of change in persistence. In the presence of GARCH and non-normality, we also undertake a Monte Carlo study of standard and recursive weighted-symmetric tests.

[2]This estimation method is known to yield more powerful t-tests than standard DF and unit root tests using Generalized Least Squares (GLS)-detrending under trend-stationarity. On related power gains see Leybourne et al. (2003a) and Pantula et al. (1994).

[3]These authors find that recursive tests are more robust to parameter instability.

2 The model

Assume the true process for T observations on y_t is

$$\begin{aligned} y_t &= d_t + u_t \quad , \quad d_t = z_t' \beta \qquad (1) \\ u_t &= \alpha u_{t-1} + \phi(L) \Delta u_{t-1} + \epsilon_t \ , \end{aligned}$$

where $z_t = [1, t]'$ and $\beta = [\beta_0, \beta_1]'$. We restrict attention to $\beta_1 = 0$, without loss of generality. Lag polynomial $\phi(L)$ is of known order $p - 1$, where the roots of $1 - \phi(L) = 0$ lie outside the unit circle. The errors follow a martingale difference sequence and the first $p - 1$ values of y_t are assumed to exist. The null hypothesis H^{11} is that y_t is $I(1)$ throughout, or $\alpha = 1$, and the alternative is that y_t undergoes a change in persistence from $I(0)$ to $I(1)$ at observation $\tau^* T$,

$$\begin{aligned} |\alpha| &< 1 \ , \qquad t \leq \tau^* T \qquad (2) \\ \alpha &= 1 \ , \qquad t > \tau^* T \end{aligned}$$

or from $I(1)$ to $I(0)$ at $\tau^* T$, implying the time-reversed series $\widetilde{y}_t = y_{T-t+1}$ changes from $I(0)$ to $I(1)$ at $(1 - \tau^*)T$, where break fraction τ^* is unknown. The two alternatives are denoted by H^{01} and H^{10}, respectively.

Our test statistics are constructed as follows. After detrending the whole series by OLS, $y_t^d = y_t - \hat{\beta}_0(\tau),\ t = 1, 2, ..., T$, an ADF regression with no deterministic trend is ran recursively on y_t^d using only the first τT observations,

$$\Delta y_t^d = \widehat{\rho}(\tau) y_{t-1}^d + \Sigma_{j=1}^{p-1} \hat{\phi}_j(\tau) \Delta y_{t-j}^d + \hat{\epsilon}_t \ , \qquad t = 1, 2, ..., \tau T \tag{3}$$

for varying break fraction τ.[4] Following Fuller (1996), weighted-symmetric estimation of $\rho(\tau)$ minimizes

$$Q(\boldsymbol{\theta}) = \sum_{t=p+1}^{T} w_t \left(\Delta y_t^d - \rho(\tau) y_{t-1}^d - \sum_{j=1}^{p-1} \phi_j(\tau) \Delta y_{t-j}^d \right)^2 \tag{4}$$

$$+ \sum_{t=1}^{T-p} (1 - w_{t+1}) \left(\Delta y_t^d - \rho(\tau) y_{t+1}^d + \sum_{j=1}^{p-1} \phi_j(\tau) \Delta y_{t+j+1}^d \right)^2 \tag{5}$$

for all τ, where $\boldsymbol{\theta} = (\rho, \boldsymbol{\phi})$, $\boldsymbol{\phi} = \{\phi_1, \phi_2, ..., \phi_{p-1}\}$ and w_t is

$$w_t = \begin{cases} 0, & 1 \leq t < p+1 \\ (t-p)/(T-2p+2), & p+1 \leq t < T-p+2 \\ 1, & T-p+2 \leq t \leq T \ . \end{cases}$$

The estimated error standard deviation is $\widehat{\sigma}(\tau) = \frac{Q(\widehat{\theta})}{T-p-2}$. The t-statistic for $\widehat{\rho}(\tau)$ is $WS(\tau) = \frac{\widehat{\rho}(\tau)}{\sqrt{\widehat{var}(\widehat{\rho}(\tau))}}$, where $\widehat{var}(\widehat{\rho}(\tau)) = \widehat{\sigma}^2(\tau) h_{PP}$ and h_{PP} is the $[1,1]$ element of $\left(\frac{\partial^2 Q(\boldsymbol{\theta})}{\partial\theta\partial\theta'}\right)^{-1}$. The statistic for testing alternative H^{01} is

$$WS^{f\ inf}(\tau) = inf_{\ \tau \in\ \Lambda}\ WS^f(\tau), \tag{6}$$

where f denotes the recursive test on the forward series, Λ is a non-empty closed interval in $(0,1)$ and the break fraction minimizing (6) is τ^*. When the alternative is a switch from $I(1)$ to $I(0)$, this statistic can be applied to the time-reversed series $\widetilde{y}_t^d$

