# Supervised Baysian SAR image Classification Using The Full Polarimetric Data 

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#### Abstract

Supervised classification procedures are developed and applied to synthetic aperture radar (SAR) in order to identify their various earth terrain components. An implementation of the maximum a posteriori (MAP) and the maximum likelihood (ML) algorithms are presented. These two techniques need a statistic model for the conditional distribution of the polarimetric complex data. Many previous studies used the classical Rayleigh distribution to characterize the earth terrain, but this model doesn't yield a good result for heterogeneous backscattering media. This study applies a new model based on the K-distribution. This distribution, based on the physical definition of the texture and its mathematical representation, will be shown as rigorous model to describe amplitudes and intensities of the backscattering signal. We also use Markov fields to enhance the results of the classifications. These classification procedures have been applied to the Flevoland site (Holland) and Landes forest (France) SAR images, supplied by the Jet Propulsion Laboratory.


Key-Words: SAR Images, Supervised Classification, K-distribution, Markov Random Field

## I. Introduction

Classification of earth terrain in SAR image is one of the many important applications of polarimetric data.
We will present a survey on the different techniques of classifications of polarimetric data: the maximum a posteriori (MAP) and the maximum likelihood (ML) algorithms. The first stage of the classification is the modeling of the radar echo. It consists of finding distribution laws that describe well the amplitudes and the absolute intensities of the backscattering signal. Certainly Gaussian models are often used to characterize the observed regions, however, they are not always able to represent the reality of the land for heterogeneous regions: forest, heat...
The K distribution, based on the physical definition of the texture and its mathematical representation, will be shown as rigorous model to describe amplitudes and the absolute intensities of the backscattering signal, that can fitto the homogeneous and heterogeneous regions at the same time.

In a second level, we will present the K distribution and we will prove the good modeling of this distribution in the different regions of imaged scene.
In many situations, radar polarimetric data are provided under shape of matrix of Muller or multi look covariance to reduce the volume of data or to minimize the presence of the speckle. We will spread the distribution laws from one look to multi look case, as well as the notion of the equivalent number of views.
At the end of this work, we will present a technique of improvement of results of the classification. It is about the improvement by Markov fields
In this paper, both MAP and ML classification techniques will be used in classification of fully polarimetric synthetic aperture radar image. Two images are used: The first was collected at P band from Flevoland forest (Holland). They are 186 pixels in the range and 256 in the azimuth with a 25 by $25-\mathrm{m}$ resolution by pixel. The second is a C band image representing Land forest (France) they are 445 pixels in the range and 1970 in the
azimuth. The first image is 4 look image; the second is one look image.

## 2. Classification procedures

Our paper deals with the supervised techniques of classification. We will present the Bayes classifier and the maximum likelihood classification. The first stage of the application of these algorithms is the choice of the model characterizing the land.
In the statistical approach of the supervised classification, each pixel is considered as the realization of a certain vector. The distribution of this vector is defined as characteristic of the density function of the class to which it belongs.
Indeed, every class of the land is described by a law of probability that describes the possibility that a given pixel belongs to this class or no. Consequently, the description of the picture derives from parameters of the law that models the land. We will compare the yields results with using Gaussian model and the K distribution
The characteristic vector of the matrix of diffusion of a pixel is given:
$\overline{\mathrm{X}}=\left[\begin{array}{c}\mathrm{HH} \\ \mathrm{HV} \\ \mathrm{VV}\end{array}\right]$ where $\mathrm{HH}=|\mathrm{HH}| \exp \left(\mathrm{i} \varphi_{\mathrm{HH}}\right)$
$|\mathrm{HH}|$ and $\varphi_{\text {нн }}$ are respectively the amplitude and the phase of the electromagnetic return at the HH polarization.