[4] In our application τ varies between 0.15 and 0.85 in 0.01 increments. Note that LKSN use GLS-detrending and trim at 0.20 employing the usual ADF statistics.

$$\Delta\widetilde{y}_t^d = \widetilde{\rho}(\tau)\widetilde{y}_{t-1}^d + \Sigma_{j=1}^{p-1}\,\widetilde{\phi}_j(\tau)\Delta\widetilde{y}_{t-j}^d + \widetilde{\epsilon}_t\ , \qquad t = 1, 2, ..., (1-\tau)T \tag{7}$$

where τ^* in forward time is $(1-\tau^*)$ in reverse time. Denoting the t-ratio for $\widetilde{\rho}(\tau)$ by $WS^r(\tau)$, the statistic for testing H^{11} against H^{10} is

$$WS^{r\ inf}(\tau) = inf_{\ \tau\in\ \Lambda}\ WS^r(\tau)\ , \tag{8}$$

with r denoting the test on the reverse series.

If one is *a priori* uncertain about the direction of change in persistence, a "two-sided" test can be constructed whose null is $I(1)$ throughout against the alternative of a change from $I(0)$ to $I(1)$ or *vice versa* at break fraction τ^*. The statistic is then the pairwise minimum of $WS^{f\ inf}$ and $WS^{r\ inf}$

$$\min(WS^{f\,\inf}, WS^{r\,\inf}) \tag{9}$$

Following LKSN and existing asymptotic results in Pantula et al (1994) and Park and Fuller (1995) it can be shown that the $WS^{f\,\inf}$ and $WS^{r\,\inf}$ tests are consistent only in the direction for which they are designed. Thus, the $\min(WS^{f\,\inf}, WS^{r\,\inf})$ test will also be consistent under H^{01} or H^{10}. Moreover, all test statistics will estimate the break fraction consistently against the true alternative.

3 Application: Treasury bond on/off spreads

Our sample period is 17.6.1991-31.12.2002, i.e. 504 weekly observations for the 1-year Treasury bill on/off spread level—the yield differential between the first off-the-run and the on-the-run issues—and 592 for the 5-year note on/off spread.[5] The spreads are expected to be positive on average. Figure 1 shows the series in basis points and Table 1 summarizes their distributional properties.

FIGURE 1 & TABLE 1 HERE

Both spreads are tightly distributed around their mean until the late 1990s when they become more volatile. There is significant excess kurtosis and also GARCH effects. The GARCH ϕ_1 coefficient (short-run variation in volatility) was estimated close to 0.1 in both cases, and $\phi_1 + \phi_2$ (persistence in volatility) was around 0.8. Table 2, Panel A reports ADF and standard WS tests on the whole sample.

TABLE 2 HERE

Lag order p is selected using the sequential 0.10 level t-tests for the longest lag coefficient's significance, recommended by Ng and Perron (1995). Nonstationarity is rejected for the 1-year bill and not rejected for the 5-year note, as ADF and standard WS tests are inconsistent in the presence of a break in persistence. Critical values for all tests are given in Appendix A.

[5]The data source is GovPX. We use Wednesday observations from the daily data to address day-of-the-week effects. Inflation-indexed and callable bond issues are excluded, as are holidays and observations more than 30 basis points, reflecting posting errors not filtered out by GovPX. The 1-year bill was discontinued in May 2001.

Although standard DF tests are asymptotically robust to GARCH, they tend to overreject when $\phi_1 + \phi_2$ is close to unity; see Kim and Schmidt (1993) and Seo (1999). Given the time-varying volatility in our series, we quantify this finding for WS-based tests in Monte Carlo simulations. When the standard WS statistic is corrected for GARCH using White's (1980) covariance matrix, nonstationarity is not rejected for either series; WS_w denotes the White-corrected statistic.

Our discussion in section 1 suggests that the alternative hypothesis is a change in persistence from $I(0)$ to $I(1)$ at observation $\tau^* T$. Thus, we apply the $WS^{f\,\inf}$ test in equation (6) to the series. The results in Table 3, Panels A and B are for the non-White-corrected and White-corrected versions of $WS^{f\,\inf}$ and $WS_w^{f\,\inf}$, and we also include the reverse and "two-sided" statistics, $WS^{r\ inf}$ and $\min(WS^{f\,\inf}, WS^{r\,\inf})$ respectively.

TABLE 3 HERE

For the 1-year Treasury bill, $WS^{f,\inf}$ and $\min(WS^{f,\inf}, WS^{r\,\inf})$ both reject the null at the 0.05 level. Supporting this, the null is not rejected under $WS^{r\ inf}$. Note that the White-corrected statistics $WS_w^{f\,\inf}$ and $WS_w^{r\,\inf}$ point in the same direction. The rejections are less significant due to lower test power, confirmed in the subsequent Monte Carlo study. For the 5-year note, $WS_w^{f,\inf}$ does not reject the null at the 0.05 level and $\min(WS^{f\,\inf}, WS^{r\,\inf})$ does not reject at the 0.10 level, both narrowly. The switch points from $I(0)$ to $I(1)$ according to the non-White-corrected statistics are found in

July 1997 for the 5-year note, and March 1999 for the 1-year bill. The corresponding changes identified by the White-corrected statistics are in May 1998 and March 1999.

Finally, in Table 4 we report standard WS tests on the pre- and post-break subsamples. Break dates are determined by forward-based recursive test (6).

TABLE 4 HERE

The pre-break and post-break subsamples are stationary and nonstationary, respectively, with and without the White-correction. Our findings suggest that a significant switch from $I(0)$ to $I(1)$ in U.S. Treasury bond on/off spread levels occurred in the late 1990s. This was likely triggered by emerging market financial crises and the Treasury's debt management policy changes.

4 Monte Carlo simulations

This section investigates the size and power properties of WS-based tests and their White-corrected versions under non-normality and conditional heteroscedasticity.We assume y_t follows the AR(1) process $y_t = ay_{t-1}+\epsilon_t$, where $\epsilon_t = h_t^{1/2}v_t$, $h_t = \phi_0 + \phi_1\epsilon_{t-1}^2 + \phi_2\ h_{t-1}$. The errors v_t are $t(5)$ or $\chi^2(3)$-distributed with the degrees of freedom based on the higher-order sample moments in Table 1. The standard normal is also considered for comparison purposes. We focus on cases of near integration, namely (ϕ_1, ϕ_2) GARCH combinations $\{(0.05, 0.9), (0.1, 0.8), (0.3, 0.6)\}$. The rejection frequencies at the nominal 0.05 level are in Table 5.

TABLE 5 HERE

The standard WS statistic is slightly oversized when $T = 100$, and more so with greater short-run volatility ϕ_1, as in Kim and Schmidt (1993) and Seo (1999). In large samples, size distortions persist for χ^2 errors only. The WS_w statistic effectively corrects these overrejections but can be somewhat undersized. As α declines, power increases across GARCH parameterizations for given sample size. Similarly for fixed α when T increases, reflecting consistency of the tests. Note that the ability of WS_w to control for size comes at some loss in power. Differences in power between WS and WS_w decline with sample size and are larger under non-normal errors.

Table 6 reports Monte Carlo simulations for recursive statistic $WS^{f\,\inf}$.[6]

TABLE 6 HERE

To some extent, the results are similar to the standard WS statistic in Table 5. At $T = 200$, size distortions are larger for greater ϕ_1 and persist for $T = 500$, but are effectively corrected using $WS_w^{f\,\inf}$.[7] In terms of power, consistency of the recursive test is apparent. Moreover, both $WS^{f\,\inf}$ and

[6] Results for the reverse and min tests are available upon request.

[7] Recursive tests can also be oversized when GARCH effects are less persistent in the case of large ϕ_1 and non-normal errors. The condition for the existence of the errors'

$WS_w^{f\,\inf}$ statistics gain power for larger τ^*; their power difference generally declines with sample size.

Finally, Table 7 reports on the accuracy of the estimated break point.

TABLE 7 HERE

Estimation becomes more accurate with sample size. Although for a larger break fraction $WS_w^{f\,\inf}$ mildly underestimates as α increases and this is more pronounced for non-normal errors and greater ϕ_1, overall $WS_w^{f\,\inf}$ is more accurate than $WS^{f\,\inf}$. The latter tends to overestimate the break point mainly for smaller sample sizes and break fractions.