### 2.1 Bayes classifier MAP (maximum a prior)

Bayes defined the general criteria that permits to optimize the probability of success in the classification of the radar image:

- $\mathrm{P}\left(\mathrm{w}_{\mathrm{i}}\right)$ is the probability of occurrence of the class i
- $\mathrm{P}\left(\mathrm{X} / \mathrm{w}_{\mathrm{i}}\right)$ the conditional probability of the vector X , held account that the class i is kept a priori.
- $P\left(w_{i} / X\right)$ the conditional probability of the class i , held account that X is kept a priori. This probability is also called Likelihood [2].
- $\mathrm{P}(\mathrm{X})$ the probability so that X occurs

A vector $X$ belongs to class wi if the probability of X belongs to wi is bigger than the probability that X belongs to the other classes.
$P\left(w_{i} / X\right)>P\left(w_{j} / X\right)$ for all $\mathrm{j} \neq \mathrm{i}$
While applying the Bayes law:

$$
\begin{equation*}
\frac{P(X / w i) P\left(w_{i}\right)}{P(X)}>\frac{P\left(X / w_{j}\right) P\left(w_{j}\right)}{P(X)} \text { for all } \mathrm{j} \neq \mathrm{i} \tag{2.4}
\end{equation*}
$$

Knowing $\mathrm{P}\left(\mathrm{w}_{\mathrm{i}}\right)$ and $\mathrm{P}\left(\mathrm{X} / \mathrm{w}_{\mathrm{i}}\right)$ we can apply the algorithm of classification pixel by pixel.

### 2.2 Maximum Likelihood (ML) classification

The maximum Likelihood classification consists in maximizing the probability $\mathrm{P}\left(\mathrm{X} / \mathrm{w}_{\mathrm{i}}\right)$. Says otherwise, a pixel possessing a polarimetric vector X is classified in class wi if only:
$P\left(X / \omega_{i}\right)=\max \left\{P\left(X / \omega_{j}\right) j=1, N\right\}$
N is the total number of classes
It is necessary to note that this technique of classification is a particular case of MAP classification. Indeed, while supposing that all classes have the same probability of occurrence, MAP classification becomes the ML classification.
For our survey, we chose two distributions to characterize the polarimetric vector $\mathrm{X}=[\mathrm{HH} \mathrm{HV}$ VV]: the Gaussian distribution and the K distribution. We present in the next section the statistic model of the K distribution and we will demonstrate that it can characterize the polarimetric SAR data.

## 3. Statistic Model For K- Distribution

The amplitude of the radar signal can be described by a Rayleigh distribution. However, many recent studies showed that this representation is valid only for the homogeneous regions and often fails for heterogeneous backscattering media [1].Several other models have been experimented to represent the land cover. Among these models, the K distribution seems to be very efficient to characterize the different types of lands: homogeneous and heterogeneous.
The K distribution has been applied for the first time in order to model the radar echo by Jao in 1984. The choice of this distribution rests on a mathematical basis and assigning it a physical signification. Indeed, it is the result of the presence of the speckle and the texture in the radar image [2]. It can be given by two ways: by a random coherent manner of while supposing that the
number of distributor in a cell follows a negative binomial law [4], or in supposing that it is the product of a gamma distribution characterizing the texture and a Gaussian distribution characterizing the speckle.
In one look case, the distribution of the amplitude of a signal is given by:

$$
\begin{equation*}
p(A)=\frac{2 b}{\Gamma(\alpha)}\left(\frac{b A}{2}\right)^{\alpha} K_{\alpha-1}(b A) \tag{3.1}
\end{equation*}
$$

A represents the amplitude of every component of the polarimetric vector, $|\mathrm{HH}|,|\mathrm{HV}|$, and $|\mathrm{VV}| . \mathrm{K}$ is the second order modified Bessel function with parameter $\alpha-1$.
$b=2 \sqrt{\frac{\alpha}{\left\langle A^{2}\right\rangle}}$ where $<\mathrm{A}^{2}>$ is the intensity of the signal
In the case of a picture intensity image, while putting $\mathrm{I}=\mathrm{A}^{2}$ :

$$
\begin{equation*}
\mathrm{p}(\mathrm{I})=\frac{2 \mathrm{~b}}{\Gamma(\alpha) \sqrt{\mathrm{I}}}\left(\frac{\mathrm{~b} \sqrt{\mathrm{I}}}{2}\right)^{\alpha} \mathrm{K}_{\alpha-1}(\mathrm{~b} \sqrt{\mathrm{I}}) \tag{3.2}
\end{equation*}
$$

The expression of the amplitude normalized is:
$p(A n)=\frac{4 \alpha^{\frac{1+\alpha}{2}}}{\Gamma(\alpha)} A n^{\alpha} K_{\alpha-1}(2 \sqrt{\alpha} A n), A n=\frac{A}{\sqrt{\left\langle A^{2}\right\rangle}}$
We notice that when the value of alpha increases, the curve of the K distribution is confounded with the Rayleigh distribution. From this fact, we can say that the distribution of Rayleigh is a particular case of the K distribution for the big values of alpha.