We conclude that employing the White-corrected version of the standard WS test is advisable to the extent that GARCH effects are persistent and ϕ_1 is large. The same applies to the recursive test when ϕ_1 is large. In both cases, correcting works relatively better in large samples. If ϕ_1 is small, White-corrected tests are not oversized but become somewhat less powerful.

5 Concluding remarks

This paper extended the recursive test procedure of LKSN by adopting weighted-symmetric estimation to detect a single change in time series persistence. An application to U.S. Treasury bond on/off spreads found a significant break from $I(0)$ to $I(1)$ in the late 1990s. Our results suggest that

fourth moment under White correction is $3\phi_1^2 + 2\phi_1\phi_2 + \phi_2^2 < 1$.

financial and macroeconomic uncertainty—the ongoing reversal in the U.S. fiscal position following the debt reduction initiative—have affected the persistence properties of Treasury bond liquidity premia.

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Appendix A

Simulated critical values

Panel A Statistic	T	0.01	0.05	0.10	*Panel B* Statistic	T	0.01	0.05	0.10
ADF	500	-3.420	-2.875	-2.578	ADF_w	500	-3.453	-2.905	-2.598
WS	100	-3.124	-2.552	-2.235	WS_w	100	-2.857	-2.299	-2.007
	250	-3.160	-2.554	-2.255		250	-2.796	-2.262	-1.982
	350	-3.111	-2.538	-2.255		350	-2.737	-2.232	-1.971
	400	-3.080	-2.543	-2.222		400	-2.733	-2.225	-1.949
	500	-3.109	-2.540	-2.228		500	-2.745	-2.220	-1.942
$WS^{f\,\inf}$	504	-3.909	-3.325	-3.030	$WS^{f\,\inf}$	504	-3.529	-3.004	-2.729
	592	-3.865	-3.318	-3.015		592	-3.515	-2.990	-2.707
$WS^{r\,\inf}$	504	-3.943	-3.323	-3.033	$WS_w^{r\,\inf}$	504	-3.578	-3.003	-2.721
	592	-3.887	-3.320	-3.027		592	-3.538	-2.993	-2.714
$\min(.,.)$	504	-4.162	-3.586	-3.309	$\min_w(.,.)$	504	-3.770	-3.252	-2.993
	592	-4.105	-3.568	-3.300		592	-3.748	-3.222	-2.975

Note: Statistic $\min(WS_w^{f,\inf}, WS_w^{r,\inf})$ is abbreviated by $\min(.,.)$.

TABLE 1

On/off bond spread statistics: 1991-2002

Statistics	1Y	5Y
Mean	-1.632	0.684
Std. Dev.	6.311	4.202
Max	28.90	12.00
Min	-26.50	-14.80
Skewness	1.518	-0.771
Kurtosis	8.437	4.947
Jarque-Bera	820.76	152.18

Note: $T = 504$ (592) weekly observations for the 1-year bill (5-year note). Units are basis points.

TABLE 2

Standard WS and ADF unit root tests: whole sample

Panel A		
Series	WS	ADF
1Y	-3.168^a	-3.009^b
5Y	-1.711	-1.409
Panel B		
Series	WS_w	ADF_w
1Y	-1.680	-2.102
5Y	-1.209	-1.299

Note: a, b, c denote 0.01, 0.05 and 0.10 significance levels.

TABLE 3
Recursive WS tests for a change in persistence

Series	$WS^{f\,\inf}$	Break date	$WS^{r\,\inf}$	Break date	min(., .)	Break date
Panel A						
1Y	-3.602^b	03/03/99	-2.808	n/a	-3.602^b	03/03/99
5Y	-3.603^b	30/07/97	-2.371	n/a	-3.603^b	30/07/97
Panel B						
Series	$WS_w^{f\,\inf}$	Break date	$WS_w^{r\,\inf}$	Break date	min(., .)	Break date
1Y	-3.282^b	24/03/99	-1.692	n/a	-3.282^b	24/03/99
5Y	-2.927^c	27/05/98	-1.555	n/a	-2.927	n/a

Note: Statistic $\min(WS_w^{f,\inf}, WS_w^{r,\inf})$ is abbreviated by min(., .). Break dates are reported only when the null is rejected. The significant break points are 395 (03/03/99), 314 (30/07/97), 398 (24/03/99), 357 (27/05/98).