Fig 1 PDF of the K distribution for different values of $\alpha$ compared to the Rayleigh distribution.
The parameter alpha of the K distribution is esteemed from moments of amplitudes or intensities normalized.
$<A m>=\int_{0}^{+\infty} A^{m} P(A) d A$
These moments are given by [4]:
$<A m>=\left(\frac{\left\langle A^{2}\right\rangle}{\alpha}\right)^{m / 2}(m / 2)!\frac{\Gamma\left(\frac{\mathrm{m}}{2}+\alpha\right)}{\Gamma(\alpha)}$
The normalized amplitudes moment is:
$A_{K}^{(m)}=(m / 2)!\frac{\Gamma\left(\frac{\mathrm{m}}{2}+\alpha\right)}{\alpha^{m / 2} \Gamma(\alpha)}$
and the normalized intensity moment is:

$$
\begin{equation*}
I_{K}^{(m)}=(m)!\frac{\Gamma(\mathrm{m}+\alpha)}{\alpha^{m} \Gamma(\alpha)} \tag{3.8}
\end{equation*}
$$

Practically, normalized intensities moments are given by:
$I_{H H}^{(m)}=\frac{E\left(|H H|^{2 m}\right)}{E\left(|H H|^{2}\right)^{m}}$

The order 2 moment constitutes the simplest means to appraise alpha, it is given by:

$$
\begin{equation*}
I_{K}^{(2)}=2\left(1+\frac{1}{\alpha}\right) \quad \text { so } \quad \alpha=\frac{1}{\frac{I_{K}^{(2)}}{2}-1} \tag{3.10}
\end{equation*}
$$

The parameter alpha is appraised for every polarimetric channel. It is bound strongly to the nature of the constituent of the scene: age, density...

For $\alpha$ values toward the infinity, $I_{K}^{(2)}$ tend to 2 what corresponds at the two-order moment of the exponential distribution. Thus, For the homogeneous regions ( $\alpha$ big), the K distribution tends toward Gaussian distribution. This affirms that the distribution doesn't characterize well that the homogeneous regions and that the K distribution is a more general case that describes well all types of land [1].


Fig. 2 Adjustment of theoretical PDF of the Kdistribution to the experimental histogram.

In the figue 2, the PDF of normalized amplitudes are plotted and compared to the experimental histogram of the Landes forest ( $\alpha=5.4$ )

For multi-look images, the expression of normalized intensity is [1]:

$$
\begin{align*}
& P\left(I_{L n}\right)=\frac{2(L \alpha)^{\frac{L+\alpha}{2}}}{\Gamma(\alpha) \Gamma(L)} I_{L N}^{\frac{L+\alpha-1}{2}} K_{\alpha-L}\left(2 \sqrt{L \alpha I_{L n}}\right)  \tag{3.11}\\
& I_{L n}=\frac{I_{L}}{\sqrt{<I>}}
\end{align*}
$$

where L represents the number of look and $\Gamma$ is the gamma function. $\alpha$ and the $m$ moment order are related by [1]:

$$
\begin{equation*}
I_{L}^{(m)}=\frac{\Gamma(\mathrm{m}+\alpha) \Gamma(\mathrm{L}+\alpha)}{\alpha^{m} L^{M} \Gamma(\alpha) \Gamma(L)} \tag{3.12}
\end{equation*}
$$

It is known that NASA/JPL multi-look polarimetric data are over sampled to get a size of the pixel finer than the spatial resolution. This situation produces an interrelationship between the neighbor pixels and generates a mistake of evaluation parameter [3]. We demonstrated by simple graphic visualization that the applied data histograms fit to the theoretical curves of the K distribution better for a different number of looks (figure 3). This number will be named equivalent number of looks (ENL).


Fig. 3 Adjustment of the PDF of normalized amplitudes the experimental histogram for two ENL (Flevoland site)


Fig. 4 Results of ML classification using HH channel and Rayleigh distribution


Fig. 5 Results of ML classification using HH channel and K-distribution

|  | ML |  | MAP |  |
| :---: | :---: | :---: | :---: | :---: |
|  | HH | Vect | HH | Vect |
| Water | 60.2 | 70.5 | 85 | 75.2 |
| Forest | 62,2 | 67 | 83 | 85 |
| Gras | 60,6 | 90 | 90 | 79 |

Table1 probability of good classification using Gaussian distribution (Flevoland).


Fig. 6 Results of MAP classification using HH channel and K-distribution


Fig. 7 Results of MAP classification using full polarimetric data and K-distribution

| ML |  |  | MAP |  |
| :---: | :---: | :---: | :---: | :---: |
|  | HH | VECT | HH | Vect |
| Water | 80.2 | 95.3 | 85.3 | 91.5 |
| Forest | 60 | 86.3 | 72.3 | 77.6 |
| Gras | 95 | 100 | 85.3 | 90.1 |

Table 2 Probability of good classification using Kdistribution (Flevoland).


Fig. 8 Results of ML classification using full polarimetric data and K-distribution.

|  | Gaussian |  | K distribution |  |
| :--- | :---: | :---: | :---: | :---: |
|  | HH | VECT | HH | VECT |
| Tree 33-46 years | 60.2 | 77.5 | 75 | 90 |
| Tree 1-5 years | 57.2 | 78.6 | 79.6 | 89.2 |

Table . 3 Comparison of the probability of good classification using Gaussian distribution and Kdistribution (Land forest).
Tables 1 and 2 show that the error of Bays classification is smaller than the ML classification, this is du to the fact that the ML technique does not account the probability of different class occurrences. This is can introduce mistakes of
classification in presence of a dominant or rare class.
To illustrate the effect of fully polarimetric data, supervised classifications were performed using various polarimetric channel and full polarimetric data.
It clearly appears, on comparing results of the classification that results obtained from full polarimetric data are better than the ones obtained from one channel data. This shows the value of the information contained in full polarimetric data. On the other hand, the K distribution yields good results of classification. The yield images are more legible than those yielded by Gaussian model. The probability of good classification is raised for the homogeneous and heterogeneous zones. It affirms the good choice of the K distribution that takes its origins from the physical definition of the texture.

## 5. Enhancement Of Classification By Markov Field:

The results of the classification aren't good in certain zones. These mistakes are owed to the presence of the speckle in the radar images and to the heterogeneity of the scene. Several techniques of improvement of results of classification have been developed: contribution multi frequency, fuzzy logic, Markov fields...
We chose the Markov fields method because it represents the mathematical interaction between the neighboring pixels. This information is rarely used in classification algorithms. It constitutes a supplementary contribution to the information extracted from the radar pictures. To improve results of the classification of a pixel $s$, we can replace the conditional distribution of the polarimetric vector Xs of s by the product of the conjoined conditional laws of whole polarimetric vector restrained in a Ns neighborhood of s: $X_{1}, \ldots, X_{N_{s}}$. This supposes that vectors are independent. This independence in law is due to the non-correlation of these vectors.

Supposing in addition that a MxM window containing the window of Ns size is homogeneous (all pixels have one same label). The conditional law of $X$ of all the window knowing that the region is L is:

$$
\begin{equation*}
p\left(X_{s} / L_{s}=l\right)=\prod_{i \in N_{s}} p\left(X i / L=l_{s}\right) \tag{5.1}
\end{equation*}
$$



Fig. 9 Results of the enhancement using full polarimetric data and K-distribution

|  | Water | Gras | Soil | Beet |
| :---: | :---: | :---: | :---: | :---: |
| Water | $\mathbf{9 8 . 6}$ | 0.46 | 0 | 0 |
| Gras | 0 | $\mathbf{1 0 0}$ | 0 | 0 |
| Soil | 0 | 0 | $\mathbf{1 0 0}$ | 0 |
| Beet | 2.72 | 0 | 0 | $\mathbf{9 7 . 2 7}$ |

Table. 4 probability of good classification using Results of the enhancement.
The classification accuracy ranged between $97 \%$ and $100 \%$.

## 6. Conclusion

A supervised classification technique has been presented in this paper. Comparisons of results using single and full polarimetric data indicates the utility of full polarimetric data. The K-distribution seems to be a good model for characterizing area. Further work is necessary to develop K-distribution for other types of area.

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