TABLE 4
Standard WS unit root tests: subsamples

Panel A	$WS^{f\,\inf}$	
Series	Pre-break	Post-break
1Y	-3.602^a	-1.847
5Y	-3.603^a	-1.736
Panel B	$WS_w^{f\,\inf}$	
Series	Pre-break	Post-break
1Y	-3.282^a	-1.651
5Y	-2.927^b	-1.261

TABLE 5

Empirical size and power of WS test at nominal 0.05-level under GARCH(1,1)

			$T=100$							
ϕ_1	ϕ_2		α: 0.70		0.80		0.90		1.00	
			WS	WS_w	WS	WS_w	WS	WS_w	WS	WS_w
0.05	0.9	$N(0,1)$	1.000	0.992	0.968	0.927	0.531	0.483	0.055	0.056
		$t(5)$	0.999	0.960	0.974	0.846	0.623	0.469	0.055	0.042
		$\chi^2(3)$	0.996	0.909	0.958	0.746	0.546	0.368	0.050	0.046
0.1	0.8	$N(0,1)$	0.999	0.986	0.964	0.898	0.536	0.465	0.058	0.053
		$t(5)$	0.998	0.936	0.971	0.808	0.625	0.438	0.057	0.040
		$\chi^2(3)$	0.994	0.885	0.956	0.706	0.564	0.351	0.059	0.045
0.3	0.6	$N(0,1)$	0.995	0.922	0.944	0.764	0.553	0.387	0.072	0.050
		$t(5)$	0.996	0.856	0.955	0.690	0.626	0.364	0.075	0.036
		$\chi^2(3)$	0.989	0.811	0.948	0.628	0.613	0.312	0.079	0.046
			$T=200$							
0.05	0.9	$N(0,1)$	1.000	1.000	1.000	0.999	0.960	0.913	0.058	0.060
		$t(5)$	1.000	0.994	1.000	0.981	0.975	0.856	0.058	0.047
		$\chi^2(3)$	1.000	0.990	0.999	0.948	0.960	0.735	0.058	0.048
0.1	0.8	$N(0,1)$	1.000	1.000	1.000	0.995	0.956	0.885	0.060	0.058
		$t(5)$	1.000	0.986	0.999	0.960	0.971	0.809	0.063	0.046
		$\chi^2(3)$	1.000	0.974	0.998	0.916	0.956	0.678	0.061	0.046
0.3	0.6	$N(0,1)$	1.000	0.982	0.998	0.942	0.934	0.716	0.076	0.051
		$t(5)$	0.999	0.942	0.998	0.874	0.955	0.648	0.074	0.043
		$\chi^2(3)$	0.998	0.916	0.995	0.800	0.949	0.535	0.082	0.044
			$T=500$							
0.05	0.9	$N(0,1)$	1.000	1.000	1.000	1.000	1.000	1.000	0.056	0.053
		$t(5)$	1.000	0.998	1.000	0.995	1.000	0.986	0.054	0.043
		$\chi^2(3)$	1.000	0.999	1.000	0.997	0.999	0.965	0.058	0.050
0.1	0.8	$N(0,1)$	1.000	1.000	1.000	1.000	1.000	1.000	0.056	0.052
		$t(5)$	1.000	0.966	1.000	0.990	1.000	0.971	0.053	0.040
		$\chi^2(3)$	1.000	0.997	1.000	0.988	0.999	0.930	0.061	0.046
0.3	0.6	$N(0,1)$	1.000	0.996	1.000	0.987	0.999	0.941	0.064	0.044
		$t(5)$	1.000	0.977	1.000	0.946	0.999	0.850	0.065	0.035
		$\chi^2(3)$	1.000	0.960	1.000	0.904	0.997	0.755	0.080	0.035

Note: The DGP is $y_t = \alpha y_{t-1} + \epsilon_t$, where $\epsilon_t = h_t^{1/2} v_t$, $h_t = \phi_0 + \phi_1 \epsilon_{t-1}^2 + \phi_2\, h_{t-1}$ and v_t is distributed as in the first column. The t and χ^2 distributions are standardized as $\frac{t(n)}{(\frac{n}{n-2})^{1/2}}$ and $\frac{\chi^2(n)-n}{(2n)^{1/2}}$ with n degrees of freedom. The unconditional variance is 1 by setting $\phi_0 = 1 - \phi_1 - \phi_2$, without loss of generality. The number of replications is 5000.

TABLE 6

Empirical size and power (under H^{01}): $WS^{f\,\inf}$ at 0.05 level under GARCH(1,1) as in Table 5

						$\tau^* = 0.5$				
						$T = 200$				
ϕ_1	ϕ_2	α:	0.70		0.80		0.90		1.00	
0.05	0.9		$WS^{f\,\inf}$	$WS_w^{f\,\inf}$	$WS^{f\,\inf}$	$WS_w^{f\,\inf}$	$WS^{f\,\inf}$	$WS_w^{f\,\inf}$	$WS^{f\,\inf}$	$WS_w^{f\,\inf}$
		$N(0,1)$	0.994	0.959	0.867	0.727	0.385	0.262	0.067	0.064
		$t(5)$	0.989	0.792	0.862	0.750	0.378	0.166	0.070	0.042
		$\chi^2(3)$	0.991	0.774	0.874	0.521	0.392	0.199	0.059	0.047
0.1	0.8	$N(0,1)$	0.991	0.935	0.859	0.704	0.387	0.262	0.066	0.059
		$t(5)$	0.984	0.780	0.846	0.509	0.386	0.171	0.071	0.039
		$\chi^2(3)$	0.990	0.766	0.871	0.514	0.415	0.203	0.060	0.044
0.3	0.6	$N(0,1)$	0.984	0.843	0.862	0.592	0.455	0.227	0.093	0.061
		$t(5)$	0.978	0.705	0.846	0.452	0.445	0.161	0.098	0.042
		$\chi^2(3)$	0.984	0.734	0.891	0.499	0.493	0.212	0.088	0.050
						$T = 500$				
0.05	0.9	$N(0,1)$	1.000	1.000	1.000	0.999	0.959	0.881	0.061	0.061
		$t(5)$	1.000	0.981	1.000	0.946	0.957	0.700	0.068	0.049
		$\chi^2(3)$	1.000	0.980	1.000	0.924	0.961	0.670	0.063	0.050
0.1	0.8	$N(0,1)$	1.000	1.000	1.000	0.996	0.952	0.855	0.061	0.056
		$t(5)$	1.000	0.975	1.000	0.928	0.950	0.664	0.070	0.042
		$\chi^2(3)$	1.000	0.971	0.999	0.897	0.958	0.621	0.066	0.045
0.3	0.6	$N(0,1)$	1.000	0.988	1.000	0.947	0.939	0.680	0.091	0.048
		$t(5)$	1.000	0.936	0.999	0.831	0.936	0.508	0.103	0.036
		$\chi^2(3)$	1.000	0.919	0.997	0.810	0.957	0.506	0.097	0.041
						$\tau^* = 0.7$				
						$T = 200$				
ϕ_1	ϕ_2	α:	0.70		0.80		0.90		1.00	
0.05	0.9		$WS^{f\,\inf}$	$WS_w^{f\,\inf}$	$WS^{f\,\inf}$	$WS_w^{f\,\inf}$	$WS^{f\,\inf}$	$WS_w^{f\,\inf}$	$WS^{f\,\inf}$	$WS_w^{f\,\inf}$
		$N(0,1)$	1.000	0.995	0.981	0.885	0.503	0.351	0.067	0.064
		$t(5)$	0.998	0.905	0.971	0.691	0.492	0.224	0.070	0.042
		$\chi^2(3)$	0.999	0.874	0.979	0.663	0.538	0.253	0.059	0.047
0.1	0.8	$N(0,1)$	1.000	0.990	0.978	0.865	0.516	0.343	0.066	0.059
		$t(5)$	0.998	0.892	0.967	0.668	0.508	0.225	0.071	0.039
		$\chi^2(3)$	0.999	0.867	0.976	0.646	0.559	0.258	0.060	0.044
0.3	0.6	$N(0,1)$	0.998	0.924	0.964	0.725	0.573	0.284	0.093	0.061
		$t(5)$	0.996	0.806	0.955	0.567	0.551	0.205	0.098	0.042
		$\chi^2(3)$	0.997	0.817	0.973	0.598	0.641	0.254	0.088	0.050
						$T = 500$				
0.05	0.9	$N(0,1)$	1.000	1.000	1.000	1.000	0.999	0.981	0.061	0.061
		$t(5)$	1.000	0.990	1.000	0.976	0.997	0.989	0.068	0.049
		$\chi^2(3)$	1.000	0.993	1.000	0.968	0.997	0.794	0.063	0.050
0.1	0.8	$N(0,1)$	1.000	1.000	1.000	1.000	0.999	0.962	0.061	0.056
		$t(5)$	1.000	0.987	1.000	0.965	0.995	0.806	0.070	0.042
		$\chi^2(3)$	1.000	0.984	1.000	0.943	0.996	0.746	0.066	0.045
0.3	0.6	$N(0,1)$	1.000	0.994	1.000	0.973	0.993	0.813	0.091	0.048
		$t(5)$	1.000	0.963	0.999	0.891	0.989	0.634	0.103	0.036
		$\chi^2(3)$	1.000	0.950	0.999	0.868	0.990	0.606	0.097	0.041

TABLE 7

Break point estimates under H^{01}: $WS^{f\,\inf}$ at 0.05 level under GARCH(1,1) as in Table 5

			$\tau^* = 0.5$					
			$T = 200$					
ϕ_1	ϕ_2	α:	0.70		0.80		0.90	
			$WS^{f\,\inf}$	$WS_w^{f\,\inf}$	$WS^{f\,\inf}$	$WS_w^{f\,\inf}$	$WS^{f\,\inf}$	$WS_w^{f\,\inf}$
0.05	0.9	$N(0,1)$	0.583	0.552	0.599	0.562	0.611	0.566
		$t(5)$	0.586	0.510	0.603	0.533	0.617	0.551
		$\chi^2(3)$	0.586	0.492	0.604	0.510	0.615	0.524
0.1	0.8	$N(0,1)$	0.566	0.535	0.582	0.549	0.598	0.553
		$t(5)$	0.568	0.501	0.584	0.521	0.601	0.538
		$\chi^2(3)$	0.569	0.481	0.587	0.497	0.599	0.512
0.3	0.6	$N(0,1)$	0.564	0.519	0.580	0.525	0.590	0.529
		$t(5)$	0.563	0.486	0.579	0.500	0.590	0.517
		$\chi^2(3)$	0.565	0.469	0.580	0.480	0.594	0.486
			$T = 500$					
0.05	0.9	$N(0,1)$	0.536	0.524	0.549	0.538	0.579	0.563
		$t(5)$	0.539	0.505	0.554	0.528	0.582	0.558
		$\chi^2(3)$	0.537	0.491	0.552	0.509	0.582	0.536
0.1	0.8	$N(0,1)$	0.528	0.516	0.541	0.528	0.568	0.551
		$t(5)$	0.531	0.499	0.546	0.519	0.570	0.549
		$\chi^2(3)$	0.530	0.481	0.543	0.499	0.571	0.526
0.3	0.6	$N(0,1)$	0.527	0.501	0.539	0.513	0.566	0.538
		$t(5)$	0.529	0.481	0.541	0.499	0.567	0.529
		$\chi^2(3)$	0.527	0.461	0.541	0.475	0.568	0.494
			$\tau^* = 0.7$					
			$T = 200$					
ϕ_1	ϕ_2	α:	0.70		0.80		0.90	
			$WS^{f\,\inf}$	$WS_w^{f\,\inf}$	$WS^{f\,\inf}$	$WS_w^{f\,\inf}$	$WS^{f\,\inf}$	$WS_w^{f\,\inf}$
0.05	0.9	$N(0,1)$	0.754	0.717	0.750	0.707	0.718	0.659
		$t(5)$	0.755	0.668	0.751	0.662	0.720	0.637
		$\chi^2(3)$	0.758	0.632	0.758	0.625	0.729	0.592
0.1	0.8	$N(0,1)$	0.739	0.702	0.737	0.691	0.709	0.651
		$t(5)$	0.741	0.655	0.738	0.649	0.709	0.622
		$\chi^2(3)$	0.745	0.620	0.745	0.609	0.719	0.586
0.3	0.6	$N(0,1)$	0.729	0.661	0.725	0.650	0.697	0.614
		$t(5)$	0.727	0.618	0.723	0.612	0.693	0.590
		$\chi^2(3)$	0.735	0.587	0.734	0.582	0.712	0.555
			$T = 500$					
0.05	0.9	$N(0,1)$	0.733	0.721	0.742	0.726	0.751	0.726
		$t(5)$	0.736	0.686	0.745	0.699	0.751	0.709
		$\chi^2(3)$	0.734	0.675	0.743	0.681	0.753	0.679
0.1	0.8	$N(0,1)$	0.726	0.710	0.733	0.715	0.741	0.713
		$t(5)$	0.727	0.676	0.735	0.687	0.741	0.694
		$\chi^2(3)$	0.726	0.661	0.735	0.665	0.742	0.661
0.3	0.6	$N(0,1)$	0.720	0.675	0.726	0.677	0.730	0.675
		$t(5)$	0.719	0.639	0.724	0.645	0.727	0.649
		$\chi^2(3)$	0.720	0.617	0.727	0.616	0.733	0.604

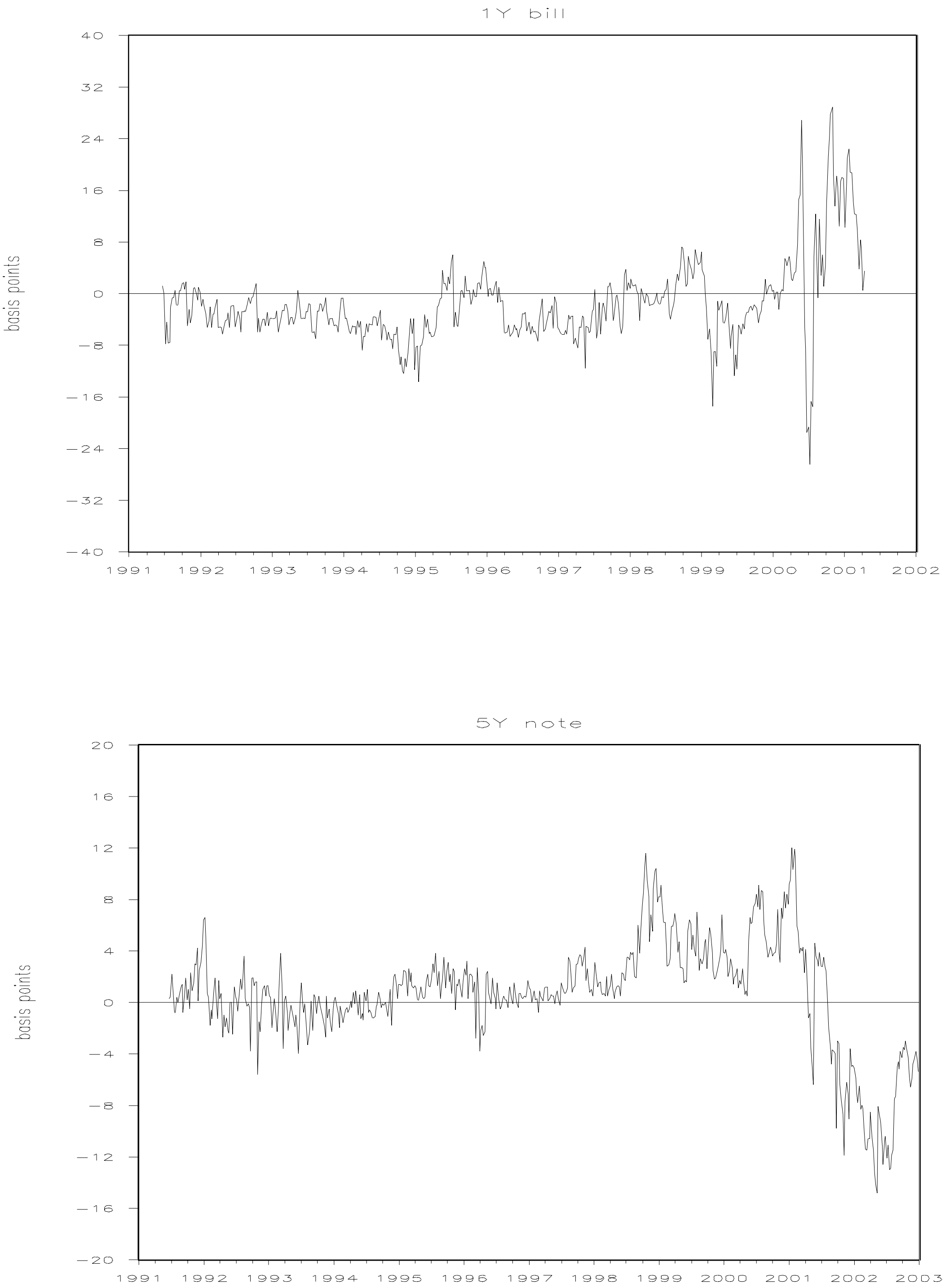

Figure 1. U.S. Treasury bond on/off spread levels: 1991-2002.